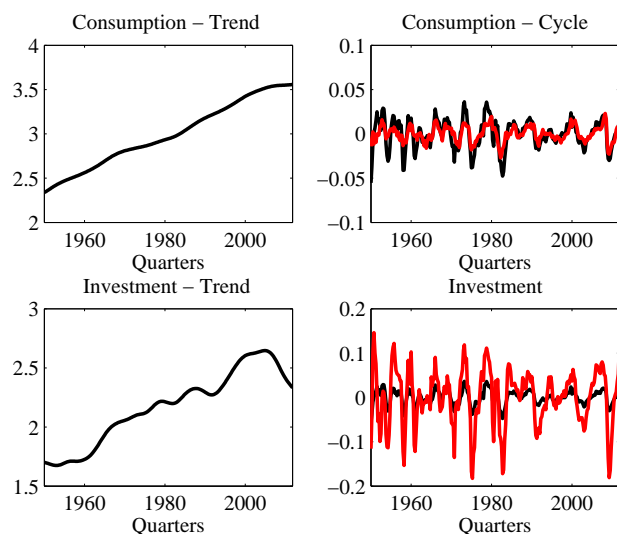# Session 4 – Assessing the fit of the RBC model*

Seminar: Macroeconomics and International Economics

Spring 2015

1 Introduction

Figure 1: Trend and cyclical component of consumption and investment, cyclical component: Black line is output.

The basic RBC model – which is just a stochastic neoclassical growth model with variable labor – is the building block of almost all modern DSGE mod-

els. It fits the data well on some dimensions, but less well on others. This session aims at describing some basic facts about US business cycles and at examining how well the basic RBC model established in previous sessions can fit those facts. *First*, we will go to the data and extract some "*business cycle moments*". *Second*, we will take our RBC model and compare the theoretical moments to those found in the data.

It is the aim of today's session to compare empirical and theoretic outcomes and to understand why the business cycle discipline has developed the basic RBC model in various directions.

2 Objectives of today's session

At the end of this session, students should be able to:

1. *load* a dataset from Excel into Matlab.
2. *extract* relevant information from the data.
3. *compute* business cycle moments.
4. *extract* relevant information from the model simulations.
5. *compare* model outcomes to stylized facts from the data.

3 Stylized US Business Cycle Facts

The basic idea underlying the measurement of business cycles is that any time series $\{x_t\}_{t=0}^T$ can be decomposed into a trend component x_t^T and a

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*Task written by Philipp Wegmüller, University of Bern

cyclical component, x_t^c, such that

$$x_t = x_t^T + x_t^c.$$

While series like output, consumption and investment display growth over time, at shorter frequencies these series display recurrent and large fluctuations about its trend, known as *business cycles*. Because many macroeconomic time series display trend growth, it is not immediately obvious what the cyclical properties of the data are. To make a comparison of the model dynamics with the cyclical properties of the empirical data, we need a procedure, a detrending method, to extract the cyclical component of the actual time series.

There are numerous ways to separate x_t^T from x_t^c. The terms *detrending* and *filtering* are often used interchangeably to describe this separation, but are distinct procedures. Detrending is the process of making economic series (covariance) stationary, which is necessary for instance to compute second moments. Filtering is a much broader concept. A filter is an operator that removes movements at particular frequencies.

3.1 The Growth Rate

It is possible to assume that the growth rate of x_t captures the cyclical component:

$$x_t^c = \Delta x_t, \qquad x_t^T = x_{t-1}$$

Task 1. *A first look at the data*

- *Download the m.file* `empirics.m`, *the* `tools.zip` *and the data file* `usdata.xlsx` *from the course webpage.*
- *Unpack the tools.zip in the folder where you would like to work.*
- *Change the path in* `empirics.m` *to your directory.*
- *Run the* `empirics.m` *and check that the data is loaded correctly.*

- *Use the function* `diff()` *and take the first difference of the* $\log()$ *of the series* `RY`, `RC`, `RI` *and* `RG`.
- *Make a subplot of the four series.*
- *Calculate the mean, standard deviation, correlation with output and the first order auto-correlation of each series.*
- *Make ratios of the standard deviation of* C, I, G *with respect to* Y.

As seen already in Session 3, King and Rebelo (1999) produce cyclical components for key US macroeconomic variables applying the *HP-Filter*. The cyclical component of output is used as a reference variable. Summary statistics for selected series are provided in Table 2

Figure 2: Business cycle statistics for the US Economy

Table 1
Business cycle statistics for the US Economy

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	−0.35
A	0.98	0.54	0.74	0.78

[a] All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson (1999), who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

Task 2. *1st-difference or HP-filter*

- *Compare your stylized facts to the ones reported by King and Rebelo (1999)*

3.2 Hodrick-Prescott filter

The Hodrick-Prescott (HP) filter is one of the most popular methods to extract cyclical components. First, trend and cycle are assumed to be uncorrelated. Second, the trend is a smooth process, which is made operational by penalizing variations in the second difference of the trend:

$$\min_{y_t^x} \left\{ \sum_{t=0}^{T} (y_t - y_t^x)^2 + \lambda \sum_{t=2}^{T-1} ((y_{t+1}^x - y_t^x) - (y_t^x - y_{t-1}^x))^2 \right\}$$

As λ increases the trend becomes smoother and for $\lambda \to \infty$ it becomes linear. For quarterly US data, $\lambda = 1600$ is typically chosen, which implies that cycles longer than six to seven years are attributed to the trend.

Task 3. *Using the HP-Filter*

- *Use the function* `hpfilter(series,lambda)` *and apply it to the* $\log()$ *of the series* `RY`, `RC`, `RI` *and* `RG`. *Extract the trend and the cyclical component.*[1] *If you face problems use the* `help` *command.*
- *Make a subplot of the four series.*
- *Calculate the mean, standard deviation, correlation with output and the first order auto-correlation of each series.*
- *Make ratios of the standard deviation of* C, I, G *with respect to* Y.
- *Compare your stylized facts to the ones reported by King and Rebelo (1999)*

Once we have identified the cyclical component, it is possible to characterize the business cycle. We would like to be able to obtain some regularities in the data, i.e.some *stylized facts*, that will help us to understand the business cycles. This amounts to get information on the joint distribution of the fluctuations of the cyclical component of output, consumption, investment, etc. This can be simply measured by computing some statistical moments

- *volatilities*, to measure the amplitude of the business cycle.

- *correlations*, to measure the co-movements of the main macroeconomic aggregates.

4 Does the basic RBC model fit the data?

Task 4. *Assessing the model performance*

- *Open the* `mainfile.m` *together with the* `rbc_growth_1.mod` *from Session3.*
- *Calculate the mean, standard deviation, correlation with output and the first order auto-correlation of output, consumption and investment.*
- *Make ratios of the standard deviation of* C *and* I *with respect to* Y.
- *Compare your theoretical predictions to the ones obtained in the data.*
- *What do you conclude?*

5 Research strategies

There is nothing misleading in the fact that different filtering techniques lead to different facts about macroeconomic time series. Alternative filters provide different windows through which economists can examine their models and data. It is an open question as to whether some of these windows are more or less interesting to look through. In the business cycle literature, two strategies for choosing the appropriate filter are usually considered.[2]

1. Theory determines the filter. Here, we consider *theoretical* moments computed with Dynare.[3]

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[1]Use per example `[hpy,ty]=hpfilter(log(RY),lambda); plot([log(RY) ty])`

[2]See http://public.econ.duke.edu/~brossi/327papers_files/ burnsideJMEcomment.pdf

[3]Theoretical moments are the ones computed based on the state space representation of the solved model. These moments are computed at the parameter vector given, i.e. either the calibrated ones when running `stoch_simul` directly or the ones from estimation if running `stoch_simul` after estimation (See e.g. Hamilton (1994): Time Series Econometrics).

2. Data sampled from an actual economy are treated symmetrically with data from a model economy. Here, we consider *empirical* moments computed with Dynare, based on simulation.[4]

Figure 3 describes an empirical methodology in which the data are pre-filtered, and theory, formal or otherwise, is used for two purposes: Through link '*A*' it is used to determine which filter is right for the model at hand. Furthermore, using either formal or informal methods, theory is used to describe the behavior data would have if they were generated by a particular model.

Choosing the approach proposed in Figure 3 implies that you calculate *theoretical moments* with your model which you then compare to the empirical moments of the data. Here you dont specify a number of periods nor a filter in your `stoch_simul()` command.

In Figure 4 however, data sampled from an actual economy are treated symmetrically with data from a model economy. These data are filtered, analyzed and compared.

If you want to go with the strategy in Figure 4, then in Dynare you have to choose the number of periods you want to simulate in `stoch_simul()`, and you have to specify the HP-filter. Doing so gives you simulated data series and hence empirical second moments.

To account for either of the strategies shown above, there are a number of options one can include behind `stoch_simul` (to do this you type `stoch_simul(options);`):

- `hp_filter = integer`: this will produce *theoretical moments* (variances, covariances, autocorrelations) after HP-filtering the data (the default is to apply no filter to the data). We typically want to look at HP filter moments, so this is an option you'll want to use. The integer is a number (i.e. 1600) corresponding to the penalty parameter in the HP-filter. So, for example, typing `stoch_simul(hp_filter=1600);` will produce theoretical (i.e. analytical) moments of the HP-filtered data.

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[4]Simulated moments are based on simulation from the model solution for a specified period of time. If you run sufficiently many periods, the simulated moments will converge to the theoretical moments (asymptotically).
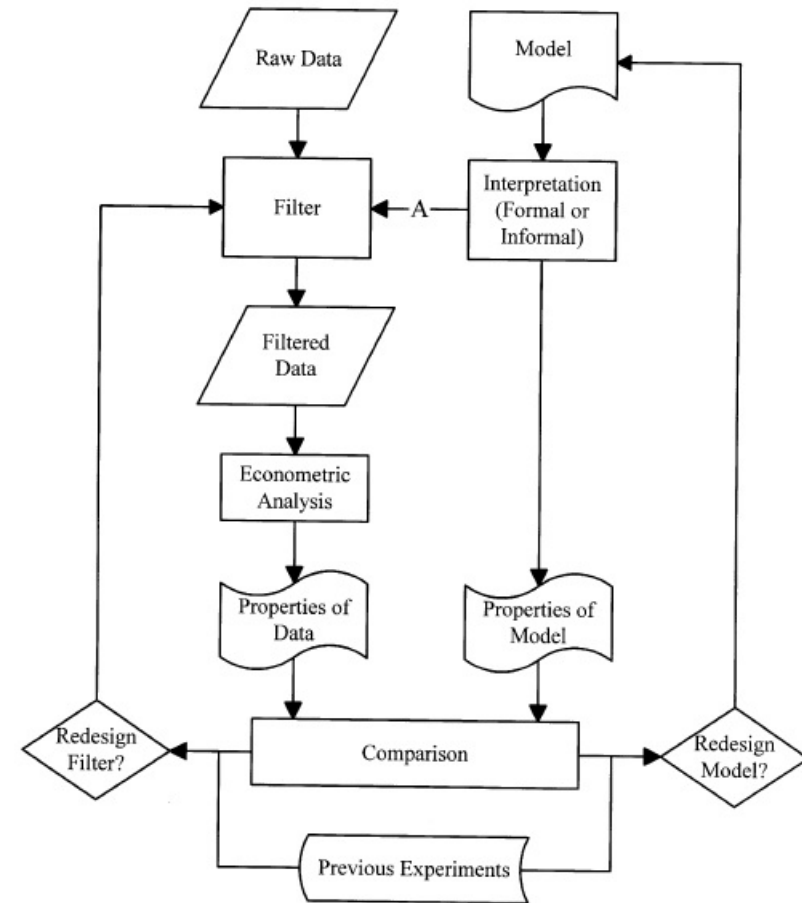
Figure 3: Theoretic moments

Fig. 5. A flow diagram for an empirical methodology.
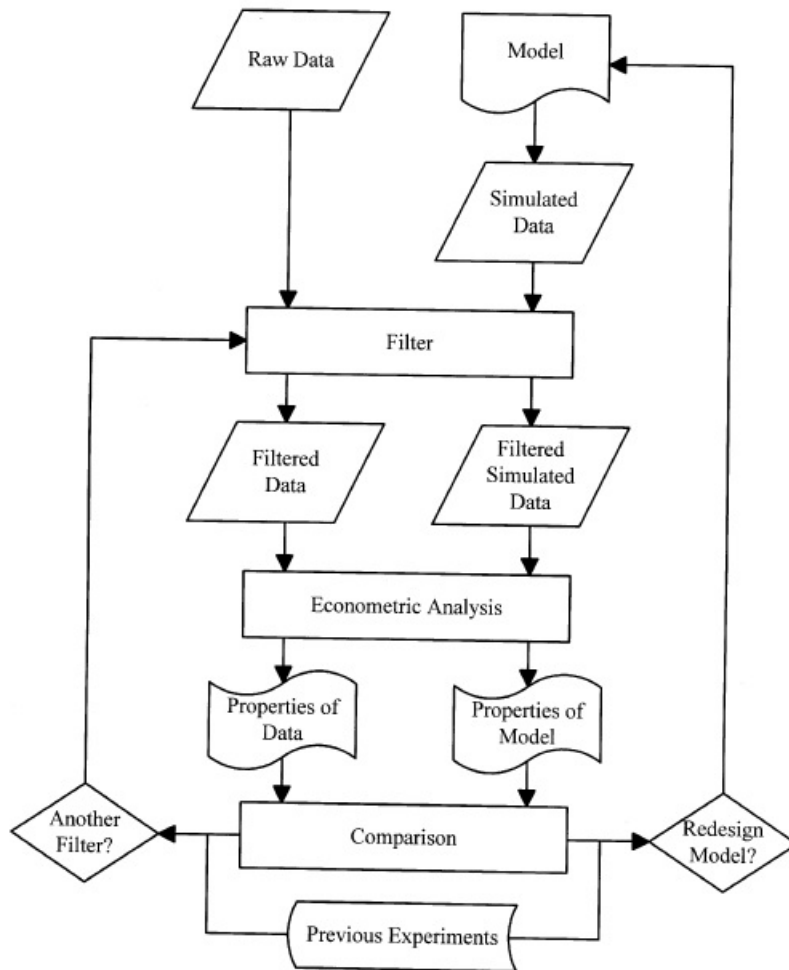
Figure 4: Simualted moments

Fig. 6. A Flow diagram for an alternative empirical methodology.

- `periods = integer`: Dynare's default is to produce *analytical/theoretical moments* of the variables. Having periods not equal to zero will instead have it simulate data and take the moments from the simulated data. By default, Dynare drops the first 100 values from a simulation, so you need to give it a number of periods greater than 100 for this to work. Hence, typing `stoch_simul(periods=300);` will produce moments based on a simulation with 200 variables.

Task 5. *Comparing moments*

- *Simulate your model using $T = 232$ and $N = 1000$, apply the HP-filter using $\lambda = 1600$.*
- *Calculate the mean, standard deviation, correlation with output and the first order auto-correlation of output, consumption and investment.*
- *Make ratios of the standard deviation of C and I with respect to Y.*
- *Compare your simulated predictions to the theoretical moments.*
- *Compare your simulated predictions to the ones obtained in the data.*
- *What do you conclude?*