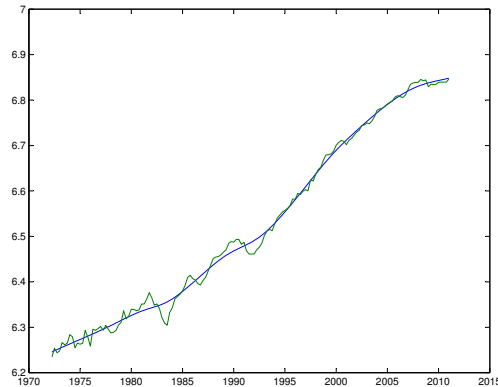# Session 3 – Introducing growth to the RBC model*

Seminar: Macroeconomics and International Economics

Spring 2015

1 Introduction

Figure 1: US GDP growth with HP-filtered trend component

In today's session we will apply the **basic RBC model** developed in the two previous sessions and introduce two new components. *First*, we will add a neutral deterministic technical progress.[1]

It is the aim of today's session to introduce growth into the traditional RBC model and to understand, how a model incorporating growth can be made stationary.

2 Objectives of today's session

At the end of this session, students should be able to:

1. *differentiate* between stochastic and deterministic technological change.
2. *make* a model stationary.
3. *understand* the role of economic growth in the real business cycle model.
4. *use* Matlab in interaction with Dynare.
5. *extract* relevant information from the model simulations.
6. *compare* model outcomes to stylized facts from the data.

3 Growth

Figure 1 displays the logarithm of current output and shows that output grows through time. To obtain a meaningful statistical measurement of business cycles, the series in our theoretical model necessarily have to be made stationary in some way. Most commonly this is done by the removal of a secular trend.

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[1] Neutral technological change refers to the behavior of technological change in models. A technological innovation is *Hicks* neutral, following John Hicks (1932), if a change in technology does not change the ratio of capital's marginal product to labor's marginal product for a given capital to labor ratio. A technological innovation is *Harrod* neutral

(following Roy Harrod) if the technology is labor-augmenting (i.e. helps labor); it is *Solow* neutral if the technology is capital-augmenting (i.e. helps capital).

In the past two sessions we have derived the **general equilibrium** representation of the baseline *real* business cycle model, calculated the *steady state* and taken a *log-linearization* around this steady state. Throughout, we have assumed that the output of the economy depends on a production function that combines labor and capital inputs. To capture the upward trend in output per capita that is shown in Figure 1, the basic neoclassical model incorporates secular improvement in factor productivity. In particular, output (Y_t) depends on the amounts of capital (K_t) and labor (N_t) according to a constant returns to scale production function satisfying the Inada conditions:

$$Y_t = A_t F(K_t, \Gamma_t N_t) = A_t K_t^\alpha (\Gamma_t N_t)^{1-\alpha}, \tag{1}$$

in which A_t is a *random* (i.e. stochastic) "productivity shock" variable, whose law of motion usually is described by an AR(1)-process. Further, Γ_t denotes *Harrod* neutral technological progress. It represents the *deterministic* component of productivity. This latter component of productivity is assumed to expand at a constant (exogenous) rate, such that

$$\Gamma_t = \gamma \Gamma_{t-1}, \qquad \text{with} \gamma \geq 1. \tag{2}$$

The form of the production function in Equation 1 is motivated by the growth facts and was widely employed in growth models after Phelps (1966) showed that steady state growth – a simulation in which all variables grow at the same constant rate – required that the deterministic component of technology be expressible in labor augmenting form in economies where the production function is of the Cobb-Douglas form. For all other production functions, the only form of technical progress consistent with steady-state growth is labor augmenting. In fact, in the feasible steady states of this model consumption, investment, output and capital all grow at the same rate – the rate of trend technical progress – so that the great ratios are stationary.

4 The RBC model with growth

In the following we will use the model of last section and add Harrod neutral technological progress to it. A short description of the decentralized general equilibrium shall recap what we have seen.

4.1 Households

The household has preferences over a bundle of consumption goods C_t and leisure l_t. He chooses the amount of consumption and labor he wants to supply to the private sector such to maximize the expected present discounted value of utility. She is subject to a budget constraint and to the capital accumulation equation. The total time endowment is normalized to unity.

4.2 Firms

The firm produces a homogenous good by means of capital and labor services according to the constant returns to scale technology of equation 1. The firm determines its production plan by maximizing its profit. The stochastic component of productivity A_t is assumed to follow an autoregressive process of order one (AR(1)). The resource constraint of the economy is given by

$$Y_t = C_t + I_t. \tag{3}$$

Remember that the steps to solving a non-linear stochastic general equilibrium were the following:

1. Find the general equilibrium representation.
2. Find the deterministic steady state.
3. Take a log-linear approximation of the model around this deterministic steady state.
4. Assign values to parameters.
5. Solve the system a stochastic linear difference equations.

4.3 General equilibrium

Assume the functional form for utility

$$U(C_t, l_t) = \log(C_t) + \chi \frac{N_t^{1+\nu}}{1+\nu}. \tag{4}$$

The equilibrium is then given by

$$\frac{1}{C_t} = \Lambda_t \tag{5}$$

$$\chi N_t^\nu = \Lambda_t W_t \tag{6}$$

$$\Lambda_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} (R_{t+1} + (1-\delta)) \right] \tag{7}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{8}$$

$$Y_t = C_t + I_t \tag{9}$$

$$Y_t = A_t K_t^\alpha (\Gamma_t N_t)^{1-\alpha} \tag{10}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{11}$$

$$R_t = \alpha \frac{Y_t}{K_t}. \tag{12}$$

together with the law of motion for the exogenous variables A_t, Γ_t.

As long as we have growth in the model, the *economy does not converge* to a steady state but keeps growing along a balanced growth path! We need to eliminate the growth component which is achieved by **deflating** the model. Therefor, lets define

$$x_t = \frac{X_t}{\Gamma_t} \tag{13}$$

for all variables that grow and are deflated. Λ_t decreases at the same rate as Γ_t, such that we define $\lambda_t = \Gamma_t \Lambda_t$.

Task 1. *Identify the variables that grow.*

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Task 2. *Characterize the deflated equilibrium (remember $\Gamma_t = \gamma \Gamma_{t-1}$):*

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Task 3. *Find the steady state:*

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Task 4. *Simulation with Dynare*

1. *Open the existing file* `rbc_logs.mod`.
2. *Save it as* `rbc_growth.mod`.
3. *Adapt the steady state and equations as found in tasks 2 and 3.*
4. *Simulate the model.*

5   Measuring the model performance

King and Rebelo (1999) produce cyclical components for key US macroe-conomic variables applying the *HP-Filter* (will be discussed in Session 4). The cyclical component of output is used as a reference variable. Summary statistics for selected series are provided in Table 2

Figure 2: Business cycle statistics for the US Economy

Table 1
Business cycle statistics for the US Economy

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	−0.35
A	0.98	0.54	0.74	0.78

' All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson (1999), who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

5.1   Dynare output

Dynare stores the output from its model simulations to a Matlab-structure, called oo_-. This information can then be used to assess the model's perfor-mance and to compare the moments obtained from the theoretical model to those observed in the data. Generally, we are interested in obtaining in-formation on the cyclical properties of the model (mean, variance, standard deviation, skewness, kurtosis, matrix of contemporaneous correlations, coef-ficients of autocorrelation) and on the model dynamics (Impulse responses). The structure oo_- contains (for example)

- The steady state ($oo_-.steady\_state$)
- The variance-covariance matrix ($oo_-.var$)
- The autocorrelations ($oo_-.autocorr$)
- The impulse responses ($oo_-.irfs$)
- The coefficients of the policy and transition functions ($oo_-.dr$)
- Results (time paths of endogenous variables) from stochastic simula-tions ($oo_-.endo\_simul$)

5.2   Matlab – Dynare interaction

If you have various Dynare-files with different types of models you would like to compare or you would like to loop over certain parameters, Matlab can be used in interaction with Dynare to facilitate these tasks.

In this session we want to build a Matlab *mainfile.m*, in which we specify the parameters and steady state, to control the flow of Dynare with an `.m` file.

Task 5. *Constructing the workflow*

1. *Download the* `sample_mainfile.txt` *and open it up in Matlab.*
2. *Use the information (parameters and steady state) from your* `rbc_growth.mod`, *and extract it to the Matlab-file.*
3. *Download the* `sample_dynare.mod` *and specify the model equations from your* `rbc_growth.mod`.
4. *Run the Matlab-file, such that it executes the dynare file.*
5. *Use the Matlab file to extract information of your model stored in the* oo_- *structure and compare your results to those of Table 2.*