

# Session 2 – Working with Dynare

## Seminar: Macroeconomics and International Economics

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# Dynare Recap

Where can I get the program and some help?

Dynare is preprocessor and a collection of Matlab (and GNU Octave) routines, which solves DSGE models

The Dynare home page is <http://www.dynare.org/>

The Dynare manual is available at

<http://www.dynare.org/documentation-and-support/manual>

Dynare can be downloaded at <http://www.dynare.org/download>

Instructions how to install Dynare can be found in the manual (Chapter 2, Installation and configuration)

See also the Dynare Tutorial

<http://www.dynare.org/documentation-and-support/tutorial>

You tell Dynare what the variables, shocks and parameters (+parameter values) of your model are

You write down the model equations

You tell Dynare to find the steady state

Dynare (log)linearizes the model around the steady state

... and solves the recursive equilibrium laws of motion of the linearized model

Finally, the model is analyzed via impulse responses, stochastic simulations and moments

# The structure of a typical Dynare (.mod) file

```
var (list of endogenous variables );  
varexo (list of shocks) ;  
parameters (parameters + parameter values + transformations);  
model;  
model equations;  
end;  
initval; (initial guesses for computing the steady state)  
steady; (compute the steady state)  
shocks; (the shock structure of the model )  
var (what variables are shocked);  
stderr (the standard error of the shocks);  
end;  
stoch__simul(order=1,irf=100) ly lh lc li lk z; (analyzing the model)
```

# Timing convention

Time indices are given in parenthesis

$X_{t+1}$  is written  $X(+1)$ ,

$X_{t-1}$  is written  $X(-1)$

$X_t$  is written  $X$  (no time index needed for the current period)

Dynare will automatically recognize predetermined and nonpredetermined variables, but you must observe a few rules:

The time index refers to the period when the value of the variable is determined

The value of the capital stock, which is used in production in period  $t$ , is determined in period  $t - 1$ .

## Example for capital accumulation

Two ways of writing the following equation

$$k_{t+1} = i_t + (1 - \delta)k_t$$

The standard way:

```
var y, k, i;  
...  
model;  
k = i + (1-delta)*k(-1);  
end;
```

Another way of doing the same:

```
var y, k, i;  
predetermined_variables k;  
...  
model;  
k(+1) = i + (1-delta)*k;  
end;
```

## Some tricks and hits

Dynare linearizes the model around the steady state

It does not log-linearize the model

Remember the advantages of log-linearizations: We are dealing with **percentage deviations** from the steady state (or the balanced growth path)

Log-deviations (or percentage deviations) are easy to interpret (is a deviation 'small' or 'large'?)

Similar measures are used in empirical examination of the data (e.g. applying HP filter to  $\log(\text{GDP})$ )

A useful trick: Write down the model in terms of logarithmic transformations of the original variables

When Dynare linearizes the model in terms of the logarithmic transformations, it log-linearizes the model in terms of the original variables

## Some tricks and hits

Adopt the notation

$$ly = \log(Y), lc = \log(C), lk = \log(K) \text{ etc.}$$

Also remember the Dynare conventions pertaining to time indexation

Then the resource constraint

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t$$

can be written as

$$\exp(ly) = \exp(lc) + \exp(lk) - (1 - \delta) * \exp(lk(-1))$$



## Some tricks and hits

The expectation operator  $\mathbb{E}_t$  is not used in Dynare code. Dynare 'knows' when one has to take expectations, we do not have to tell this explicitly.

Thus the consumption Euler equation

$$\mathbb{E}_t \left[ \frac{C_{t+1}}{C_t} \right] = \beta(1 + r_t)$$

is written as

$$\exp(\ln C_{t+1} - \ln C_t) = \beta(1 + \exp(r));$$

# The general problem

$$\mathbb{E}_t\{f(x_{t+1}, x_t, x_{t-1}, u_t; \theta)\} = 0$$

where  $f()$  are functions

$x_t$  is a vector of endogenous variables that contains both forward looking variables and predetermined variables

$u_t$  is a vector of exogenous shocks

$\theta$  are exogenous parameters

In a stochastic framework, the unknowns are the policy functions:

$$x_t = g(x_{t-1}; u_t)$$

this equation is approximated by

$$x_t = \bar{x} + A\hat{x}_{t-1} + Bu_t$$

where  $\hat{x}_t = x_t - \bar{x}$  and  $\bar{x}$  is the steady state.

# Neoclassical growth model as example

## Everything is given in levels!

We are given the equations:

$$\begin{aligned} \max \sum_{s=0}^{\infty} \{ \beta^s U(c_{t+s}) \} \quad & \text{with} \quad U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{s.t.} \quad & y_t = c_t + i_t \\ & k_{t+1} = i_t + (1 - \delta)k_t \\ & y_t = z_t k_t^\alpha \\ & z_t = \rho_z z_{t-1} + \epsilon_t \quad \text{with} \quad \epsilon_t \sim \text{iid} \quad N(0, \sigma_\epsilon^2) \end{aligned} \quad (1)$$

Objective: Obtain a linear approximation to the policy functions that satisfy the 1st order conditions.

# Neoclassical growth model

To start, set up the Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{c_t^{1-\sigma}}{1-\sigma} \right) + \lambda_t [z_t k_t^\alpha + (1-\delta)k_t - c_t - k_{t+1}] \right\}$$

Optimize with respect to  $c_t$ ,  $k_{t+1}$ ,  $\lambda_t$ , note that  $U'(c_t) = \lambda_t = c_t^{-\sigma}$ .  
The model equations then are

$$c_t^{-\sigma} = \beta \mathbb{E}_t [c_{t+1}^{-\sigma} (\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta)] \quad (2)$$

$$c_t + k_{t+1} = z_t k_t^\alpha + (1 - \delta)k_t \quad (3)$$

$$z_t = \rho_z z_{t-1} + \epsilon_t \quad (4)$$

As a sidemark: If you type your model in levels only, then the shock has to be specified as  $z_t = (1 - \rho_z) + \rho_z z_{t-1} + \epsilon_t$  such that in steady state  $z \neq 0$ .

## Neoclassical growth model

Policy functions: how the period  $t$  values of the variables depend on the period  $t - 1$  values of the state variables, and the shock.

Transition functions: how the period  $t$  values of the state variables depend on the period  $t - 1$  values of the state variables, and the shock

Using the linearization technique in Annex 3 we will get:

$$\begin{aligned}c_t &= \bar{c} + a_{c,k}(k_{t-1} - \bar{k}) + a_{c,z}(z_t - \bar{z}) \\k_t &= \bar{k} + a_{k,k}(k_{t-1} - \bar{k}) + a_{k,z}(z_t - \bar{z}) \\z_t &= \rho_z z_{t-1} + \epsilon_t\end{aligned}\tag{5}$$

or applying  $\hat{x}_t = x_t - \bar{x}$  (%age deviation from steady state) :

$$\begin{aligned}\hat{c}_t &= a_{c,k}\hat{k}_{t-1} + a_{c,z}\hat{z}_t \\ \hat{k}_t &= a_{k,k}\hat{k}_{t-1} + a_{k,z}\hat{z}_t \\ \hat{z}_t &= \rho_z\hat{z}_{t-1} + \epsilon_t\end{aligned}\tag{6}$$

# Linearization Technique, Log conversion

Linearization around (log-)linear steady state.

define:  $x_t = \log(X_t) \Rightarrow \exp(x_t) = \exp(\log(X_t)) = X_t$ .

Taylor expansion:

$f(x_t) \approx f(\bar{x}) + f_x(\bar{x})(x_t - \bar{x}) + \frac{1}{2}f_{xx}(\bar{x})(x_t - \bar{x})^2 + \mathcal{O}$ , with:  $\hat{x}_t = x_t - \bar{x}$ .

$$f(x_t) \simeq \sum_{i=1}^n \bar{x}_i \left. \frac{\partial f(x)}{\partial x_i} \right|_{x=\bar{x}} \hat{x}_i$$

Example:

$$f(c_t, \lambda_t) \equiv c_t^{-\sigma} - \lambda_t = 0$$

$$\frac{\partial f(c_t, \lambda_t)}{\partial c_t} = -\sigma c_t^{-\sigma-1}, \quad \text{and} \quad \frac{\partial f(c_t, \lambda_t)}{\partial \lambda_t} = -1$$

$$f(c_t, \lambda_t) \simeq -\sigma \bar{c}^{-\sigma-1} * \bar{c} * \hat{c}_t + (-1) * \bar{\lambda} * \hat{\lambda}_t = 0$$

The steady state  $\bar{c}^{-\sigma} = \bar{\lambda}$  cancels out, so:

$$-\sigma \hat{c}_t = \hat{\lambda}_t$$

# Neoclassical growth model

Let's do the simulation!

$$\delta = 0.025, \sigma = 2, \alpha = 0.36, \beta = 0.99, \rho = 0.95, \sigma_e = 0.07$$

Now do the same with productivity in logs!

That is write  $\exp(z_t)$

Now do the same with all the variables in logs!

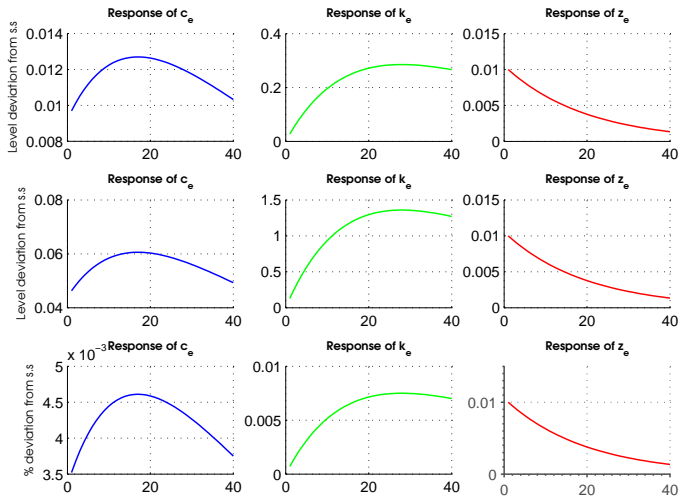
That is write  $\exp(c_t)$ ,  $\exp(k_t)$ ,  $\exp(z_t)$

Compare the IRFs of the three different solutions

**Dynare gives linear system in what you specify the vars to be!**

# Dynare output

Now we can have a look at IRFs:





# Dynare output

## Output includes

Policy and transition functions of the (log)linearized model

Moments of the endogenous variables mean, variance, standard deviation, skewness, kurtosis matrix of contemporaneous correlations coefficients of autocorrelation

Impulse responses If if you have included the periods option in *stoch\_simul(order = 1, periods = 1000, irf = 100)*

Output also includes the results (time paths of endogenous variables) from the stochastic simulation.