Session 2 – Working with Dynare[*]

Seminar: Macroeconomics and International Economics

Spring 2015

1 Introduction

In today's session we will apply the tools learnt in *Session 1 – Introduction to Dynare* and work towards a deeper understanding of the **basic RBC model**. We have seen how to obtain a **general equilibrium** representation of a *decentralized economy* (households and firms optimization) and how we can derive the **steady state** of the economy. Once we know the steady state, we can analyze the **impact of shocks** to the economy and scutinize how the endogenous variables deviate from the steady state. Recall the following:

Endogenous variables are determined within the system.

Exogenous variables are the drivers of the economy and are determined outside of the system[1]

Parameters can be either *calibrated* or set to some specific value. In the former, the parameters are fitted to empirical regularities, in the latter their values are borrowed from micro evidence and macro-data.

Steady State is the stable equilibrium of the model and long run solution.

In today's class we will use the basic real business cycle model and analyze the impact of shocks in **levels** and shocks in the log-specification. Finally we will linearize the model around the deterministic steady state and interpret the impulse responses.

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[*]Task written by Philipp Wegmüller, University of Bern
[1]Usually we assume a normal distribution with mean zero and some variance σ^2

2 Objectives of today's session

At the end of this session, students should be able to:
1. *understand* the timing convention of Dynare.
2. *take* a first order Taylor expansion around the steady state.
3. *simulate* the model in levels and in logs.
4. *read* the Dynare output and extract information out of it.

3 The RBC model from Session 1

Recall the general equilibrium representation of the basic RBC model we know from Session 1:

$$c_t^{-\sigma} = \Lambda_t \tag{1}$$

$$\chi n_t^{\nu} = \Lambda_t W_t \tag{2}$$

$$\Lambda_t = \beta \mathbb{E}_t \left[\Lambda_{t+1}(R_{t+1} + (1-\delta)) \right] \tag{3}$$

$$k_{t+1} = i_t + (1-\delta)k_t \tag{4}$$

$$y_t = c_t + i_t \tag{5}$$

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha} \tag{6}$$

$$W_t = (1-\alpha)a_t \frac{y_t}{n_t} \tag{7}$$

$$R_t = \alpha a_t \frac{y_t}{k_t} \tag{8}$$

The general equlibrium is defined as a sequence of endogenous variables $\mathcal{Q}_t = \{c_t, n_t, \Lambda_t, W_t, R_t, k_t, i_t, y_t\}_{t=0}^{\infty}$ and exogenous variables $\mathcal{P}_t = \{a_t\}_{t=0}^{\infty}$. The set of parameters is given by $\{\sigma, \nu, \chi, \beta, \delta, \alpha, \rho_a, \epsilon_a\}$.

Note that Dynare linearizes the system of equations for you, that is the reason why we have to give it the correct initial conditions and why we hove to solve analytically for the steady state.

3.1 Timing convention

There is a timing convention in Dynare that requires that we (slightly) rewrite the model. In particular, Dynare requires that **predetermined variables** (like the *capital stock*) show up as dated $t-1$ in the time t equations and t in the $t+1$ equations. This is how you tell Dynare if a variable is a state variable or not.

- Time indices are given in parenthesis;
- X_{t+1} is written X(+1);
- X_{t-1} is written X(-1);
- X_t is written X (no time index needed for the current period);

Dynare will automatically recognize predetermined and nonpredetermined variables, but you must observe a few rules: The time index refers to the period when the value of the variable is determined. The value of the capital stock, which is used in production in period t, is determined in period $t-1$. As an example, let us consider the equations for the capital stock and output. The capital stock in period t is given by the amount of capital we bring into period t from $t-1$ minus depreciation, δ, and by how much we decide to invest in period t, that is

$$k_{t+1} = i_t + (1-\delta)k_t.$$

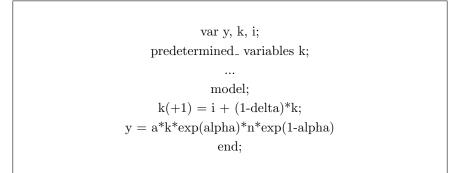Output is a combination of labor and capital input,

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha}.$$

In Dynare, there are two ways to write these two equations.

(1) The standard way, where capital is implicitly defined as a predetermined variable:

```
var y, k, i;
...
model;
k = i + (1-delta)*k(-1);
y = a*(k-1)*exp(alpha)*n*exp(1-alpha);
end;
```

(2) The alternative way, where capital is explicitly defined as a predetermined variable:

```
var y, k, i;
predetermined_ variables k;
...
model;
k(+1) = i + (1-delta)*k;
y = a*k*exp(alpha)*n*exp(1-alpha)
end;
```

3.2 The model in levels

In a first step, we assume that everything is specified in **levels**. This amounts to ask the following: If technology is increased by a certain amount of units, by how many units does output increase? To this end, the process for technology is given by

$$a_t = (1-\rho_a)\bar{A} + \rho_a a_{t-1} + \epsilon_{a,t} \tag{9}$$

Task 1

*A model in **levels**:*

1. Open up the file `rbc.mod` from Session1.
2. Specify the productivity shock as given in equation 9. Run the model and have a look at the output.
3. What do the impulse responses tell you?

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4. What does the model summary tell you?

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3.3 The general problem

Dynare will produce *policy and transition functions* for the endogenous variables of the model in *state space* form. In order to solve for the state space form, we have to remember that the general problem of the general equilibrium solution is written in the following form:

$$\mathbb{E}_t\{f(x_{t+1}, x_t, x_{t-1}, u_t; \theta)\} = 0,$$

- where $f()$ are functions
- x_t is a vector of endogenous variables that contains both forward looking variables and predetermined variables
- u_t is a vector of exogenous shocks

- θ are exogenous parameters
- In a stochastic framework, the unknowns are the policy functions:

$$x_t = g(x_{t-1}; u_t)$$

- this equation is approximated by

$$x_t = \bar{x} + A\hat{x}_{t-1} + Bu_t$$

  where $\hat{x}_t = x_t - \bar{x}$ and $\bar{x}$ is the steady state.

Policy functions tell you, how the period t values of the variables depend on the period $t-1$ values of the state variables, and the shock.

Transition functions tell you how the period t values of the state variables depend on the period $t-1$ values of the state variables, and the shock.

4. Given an interpretation for the policy and transition function of c_t.

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3.4 Shock in logs

Now let us ask the following question: By how many units does output rise, if technology increases by one percent? To give an answer to this question, we have to transform the shock process into **logs**.

Task 2

*A model with the shock in **logs**:*

1. Generate a new file `rbc_log.mod`.
2. Specify a process for the productivity shock which is

$$\log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_{a,t} \tag{10}$$

Run the model and have a look at the output.

3. What do the impulse responses tell you?

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4. What does the model summary tell you?

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4 Towards a linearized model

It is important that you understand that each equation belonging to the system that characterizes the equilibrium can be written as:

$$\mathbb{E}_t[\phi(X)] = 0, \tag{11}$$

where X denotes the vector of variables dated either t or $t+1$. Let us denote X^* the value of X in the deterministic steady state. Note that by definition of the steady state, the $\phi(\cdot)$ function satisfies:

$$\phi(X^*) = 0. \tag{12}$$

There are two ways to obtain the solution to the linearized model with Dynare. As Dynare can linearize the model for you, one way to obtain the solution is to redefine all the variables by using the following specification:

$$x_t \equiv \log(X_t) \Rightarrow \exp(x_t) = \exp(\log(X_t)) = X_t$$

That amounts to say that we **log**-transform every variable in the system and let Dynare perform a first order Taylor approximation around these variables.

4.1 Linearization Technique, Log conversion

Another way to do the same is to **log**-linearize the equilibrium equations around its steady state by hand and give Dynare the system of linearized equations. To do so, we rely on the principle of the Taylor-expansion:

$$f(x_t) \approx f(\bar{x}) + f_x(\bar{x})(x_t - \bar{x}) + \frac{1}{2}f_{xx}(\bar{x})(x_t - \bar{x})^2 + \mathcal{O},$$

with:

$$\hat{x}_t = x_t - \bar{x}$$

Then we can write in general form:

$$f(x_t) \simeq \sum_{i=1}^{n} \bar{x}_i \left.\frac{\partial f(x)}{\partial x_i}\right|_{x=\bar{x}} \hat{x}_i \tag{13}$$

Let us take as an example the first equation of our RBC model which links householsd's consumption to the marginal utility of consumption.We have the following equation:

$$f(c_t, \lambda_t) \equiv c_t^{-\sigma} - \lambda_t = 0$$

Then by applying equation 13, we get

$$\frac{\partial f(c_t, \lambda_t)}{\partial c_t} = -\sigma c_t^{-\sigma-1}, \quad \text{and} \quad \frac{\partial f(c_t, \lambda_t)}{\partial \lambda_t} = -1, \tag{14}$$

such that by combining everything we have the log-linearized equation:

$$f(c_t, \lambda_t) \simeq -\sigma\bar{c}^{-\sigma-1} * \bar{c} * \hat{c}_t + (-1) * \bar{\lambda} * \hat{\lambda}_t = 0$$

where the steady state $\bar{c}^{-\sigma} = \bar{\lambda}$ cancels out, so:

$$-\sigma\hat{c}_t = \hat{\lambda}_t$$

Task 3

Take the general equilibrium from Session1 and log-linearize it around the steady state:

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Task 4

*A model with everything in **logs**:*

1. Generate a new file `rbc_all_log.mod`.
2. Change all your equations by using the definition of Section 4 above. Note that the shock has to be specified as in the level-model. Further, the initial values have to be adjusted to $c = \log(css), y = \log(yss)$ etc. As an example lets look at equation 1, which will now read:
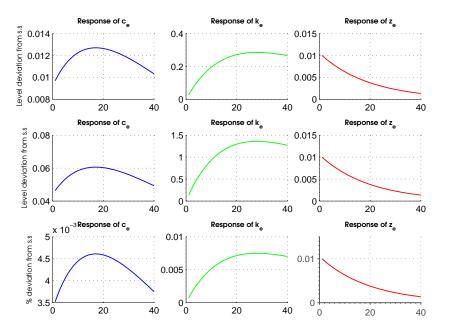
$$(1/\exp(c))=\exp(lb);$$

   Run the model and have a look at the output.
3. What do the impulse responses tell you?

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4. What does the model summary tell you?

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Task 5

If you have time left, write a new file called `rbc_linear.mod` and specify the model given the linearized equations from task 3.

The resulting impulse response functions from the three experiments should approximately look like the following (they are computed for a more stylized RBC model, but qualitatively the implications are the same).