Introduction to Dynare — Solutions

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In this document I will comment on the solutions to the AR(1) model and the basic real business cycle (RBC) model which we encountered in the first seminar session.

1 The AR(1)- model

Task 1

What are the endogenous variables? x

What are the exogenous variables? ϵ_x

What are the parameters? $\rho_x, \sigma_{\epsilon,x}$

The Dynare code reads:

```
1   //
2   // AR(1) model
3   //
4   var x; // Name of the variable
5   varexo e; // Name of the exogenous variable
6   parameters rho, se; // Parameters of the model
7   rho = 0.95;
8   se = 0.02;
9   model;
10  x = rho*x(-1)+e;
11  end;
12  initval;
13  e=0;
14  x=0;
15  end;
16  steady;
17  check;
18  shocks;
19  var e; stderr se;
20  end;
21  stoch_simul(linear);
```

[*]Solutions to Session 1, written by Philipp Wegmüller, University of Bern

2   The basic RBC model

The present model is characterized by two **building blocks**: (1) the **households** and (2) the **firms**. Households own the firms, but management and ownership are distinct, and so households behave as though firm profits are given. Households own the capital stock K_t and rent it to firms. Firms issue debt B_t and equity to households. We abstract from trend growth. As all agents are identical, in equilibrium, they have no reason to trade with each other, that is, they have no reason to hold bonds in the first place, hence $B_t = 0$

The following steps are crucial before taking the model to Dynare and simulate it:

1. Find the general equilibrium representation.

2. Find the deterministic steady state.

3. Take a log-linear approximation of the model around this deterministic steady state.

4. Assign values to parameters.

5. Solve the system a stochastic linear difference equations.

2.1   The household

The household has preferences over a bundle of consumption goods c_t and leisure l_t. He chooses the amount of consumption and labor he wants to supply to the private sector such to maximize the expected present discounted value of utility,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \tag{1}$$

subject to the budget constraint:

$$c_t + i_t \leq W_t n_t + R_t k_t + \Pi_t, \tag{2}$$

in which c_t is consumption expenditure, i_t is investment and $W_t n_t$ is the labor income, $R_t k_t$ the rent from holding capital and Π_t the profits from the firms. The household is also subject to the capital accumulation equation

$$k_{t+1} = i_t + (1 - \delta)k_t. \tag{3}$$

The total time endowment is normalized to unity and the household is subject to the time constraint

$$l_t = 1 - n_t.$$

Task 2

Derive the first order condition of the household, assuming that

$$U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\nu}}{1+\nu}.$$

Hint: Insert the capital accumulation equation for investment into the budget constraint and use Lagrangian optimization.

Lagrange:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\nu}}{1+\nu} + \Lambda \left[W_t n_t + R_t k_t + \Pi_t + (1-\delta)k_t - c_t - k_{t+1} \right] \right\}$$

First order conditions:

$$c_t^{-\sigma} = \Lambda_t \tag{4}$$

$$\chi n_t^{\nu} = \Lambda_t W_t \tag{5}$$

$$\Lambda_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} (R_{t+1} + (1 - \delta)) \right] \tag{6}$$

BC holding with equality $\qquad$ (7)

transversality condition $\qquad$ (8)

2.2 The firm

The firm then determines its production plan by maximizing its profit,

$$\max \Pi_t(k_t, n_t) = a_t F(k_t, n_t) - W_t n_t - R_t k_t.$$

The production function is Cobb-Douglas

$$y_t = F(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}. \tag{9}$$

Task 3

Derive the first order conditions of the firm.

$$W_t = (1 - \alpha) a_t \frac{y_t}{n_t} \tag{10}$$

$$R_t = \alpha a_t \frac{y_t}{k_t} \tag{11}$$

From the firm's profit maximization and the household's budget optimization we can derive the aggregate market clearing equation (check this!):

$$y_t = c_t + i_t, \tag{12}$$

and to close the model we have that productivity A_t follows an exogenous AR(1) process as seen in the previous exercise.

In order to define the general equilibrium, we have to identify the endogenous and exogenous variables.

Task 4

What are the endogenous variables? $y, c, i, k, a, \Lambda, n, W, R$

What are the exogenous variables? ϵ_a

What are the parameters? $\sigma, \beta, \nu, \delta, \alpha, \rho_a, \sigma_\epsilon$

Task 5

Write down the equilibrium equations (should be nine):

$$c_t^{-\sigma} = \Lambda_t \tag{13}$$

$$\chi n_t^{\nu} = \Lambda_t W_t \tag{14}$$

$$\Lambda_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} (R_{t+1} + (1 - \delta)) \right] \tag{15}$$

$$k_{t+1} = i_t + (1 - \delta) k_t \tag{16}$$

$$y_t = c_t + i_t \tag{17}$$

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha} \tag{18}$$

$$W_t = (1 - \alpha) a_t \frac{y_t}{n_t} \tag{19}$$

$$R_t = \alpha a_t \frac{y_t}{k_t} \tag{20}$$

$$\log\{a_t\} = (1 - \rho_a)\bar{a} + \rho_a \log\{a_{t-1}\} + \epsilon_a \tag{21}$$

Task 6

Find the deterministic steady state. Hints: It is helpful work with the great ratios, that is variable x over output y. To find the steady state, start by setting $k_{t+1} = k_t = k$ and $\Lambda_{t+1} = \Lambda_t = \Lambda$. Then find the steady state value for R. Then you solve for $\frac{k}{y}$, afterward for $\frac{i}{k}$ and hence for $\frac{i}{y}$. Once you have that you can find $\frac{c}{y}$. We calibrate steady state labor to be 1/3 and have to get an expression for χ in terms of steady state labor which finally recovers all the variables.

$$R^* = \frac{1 - \beta(1 - \delta)}{\beta}$$

$$\frac{k}{y} = \frac{\alpha}{R^*}$$

$$\frac{i}{y} = \frac{i}{k}\frac{k}{y} = \delta\alpha R^*$$

$$\frac{c}{y} = 1 - \frac{i}{y}$$

$$n^* = 1/3$$

$$y^* = \left(\frac{k}{y}\right)^{\frac{\alpha}{1-\alpha}} n^*$$

From here Dynare does the rest for us. The only thing we need to do is assign parameter values and give Dynare the correct initial conditions (steady state). Dynare will do the log-linearization for us.

Task 7

Let's go to Dynare and simulate the basic RBC model!

```
1   var y,c,i,k,n,lb,a,w,r; // Name of the variables
2   varexo ea; // Name of the exogenous variable
3   parameters beta,nu,sigma,chi,alpha, Δ, ra, sa; //Declaration
4   beta = 0.99;        // Discount factor
5   sigma = 2;          // Intertemporal elasticity of subst.
6   nu    = 2;          // Inverse of labor supply elasticity
7   alpha = 0.35;    // capital elasticity
8   Δ = 0.025;   // depreciation rate
9   ra = 0.95;          // Persistence of Technology Shock
10  sa = 0.01;          // Std. dev. of Technology Shock
11  rs = (1-beta*(1-Δ))/beta;
12  ksy = alpha/rs;
13  isy = Δ*ksy;
14  csy = 1-isy;
15  ns = 1/3;
16  ys = ksy^(alpha/(1-alpha))*ns;
17  cs = csy*ys;
18  is = isy*ys;
19  ks = ksy*ys;
20  ws = (1-alpha)*ys/ns;
21  lbs = cs^(-sigma);
22  chi = lbs*ws*ns^(-nu);
23  model;
24  c^(-sigma)=lb; // Consumption
25  chi*n^(nu)=lb*w; // Labor supply
26  lb=beta*lb(+1)*(r(+1)+(1-Δ)); //Euler equation
27  k=i+(1-Δ)*k(-1); //capital accumulation
28  y=c+i; //market clearing
29  y=a*(k(-1))^alpha*n^(1-alpha); // Production function
30  w=(1-alpha)*a*y/n; //labor demand
31  r=alpha*a*y/(k(-1)); //capital demand
32  log(a) = ra*log(a(-1))+ea; // Supply Shock
33  end;
34  initval;
35  ea = 0;
36  a = 1;
37  y = ys;
38  c = cs;
39  i = is;
40  k = ks;
41  n = ns;
42  lb = lbs;
43  w = ws;
44  r =rs;
45  end;
46  steady; check;
47  shocks;
48  var ea; stderr sa;
49  end;
50  stoch_simul(order=1,irf=20) y,c,i,k,n;
```