Introduction to Dynare*

Seminar: Macroeconomics and International Economics

Spring 2015

1 Introduction

One of the key competences the students should acquire while attending the seminar Macroeconomics and International Economics is to solve correctly a dynamic stochastic general equilibrium model, first on paper and second on with the help of the computer. In this session, we will start with the most basic example of a dynamic model, the autoregressive process of order one (AR(1)-process) and then take a step towards our baseline real business cycle model.

In order to solve and simulate dynamic macroeconomic models, we can use the computer and make use of **Dynare**[1], a **Matlab front-end**[2] These models can be either stochastic or deterministic.[3]

A dynamic model is different from the traditional static IS-LM model, as the latter is based on the assumption that there are barriers to the instantaneous adjustment of nominal prices. Instead, here we are interested in studying economic models which experience economic adjustments to random shocks over time, under the assumption that prices are either fully flexible (**RBC** models) or adjust gradually (**New Keynesian** models).

In today's seminar session we will develop a first basic flexible price dynamic stochastic general equilibrium model (**DSGE**). The baseline RBC model builds on the optimal growth model proposed by Cass [1965] and has been extended to a stochastic environment by Brock and Mirman [1972]. However, rather than thinking of an optimal growth model, one should think about the baseline RBC model as the neo-classical model of capital accumulation.

2 Objectives of today's session

At the end of this session, students should be able to:

1. *understand* the concept of a dynamic process and the steady state.
2. *distinguish* between endogenous and exogenous variables.
3. *set up* the structure and the components of a `.mod` file.
4. *solve* the basic real business cycle model on paper.
5. *implement* a simple macroeconomic model with Dynare.

3 A first example - the AR(1)-process

Before taking the model to the computer and simulate it, we want to think about its main properties first. This will also be your first step, when you

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[1]Developed by Michel Juillard, CEPREMAP, `http://www.cepremap.cnrs.fr/dynare`

[2]In computer science, the front end is responsible for collecting input in various forms from the user and processing it to conform to a specification the back end can use. The front-end is an interface between the user and the back end.

[3]This is an important distinction: In deterministic models, the occurrence of all future shocks is known exactly at the time of computing the model's solution. In stochastic models, instead, only the distribution of future shocks is known. In this session we only deal with **stochastic models**.

read your seminar paper. Before you make use of the computer, read the model section of your paper first and solve it with paper and pencil. Once you have all the equations you need and understood the framework, then you can make help of the computer.

Assume that your model takes the following linear form:

$$x_t = \rho_x x_{t-1} + \epsilon_{x,t} \tag{1}$$

with $\epsilon_{x,t} \sim \text{iiD}\mathcal{N}(0, \sigma_x^2)$. This is called an auto-regressive process of order one (as x has only one lag). The two parameters in this model, ρ_x, σ_x^2 are set such that $\rho_x = 0.95$ and $\sigma_x^2 = 0.02$.

Task 1
Thinking about the problem first:

1. What is the endogenous variable?

   _____

2. What is the exogenous variable?

   _____

3. What are the steady state values for x, ϵ_x?

   _____

Now we are at the point where we have understood the key elements our first simple one equation model and are able to take it to the computer to make our first simulation.
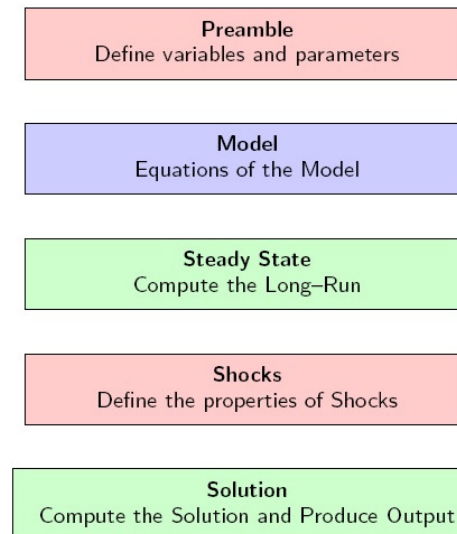
In order to run Dynare, it must first be installed on the computer. In a second step, we must add the path containing the Dynare functions to the Matlab directory.

Task 2
Check the following on your workstation:

1. Dynare is installed (usually it is `C:\dynare`).
2. The path is added to Matlab: `addpath c:\dynare\4.x.y\matlab`

In a next step, we want to create a `.mod-file`, where we can write the code of the AR(1)-model. This file will then parse the model to Dynare and produce its output in the Matlab command window. The usual **structure** of a `.mod`-file is the following:

| Preamble |
| Define variables and parameters |

| Model |
| Equations of the Model |

| Steady State |
| Compute the Long–Run |

| Shocks |
| Define the properties of Shocks |

| Solution |
| Compute the Solution and Produce Output |

Task 3
Setting up the first `.mod` *file:*

1. Go to `File`⇒ `New`⇒ `Blank M-File`.
2. Save the empty file as `ar1.mod`
3. Use \\ to comment in your .mod-file. In the first line of your .mod-file, comment your name and the name of this exercise.

Task 4

The simulation of an AR(1) process with Dynare:

A typical `.mod` file has the following structure:

```
1   // Preamble
2   var x,y,z;        //(endogenous variables)
3   varexo e,u,o; //(exogenous variables)
4
5   parameters a,b,c; //(parameters)
6   a=value of a;          //(values of parameters)
7
8   // Assisting Steady state
9   xss=value of stst x;
10   //(the steady states give the initial values)
11
12  // Model
13  model;
14   //(here you write the model equations)
15  end;
16
17  // Starting value(s) of model
18  initval;
19      x=xss;
20      y=yss;
21      z=zss;
22  end;
23
24  //Is my steady state correct?
25  steady;
26  check;
27
28  // Shocks
29  shocks;
30  var e; stderr value of e;
31  end;
32
33  // Solution
34  stoch_simul([some options]);
```

1. Use your **ar1.mod file**.
2. Define variables and parameters in the **preamble**.
3. Define the model equation in the **model** section.
4. Set the initial values (long-run values) of the model in the **Steady State** section.
5. In the **shocks** block, define the properties of the exogenous shock.
6. Compute the **solution** and produce some output. By typing

<div align="center">

`dynare ar1`

</div>

in the **matlab command window** you execute the .mod-file.

4 The basic RBC model

The AR(1)-model we have looked at is a linear dynamic model. However, most modern economic models are **not linear**. Dynare is able to deal with dynamic non-linear equation systems. The program (log-)linearizes (or takes a higher order Taylor expansion) around the deterministic steady state. In order to illustrate this, we will use a basic real business cycle model, which is the underlying framework of the models in the papers that you are replicating.

The present model is a decentralized economy, where the **building blocks** are on the one hand the behavior of **households** and on the other hand the behavior of **firms**. Households own the firms, but management and ownership are distinct, and so households behave as though firm profits are given. Households own the capital stock K_t and rent it to firms. Firms issue debt B_t and equity to households. We abstract from trend growth. As all agents are identical, in equilibrium, they have no reason to trade with each other, that is, they have no reason to hold bonds in the first place, hence $B_t = 0$

The RBC-model does not admit an analytical solution as it is non-linear. There are many ways to solve the model numerically and Dynare will help us do this. We will rely on a log-linear approximation of the model around the steady state. In order to understand what Dynare does, we will first do the calculations manually.

The calculations involve the following steps:

Usual steps to solve a DSGE-model
1. Find the general equilibrium representation.
2. Solve for the deterministic steady state.
3. Log-linearize the model around this deterministic steady state.
4. Assign values to parameters.
5. Numerically solve the system of linear difference equations.

Next we want to solve for the general equilibrium, starting with the prob-

lem of the households.

4.1 The household

The household has preferences over a bundle of consumption goods c_t and leisure l_t. He chooses the amount of consumption and labor he wants to supply to the private sector such to maximize the expected present discounted value of utility,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \tag{2}$$

subject to the budget constraint

$$c_t + i_t \leq W_t n_t + R_t k_t + \Pi_t, \tag{3}$$

where c_t is consumption expenditure, i_t is investment and $W_t n_t$ is the labor income, $R_t k_t$ the rent from holding capital and Π_t the profits from the firms.

The household is also subject to the capital accumulation equation

$$k_{t+1} = i_t + (1 - \delta)k_t \tag{4}$$

The total time endowment is normalized to unity and the household is subject to the time constraint

$$l_t = 1 - n_t.$$

Task 5
Derive the first order condition of the household, assuming that

$$U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\nu}}{1+\nu}. \tag{5}$$

Hint: Insert the capital accumulation equation for investment into the budget constraint and use Lagrangian optimization.

4.2 The firm

The firm then determines its production plan by maximizing its profit,

$$\max \Pi_t(k_t, n_t) = A_t F(k_t, n_t) - W_t n_t - R_t k_t. \tag{6}$$

The production function is Cobb-Douglas

$$y_t = F(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}. \tag{7}$$

Task 6
Derive the first order conditions of the firm.

From the firm's profit maximization and the household's budget optimization we can derive the aggregate market clearing equation (check this!):

$$y_t = c_t + i_t, \tag{8}$$

and to close the model we have that productivity A_t follows an exogenous AR(1) process as seen in the previous exercise.

Task 7
In order to define the general equilibrium, we have to identify the endogenous and exogenous variables, as well as the parameters.

What are the endogenous variables?

_____

What are the exogenous variables?

_____

What are the parameters?

_____

Now we are able to characterize the general equilibrium by collecting equations.

Task 8

Write down the equilibrium equations (should be nine):

_____

_____

_____

_____

_____

_____

Task 9

Find the deterministic steady state. Hints: It is helpful work with the great ratios, that is variable x over output y. To find the steady state, start by setting $k_{t+1} = k_t = k$ and $\Lambda_{t+1} = \Lambda_t = \Lambda$. Then find the steady state value for R. Then you solve for $\frac{k}{y}$, afterward for $\frac{i}{k}$ and hence for $\frac{i}{y}$. Once you have that you can find $\frac{c}{y}$. We calibrate steady state labor to be 1/3 and have to get an expression for χ in terms of steady state labor which finally recovers all the variables.

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_____

_____

From here on, Dynare does the rest for us. The only thing we need to do is assign parameter values and give Dynare the correct initial conditions (steady state). Dynare will do the log-linearization and simulation for us.

Task 10

Let's go to Dynare and simulate the basic RBC model!

For the calibration of the parameters use:

$$\beta = 0.99, \alpha = 0.35, \sigma = 2, \nu = 1, \delta = 0.025, \rho_A = 0.95, \sigma_A = 0.01.$$

1. Open up a new file and name it **rbc.mod**.
2. Define variables and parameters in the **preamble**.
3. Define the model equation in the **model** section.
4. Set the initial values (long-run values) of the model in the **Steady State** section.
5. In the **shocks** block, define the properties of the exogenous shock.
6. Compute the **solution** and produce some output. By typing

   ```
   dynare rbc
   ```

   in the **matlab command window** you execute the .mod-file.
7. Interpret the impulse responses in a few words