Doing Economics with the Computer

Lecture 4: SOLUTIONS

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Matlab Application: Financial Evaluation 1

In this lecture we will explain how to use MATLAB to solve amortization problems, interest compounding and defaulting on a mortgage.

Compulsory Readings

Up to now, you should have read chapters 1 - 6 in Attaway (2019). The book is provided on Ilias.

Further Exercises

Up to now, you should have solved the practice questions in chapters 1 - 6 in Attaway (2019). Solutions to the practice questions in chapter 3 - 6 are available on Ilias.

Housekeeping

When you write a script, you should include these commands at the start.

```
clear all % Deletes memory
close all % closes open windows (figures etc.)
clc       % clears command window
```

Number format

Change number format

```
format bank % Format numbers in currency format with 2 digits after the decimal point
```

Fixed Rate Mortgage Payments

Let us consider a fixed rate mortgage, that is, a mortgage that carries a fixed interest rate over its lifetime. We would like to understand how mortgage payments relate to the initial amount and the evolution of the principal, and the role of the interest rate and the length of the loan period. Let us assume that payments are made monthly until the principal has been paid out and that all the payments are of the same size.

Definitions

- $T =$ maturity of loans (number of years)
- $N = T * 12 =$ total number of payments
- $n =$ is number of payment
- $p(t) =$ principal, $t = 1, 2, ..., N$,
- $A =$ initial principal
- $R =$ constant, monthly payment
- $ra =$ annual interest rate
- $r = ra/12 =$ monthly interest rate

Example:

A 20-year loan of size $A = 350000$, at an annual interest of $ra = 0.05$, that is 5%. The principal then evolves according to $p(n+1) = (1+r)p(n) - R$. We can write the principal on the final date as (see table)

$p(N) = A * (1+r)^N - R * \sum_{j=0}^{N-1}(1+r)^j$.

The principal on the final date must be zero (the loan will have been fully repaid), that is $p(N) = 0$. Using this and solving for R (utilizing the definition of the sum of a finite geometric series) gives the required amount of monthly payment R:

$R = \frac{r*A*(1+r)^N}{(1+r)^N-1}$.

Table 1: No of payments and remaining principal

No payments	Remaining Principal	Remaining Principal
0	$A =$	A
1	$A * (1+r) - R =$	$-R + A * (1+r)$
2	$(A * (1+r) - R) * (1+r) - R =$	$-R - R * (1+r) + A * (1+r)^2$
3	$((A * (1+r) - R) * (1+r) - R) * (1+r) - R =$	$-R - R * (1+r) - R * (1+r)^2 + A * (1+r)^3$
4	$(((A * (1+r) - R) * (1+r) - R) * (1+r) - R) * (1+r) - R =$	$-R - R * (1+r) - R * (1+r)^2 - R(1+r)^3 + A * (1+r)^4$

Exercises:

A. Fixed Rate Mortgage Payments

a)

Suppose that $T = 30, ra = 0.05, A = 400000$. Using MATLAB determine the required monthly payments R. How much principal is left after 10 years? After 20 years? Use a for loop to answer the latter questions. Do it both on line (i.e. 'in script form') and using a function file.

a.i) Script form

```
A  = 400000;
T  = 30;
ra = 0.05;
N  = 12*T;
r  = ra/12;

% Determine monthly payment:
R=(r*A*(1+r)^N)/((1+r)^N-1);

% Determine principal:
p=[];
p(1)=A*(1+r)-R;

for n=1:N-1
    p(n+1)=(1+r)*p(n)-R;
end

% Monthly Payment
fprintf('Monthly Payment: %.3f USD \n',R);

% Principal left after 10 years
fprintf('Principal left after 10 years: %.3f USD \n', p(120));

% Principal left after 20 years
fprintf('Principal left after 20 years: %.3f USD \n', p(240));

% You can also access the principals for 10 and 20 years directly:
 [p(120) p(240)]
```

a.ii) Function file

Write the function file, pay.m that calculates the required monthly payments and the principal for each month. Give the parameter values from above and call the function. The result should be the same as in a.i).

```
A  = 400000;
T  = 30;
ra = 0.05;
N  = 12*T;
r  = ra/12;

[p, R] = pay(A,r,N);

% Monthly Payment
fprintf('Monthly Payment: %.3f USD \n',R);

% Principal left after 10 years
fprintf('Principal left after 10 years: %.3f USD \n', p(120));

% Principal left after 20 years
fprintf('Principal left after 20 years: %.3f USD \n', p(240));
```

b)

Let the annual interest rate now be higher, namely 0.06 and answer part (a) with the function file pay.m.

```
clear;

A  = 400000;
T  = 30;
ra = 0.06; % Higher interest rate
N  = 12*T;
r  = ra/12;

[p, R] = pay(A,r,N);

% Monthly Payment
fprintf('Monthly Payment: %.3f USD \n',R);

% Principal left after 10 years
fprintf('Principal left after 10 years: %.3f USD \n', p(120));

% Principal left after 20 years
fprintf('Principal left after 20 years: %.3f USD \n', p(240));
```

c)

Suppose that given your income, you can afford to make a maximum monthly payment $R = 500$. If the annual interest rate is $ra = 0.05$ what is the maximum amount of a 20 year loan that you can afford to borrow? Of a 30 year loan?

c.i) Using a script file: 20 year loan

```
R  = 500;
T  = 20;
ra = 0.05;
N  = 12*T;
r  = ra/12;

A=fzero(@(A) R-(r*A*(1+r)^N)/((1+r)^N-1), 200000);

% Maximum amount of a 20 year loan that you can afford to borrow
fprintf('Maximum amount of a 20 year loan that you can afford to borrow: %.3f USD \n', A);

% Same as if we were to rewrite the formula for R:
A_alt = R*((1+r)^N-1)/(r*(1+r)^N);
fprintf('Maximum amount of a 20 year loan that you can afford to borrow: %.3f USD \n', A_alt);
```

c.i) Using a script file: 30 year loan

```
R  = 500;
T  = 30;
ra = 0.05;
N  = 12*T;
r  = ra/12;

A=fzero(@(A) R-(r*A*(1+r)^N)/((1+r)^N-1), 200000);

% Maximum amount of a 30 year loan that you can afford to borrow
fprintf('Maximum amount of a 30 year loan that you can afford to borrow: %.3f USD \n', A);
```

c.ii) Using a function file: 30 year loan

```
R     = 500;
T     = 30;
ra    = 0.05;
param = [R,T,ra];
A0    = 200000; %Initial guess for A (the amount of the maximum loan)
% options=[1e9;1e-6;1e-8;1e-3;0];

A=fcsolve(@heidi,A0,[],param);
% See the function file heidi.m

% Maximum amount of a 30 year loan that you can afford to borrow
fprintf('Maximum amount of a 30 year loan that you can afford to borrow: %.3f USD \n', A);
```

d)

Suppose that you have an arrangement that during the first 5 years of the 30 year mortgage of 400'000 you only have to pay the interest due and nothing of the capital. What is the monthly payment you will have to make over the remaining 25 years? Is it smaller or larger than if you did not have this deal? Compute the principal after 10 years. Plot the evolution of the principal with and without the deal in the same graph.

```
% With the deal:
A=400000; T=30; ra=0.05; N=12*T; r=ra/12;
R=(r*A*(1+r)^(N-60))/((1+r)^(N-60)-1); % pay the capital over 25 years

p=[];
p(61)=A*(1+r)-R;

for n=61:N-1;
    p(n+1)=(1+r)*p(n)-R;
end

p(120)
pb=p;

% Without the deal:
A=400000; T=30; ra=0.05; N=12*T; r=ra/12;
[pa, R] = pay(A,r,N);

plot(1:N,[pa; pb])
```

e)

Suppose now that you pay nothing during the first 5 years and then you pay 500 over the following 5 years. What must the monthly payment R over the remaining 20 years be in order for the mortgage to be fully repaid?

```
A=400000; T=30; ra=0.05; N=12*T; r=ra/12; p=[];

% Pay R1 = 500 from year 5 to 10
R1   = 500;
A5   = A*(1+r)^60;
p(61)= A5*(1+r)-R1;

for n=61:120
    p(n+1)=(1+r)*p(n)-R1;
end

% Payment over the remaining 20 years
R=(r*p(120)*(1+r)^(N-120))/((1+r)^(N-120)-1); % pay the capital over 20 years

fprintf('Monthly payment over the remaining 20 years: %.3f USD \n', R);
```

e.i)

Suppose that the bank unexpectedly forgives the remaining amount of the loan once the principal has fallen below 50000. After how many months will this happen?

```
A=400000;T=30;ra=0.05; N=12*T; r=ra/12;
R=(r*A*(1+r)^N)/((1+r)^N-1);

p=[];
p(1)=A*(1+r)-R;

for n=1:N-1
    p(n+1)=(1+r)*p(n)-R;
    if p(n+1) < 50000
        break;
    end
end

l = size(p)
p(l)
% p(l-1)

fprintf('Principal has fallen below 50000 after: %.f Months \n', l(2));
```

e.ii)

Find an alternative way to solve the task above

```
A=400000; T=30; ra=0.05; N=12*T; r=ra/12;
R=(r*A*(1+r)^N)/((1+r)^N-1);

p=[]; p0=A*(1+r)-R; crit=1; n=1;

while crit>0
    p=(1+r)*p0-R;
    crit=p-50000;
    p0=p;
    n=n+1;
end

fprintf('Principal has fallen below 50000 after: %.f Months \n', n);
```

B. Flexible Rate Mortgage Payment

Let us now consider a flexible, 30 year mortgage, that is, a mortgage for which the interest rate changes from one month to the next. Let the monthly rate be r(n) and suppose, for simplicity, that the path of interest rates is known when the mortgage is issued and that the borrower still makes a fixed payment of 2000 every month. At the end of the 30 years period, the borrower pays the balance to the creditor. How much is this balance? Let $T = 30, A = 400000$ so that $N = 360$, $R = 2000$ and r be in the interval [a,b] of a uniform distribution, $a = 0.003, b = 0.005$, so that $r = a + (b - a) * rand(360, 1)$.

Exercises:

B. Flexible Rate Mortgage Payments

a)

Using MATLAB determine the amount of principal left after: 10 years; after 30 years.

```
A=400000;T=30; N=12*T; a=0.003; b=0.005; R=2000; r = a + (b-a)*rand(360,1);
p=[];

p(1)=A*(1+r(1))-R;

for n=1:N-1
    p(n+1)=(1+r(n+1))*p(n)-R;
end

% Principal left after 10 years
fprintf('Principal left after 10 years: %.3f USD \n', p(10*12));

% Principal left after 30 years
fprintf('Principal left after 30 years: %.3f USD \n', p(12*30));
```

b.i)

Now suppose that in each period you pay the full interest cost plus one tenth of the principal owed in that period. How much will you owe after 5 years? How many months will it take you before your principal drops below 50000?

```
A=400000;T=30; N=12*T; a=0.003; b=0.005; r = a + (b-a)*rand(360,1);
p=[];p(1)=A*(1+r(1))-0.1*A;

for n=1:N-1
    p(n+1)=(1+r(n+1))*p(n)-0.1*p(n);
end

% How much will you owe after 5 years?
fprintf('How much will you owe after 5 years: %.3f USD \n', p(5*12));

% How many months will it take you before your principal drops below 50000?
k=p(p>50000);
[i, j] = size(k);
fprintf('Months before principal drops below 50000: %.f Months \n', j);

% Note: After 20 Months, principal is:
fprintf('\nNote: \n')
fprintf('Principal after 20 Months: %.f USD \n',  p(20));

% Note: After 21 Months, principal is:
fprintf('Principal after 21 Months: %.f USD \n',  p(21));
```

b.ii)

Find an alternative way to solve the task above

```
[~, j]=(min(abs(p-50000)));
fprintf('Months before principal drops below 50000: %.f Months \n', j-1);
```