

Session 6: The Two-Country Mundell-Fleming Model

Doing Economics with the Computer

1 The Topic

Today's model extends the IS-LM model to the case of an open economy. The most crucial assumptions are that there is *perfect capital mobility* and that investors do not expect the exchange rate to change. Assumptions concerning the exchange rate regime (floating or fixed) may differ. The original model, with an exogenous foreign country, was proposed by Robert A. Mundell¹ and J. Marcus Fleming in the early 60s. For an introduction to the Small-Country, Mundell-Fleming (MF) model see e.g. Mankiw (1997, pp. 299, 2000, pp. 312); for an introduction to the Large-Country or Two-Country MF model see the supplementary material handed out in class.

A simple version of the Two-Country MF model looks as follows. Suppose a world consisting of two countries. One country is the Home country, the other country is the Foreign country. Both economies can be described by means of a (slightly modified) static IS-LM model. Goods and bonds can be traded across countries. Goods and bonds sold by agents living in H must be paid in the domestic currency; goods and bonds sold by agents

living in F must be paid in the foreign currency. There is a market where the currencies of the two countries can be exchanged; it is called *foreign exchange market*. Nominal goods prices in both countries are fixed. The imports of H are the exports of F and vice versa. Net exports of H decrease with its own output, but rise with F 's output. A rising real exchange rate (a deterioration of the terms of trade) has negative effects on net exports. Given perfect capital mobility and that investors do not expect the exchange rate to change, in a equilibrium situation the interest rate on both islands must be equal. Otherwise an arbitrageur could make profits by borrowing money in the country with lower interest rate and invest the money on the other island.

1.1 An Algebraic Version of the Two Country Mundell-Fleming Model

This Section describes a linear version of the Two Country MF model.

Parameters:

$a, b, c, d, e, f, g, h, i, a_w, b_w, c_w, d_w, e_w, f_w$

¹He got the 1999 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel.

Exogenous variables:

 G, G_w government spending (home and foreign) T, T_w tax on income M, M_w money supply P, P_w price level (fixed)

Endogenous variables:

 Y, Y_w production (output) C, C_w consumption I, I_w investments NX, NX_w net exports (i.e., exports minus imports) R, R_w interest rate (real and nominal) E exchange rate (real and nominal) – as usual the home price for foreign currency (e.g. 1.50 CHF/EUR) \vdots

$$Y_w = C_w + I_w + G_w + NX_w \quad (6)$$

$$C_w = a_w + b_w(Y_w - T_w) \quad (7)$$

$$I_w = c_w - d_w R_w \quad (8)$$

$$M_w/P_w = e_w Y_w - f_w R_w \quad (9)$$

$$NX_w = -NX \quad (10)$$

$$R = R_w \quad (11)$$

$$G = T$$

$$G_w = T_w$$

Model equations:

$$Y = C + I + G + NX \quad (1)$$

$$C = a + b(Y - T) \quad (2)$$

$$I = c - dR \quad (3)$$

$$M/P = eY - fR \quad (4)$$

$$NX = g - h(Y - Y_w) + iEP_w/P \quad (5)$$

 \vdots

1.2 Numerical Example

Start the analysis with two identical countries (see Hall 1991, exercise 4, chapter 19):

$$a = a_w = 55 \quad b = b_w = 0.63 \quad c = c_w = 47 \quad d = d_w = 1500$$

$$e = e_w = 0.6 \quad f = f_w = 2700 \quad g = -10, \quad h = 0.1, \quad i = 10$$

$$G = G_w = 150 \quad M = M_w = 210 \quad T = T_w = 150 \quad P = P_w = 1.04$$

2 The Method

The above model as well as the *IS-LM model in Session 1* are linear models.² They are defined by a system of linear equations. We use *matrix algebra* to deal with these models.³ Matrix algebra gives a compact notation, an exact criterion for testing the existence of a unique solution and an elegant way to solve the system.

Consider the following system of linear equations with n equations and m unknowns x_1, \dots, x_m :

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nm}x_m &= b_n \end{aligned}$$

Let us call x_m a *variable*, a_{ij} a *coefficient*, and b_n a *constant term*. There are $n \times m$ known coefficients, a_1, \dots, a_{nm} , and n known constant terms, b_1, \dots, b_n , of the column vector b .

The above linear equation system can be written in matrix form:

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}}_{\substack{(n \times m) \\ A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}}_{\substack{(m \times 1) \\ X}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}_{\substack{(n \times 1) \\ B}}$$

or in short

$$AX = B \quad (12)$$

where A is called the *coefficient matrix* and x is the vector of unknowns (or variables).

There is a *unique non-trivial solution* to a linear equation system if there are exactly as many unknown variables as equations and if there is no redundant information in the equations. These conditions can be expressed in mathematical terms: A has to be *quadratic* and *nonsingular*, and $B \neq 0$. (A is nonsingular if its determinant differs from zero, $|A| \neq 0$.)

Premultiplying both sides of equation (12) with A^{-1} yields the unique solution of the equation system:

$$X = A^{-1}B \quad (13)$$

If the numerical values of the parameters a_{ij} and b_i are known, the inverse of A can be calculated numerically. Computers can calculate the inverse of big matrices. Computing the inverse of a matrix, however, is a costly problem and is therefore avoidable whenever possible (Judd 1998, p. 84). A more efficient and numerically more accurate ways of solving the equation $AX = B$ is for instance Gaussian elimination (Strang 1986, chapter 1).⁴

²They are linear with respect to the unknown (endogenous) variables. However, they are not linear in prices. Therefore, the model becomes a non-linear model as soon as we skip the assumption that prices are fixed. This is e.g. the case in the traditional IS-LM-Phillips curve framework. IS-LM models that are non-linear in prices cannot be solved with matrix algebra. Programs such as GAMS use iterative algorithms to find the solution numerically.

³Matrix algebra is introduced in Chiang (1984). The department of econometrics provides a short introduction to matrix algebra.

⁴Strang (1986, p. 56) shows how bad algorithms ruin the numerical accuracy of the

Note that equation (13) can also be used to determine the analytical solution (well, it is already the analytical solution, but not as explicit as we like.)

3 The Software

We will use Matlab to solve linear equation systems numerically. We will use backslash or *matrix left division* in Matlab for solving linear equation systems, e.g.:

$$\gg X = A \backslash B$$

If A is a square matrix, $A \backslash B$ is roughly the same as $\text{inv}(A) * B$, except it is computed in a different way. If A is an $n - by - n$ matrix and B is a column vector with n components, or a matrix with several such columns, then $X = A \backslash B$ is the solution to the equation $AX = B$ computed by Gaussian elimination. A warning message prints if A is badly scaled or nearly singular.

4 References

Chiang, Alpha C. (1984). *Fundamental Methods of Mathematical Economics*, Auckland: McGraw-Hill, chapter 4 and 5.

Hall, Robert E. and John B. Taylor (1991). *Macroeconomics*, 3rd edition, New York: Norton.

Mankiw, N. Gregory (1997). *Macroeconomics*, 3rd edition, New York:

solution. The Matlab matrix left division (\backslash) uses Gaussian elimination with partial pivoting as described in Strang. Judd (1998, chapter 3) describes a variety of sophisticated numerical solving strategies in detail.

Worth.

Strang, G. (1986). *Linear Algebra and its Applications*, 3rd Edition, San Diego: Harcourt Brace Jovanovich.

5 Today's Task

Exercise 1: solve the static IS-LM model numerically

We start with the simple IS-LM Model from Session 1.

- a) Rewrite equation (1) to (4) from session 1, such that all endogenous variables are on the left hand side (lhs) and only exogenous variables are on the right hand side (rhs). Now, rewrite the 4 by 4 equation system as a matrix equation of the form $AX = B$
- b) Let us open a new *Editor* window in Matlab. Let us proceed as follows. First, we define the parameters and then use the parameter names in the definition of the matrix A and the vector B . Let us use the parameters from session 1.
- c) Test if the model has a non-trivial unique solution. Calculate the inverse of the matrix A . Calculate the solution vector $X = A^{-1}B$. Compare it with the results in Session 1.
- d) Read the notes on the `inv` function in the Matlab help.
- e) Calculate the solution using *Gaussian Elimination*. Matlab performs Gaussian elimination with matrix left division (`\`).
- f) Suppose, the government cuts non-distorting (lump sum) taxes without changing government spending; the resulting budget deficit is financed by borrowing to the public (i.e., a deficit-financed tax cut). If taxes are cut by 1 billion CHF (without changing government spending), by how much do GDP, consumption, investment, and the interest rate change?

Exercise 2: the two country IS-LM Model under a floating exchange rate

Consider the two country model defined in section 1.1. Assume a *floating* exchange rate regime

- a) Which variables are exogenous and which endogenous? How many unknown variables are there? And how many equations? Is the system identified?
- b) Use the above parameters and solve the model numerically in Matlab. Follow the steps in Exercise 1. Look at your solution for the variable NX : How do you interpret the result?
- c) How does a 20 billion increase in government spending of the country affect its output, net exports and both countries' consumption. Compare this to the outcome in a closed economy. This answer was worth a Nobel price.
- d) How does a 20 billion increase in the money supply of one country affect net exports and consumption in the two countries.
- e) Use the model to study the effects of other policy changes and shifts in parameters.
- f) Solve the two country Mundell-Fleming Model under the assumption that the Home country unilaterally pegs its currency to the Foreign country's currency. Assume for instance that in the benchmark solution Home's endogenous money supply equals the level of the Foreign money supply (210); the implied value for E then is 1. Redo problem c.