Lecture 1 in Monetary Economics

Harris Dellas

February 27, 2010

THE NEW KEYNESIAN MODEL

A key issue in macroeconomics: The existence of a trade off between inflation and economic activity (the **slope** of the AS -Phillips- curve)

Various stories exist in the literature about how different frictions (nominal wage or price rigidity, limited participation..) can lead to a positively sloped AS curve in the short run

The new Keynesian model relies on the familiar story (price or wage rigidity) but uses a sophisticated modern modelling (RBC) technology

The RBC model

- The RBC constitutes the methodological foundation of the NK model.
- It is a micro-founded DSGE model with rational agents, flexible prices and competitive markets. Representative agent.
- It has decent empirical properties (in terms of the match between the model implied and empirical pdf of the data).

The objective of any macro model is to determine the aggregate quantities (Y,C, I,..) and prices (R, q, ..) in period t as a function of the state variables (exogenous -shocks- and endogenous) in that period.

Steps

Step 1. Specification of the economic environment

1a. Number and type of agents (consumers, producers, workers, fiscal authorities, monetary authorities,..). Number and types of goods.

For simplicity (in order to avoid wealth distribution issues) the assumption of a representative agent is made. This does not necessarily mean a single agent who is all of the above at the same time, but rather a "family" that consists of agents of each of these types who completely pool and share the family resources at the end of the day. All families are identical.

1b. Specification of the objectives for the agents (the max. of utility for consumers-workers, the max. of profits for firms, the max. of social welfare for the govt, ...).
1c. Specification of the constraints faced by the agents (production, time endowment, ..).
1d. Specification of other "institutional" aspects of the economic environment (market structure, degree of completeness of asset markets, presence of uncertainty..).
1e. Mode of behavior for the agents. The standard assumption is of the rational pursuit of self-interest.

Step 2. Derivation of the individual demand and supply schedules for all the goods, assets and inputs in the economy. Aggregation to get the aggregate demands and supplies.
Step 3: D = S for each market $\Rightarrow$. Put all market clearing conditions together in order to solve for the equilibrium price and quantity in each market.
Note: Steps 2 and 3 typically require the use of specific functional forms for the objectives and constraints and reliance on numerical methods (more on this later) to the the solution of the model (equilibrium quantities and prices).

An example: A 2 period, endowment (no production), single, perishable good, closed economy.

Consumption-savings choice

Utility:

$$u(C_1) + \beta u(C_2) \tag{1}$$

Budget constraint:

$$\begin{aligned} P_1 Y_1 &= P_1 C_1 + B && (2) \\ P_2 Y_2 + RB &= P_2 C_2 && (3) \end{aligned}$$

$B > 0$ means lending in the first period.

$$P_2 C_2 = P_2 Y_2 + R(P_1 Y_1 - P_1 C_1) \tag{4}$$

The supply of output is fixed. The demand for output consists of the demand for consumption (or equivalently, savings as $C + S = Y$). It is given by the Euler equation (or dynamic IS curve)

$$u_{c1} = \beta R u_{c2} \frac{P_1}{P_2} = \beta R u_{c2} \frac{1}{\pi} \Rightarrow 1 = \beta r \frac{u_{c2}}{u_{c1}} \tag{5}$$

The Euler equation plays a critical role in the monetary transmission mechanism:
An increase in the real interest rate decreases current spending (consumption). Under sticky prices the CB may be able to influence the real interest and thus aggregate demand.
The solution of the model:
$C_1 = Y_1,\ Y_2 = C_2,\ B = 0,\ r = r\frac{u_{c_1}}{u_{c_2}\beta}$
To determine the nominal interest rate R and the inflation rate π we need to specify the monetary side of the economy (monetary policy, use of money, etc.).

A one period model with production and a work decision

Utility

$$u(C, h) \tag{6}$$

The budget constraint

$$Wh + \Omega = PC \tag{7}$$

The supply of labor

$$-\frac{u_h}{u_C} = \frac{W}{P} = w \tag{8}$$

The marginal rate of substitution between consumption and leisure equals the real wage.

The demand for labor. The firm faces the production constraint

$$Y = f(h)$$

Profit maximization ($\Omega = PY - Wh$) generates the demand for labor

$$\frac{dY}{dh} = MPL = w \tag{9}$$

Market clearing in the labor market (demand = supply of labor)

$$-\frac{u_h}{u_c} = MPL \tag{10}$$

Market clearing in the market for goods, $Y = C$
Using 10 and $Y = C$ in the labor market clearing condition gives Y and thus C, h and w.
Again the standard dichotomy between the real and nominal sectors obtains and we need more information to determine the nominal variables (W and P).

OPTIONAL: More general cases

A multi-period version of the RBC model with uncertainty and a representative agent.

$$V = E_0\left[\sum_{t=0}^{\infty} \beta^t U(C_t, h_t)\right] \tag{11}$$

Flow budget constraint:

$$P_t C_t + B_t = R_{t-1} B_{t-1} + W_t h_t + \Pi_t \tag{12}$$

$$U(C_t, h_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{N_t}{1+\sigma} h_t^{1+\sigma} \tag{13}$$

$$Y_t = A_t h_t^{1-\alpha} \tag{14}$$

N_t is a preference and A_t a technology (productivity) shock.

The general equilibrium solution of the model for $\{Y, C, h, w = W/P, r = R/\pi\}$ is obtained by solving

$$\begin{array}{rcl}
\frac{N_t h_t^{\sigma}}{C_t^{-\gamma}} & = & w_t \\
C_t^{-\gamma} & = & \beta E_t r_t C_{t+1}^{-\gamma} \\
w_t & = & (1-\alpha) A_t h_t^{-\alpha} \\
Y_t & = & A_t h_t^{1-\alpha} \\
Y_t & = & C_t \\
A_{t+1} & = & \overline{A}^{1-\rho_a} A_t^{\rho_a} \epsilon_{a,t+1} \\
N_{t+1} & = & \overline{N}^{1-\rho_\nu} N_t^{\rho_\nu} \epsilon_{g,t+1}
\end{array} \qquad (15)$$

$b_t \equiv B_t / P_t = 0$

Nonlinear, dynamic, stochastic equations that can only be solved analytically in special cases.

In practice we solve an approximate version of the system, typically a log-linear approximation around the steady state.

The steady state can be "easily" derived by setting $A_t = \overline{A}, N_t = \overline{N}, C_t = C_{t+1} = \overline{C}$ ...

The log-linearized system around the steady state takes the form

$$
\begin{aligned}
\hat{w}_t &= \sigma \hat{h}_t + \gamma \hat{c}_t + \hat{\nu}_t \\
0 &= \gamma E_t \hat{c}_{t+1} - \gamma \hat{c}_t + E_t \hat{r}_t \\
\hat{w}_t &= \hat{a}_t - \alpha \hat{h}_t \\
\hat{y}_t &= \hat{a}_t + (1-\alpha)\hat{h}_t \\
\hat{y}_t &= \hat{c}_t \\
\hat{a}_{t+1} &= \rho_a \hat{a}_t + \hat{\epsilon}_{a,t+1} \\
\hat{\nu}_{t+1} &= \rho_\nu \hat{\nu}_t + \hat{\epsilon}_{\nu,t+1}
\end{aligned}
\tag{16}
$$

where for variable x we define $\hat{x} = \frac{x-x^*}{x^*} \approx \log x - \log x^*$ as the percentage deviation of x from its steady state value, x^*.

In state space form

$$A_0 E_t x_{t+1} = A_1 x_t + B_0 e_{t+1} \tag{17}$$

When A_0 is invertible,

$$\begin{aligned} E_t x_{t+1} &= A_0^{-1} A_1 x_t + A_0^{-1} B_0 e_{t+1} \Rightarrow \\ E_t x_{t+1} &= A x_t + B e_{t+1} \end{aligned} \tag{18}$$

The Blanchard-Khan (1980) method: Partition the state variables of the system into backward (s) and forward looking (z) variables.

$$\begin{bmatrix} s_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} s_t \\ z_t \end{bmatrix} + B e_{t+1} \tag{19}$$

The properties of the solution The Blanchard-Khan criterion :

n = the number of eigenvalues of A that lie outside the unit circle
f = number of the forward looking variables.

- If n=f there exists a unique rational expectations solution to the system
- If n< f the system has multiple solutions[1] (indeterminacy).
- If If n> f then the system has no solution (all dynamic paths are explosive, violating the transversality condition).

[1] In this case one needs to use alternative methods to solve the system, for instance, Sims, 2000.

If A_0 is not invertible then the system

$$A_0 E_t x_{t+1} = A_1 x_t + B_0 e_{t+1}$$

can be solved using the QZ decomposition:

$\exists Q, \Lambda, Z, \Omega \; s.t. Q'\Lambda Z' = A_0, Q'\Omega Z' = A_1$, Λ, Ω upper triangular (see Klein, 2002, Sims, 2000).

The software of choice: Dynare

Basic structure of Dynare

// **declarations**

$var\, x, y, \ldots;$

$varexo\, ea, ev, \ldots;$

$parameters\, alpha, beta, \ldots;$

// **parameter values**

$alp =; bet =; \ldots$

// **model equations**

model;

$exp(v) * exp(c * gam) * exp(h * sig) - exp(w) = 0;$ // consumption

$(1 - alp) * exp(a) * exp(h * (-alp)) - exp(w) = 0;$ // work

$\ldots$

end;

//**steady state solution**
initval;
$c = log(..); h = log(..); \ldots$
end;
steady;
check;
// **stochastic structure** shocks;
$varea = ..; varev =; end;$
//**simulations**
stoch_simul(dr_algo=0,periods=1000, irf=20, nocorr, nofunctions, order=1) c y h w;

POLICY AND TRANSITION FUNCTIONS:

	c	y	h	w	r
Constant	-0.12	-0.12	-0.18	-0.36	0.01
a(-1)	0.81	0.81	-0.20	1.02	-0.06
v(-1)	-0.26	-0.26	-0.40	0.14	0.01
ea	0.86	0.86	-0.21	1.07	-0.06
ev	-0.27	-0.27	-0.43	0.15	0.02

MOMENTS OF SIMULATED VARIABLES:

VARIABLE	STD. DEV.	AUTOCOR
c	0.028145	0.9568
y	0.028145	0.9568
h	0.007386	0.9578
w	0.035377	0.9571
r	0.002111	0.9568