

Monetary Misperceptions, Output and Inflation Dynamics*

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Abstract

We revisit the contribution of misperceived money to business cycles, and in particular to the inertial dynamics of inflation following a monetary policy shock. We establish three things. First, the difference between preliminary and revised money data captures monetary misperceptions well. Second, misperceived money is quantitatively substantial and also matters significantly for economic activity. And third, imperfect information about monetary aggregates can help the standard NK model exhibit inertial inflation dynamics.

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Introduction

The key element in Lucas' (Lucas, 1977) flexible price, monetary business cycle model is the distinction between perceived and misperceived monetary aggregates. Fluctuations in the supply of money matter for economic activity only to the extent that they are not perceived. The Keynesian version of Lucas's model (Fischer, 1977, Gray, 1976, Taylor, 1980) emphasizes a different type of variation in the money supply as a driving force of business cycles, namely unanticipated changes. With fixed nominal prices or wages, even perfectly perceived movements in the supply of money that differ from those that had been anticipated can induce movements in aggregate economic activity.

A key macroeconomic debate of the second half of the seventies and most of the eighties evolved around the relative contribution of anticipated vs unanticipated and perceived vs misperceived money. The empirical evidence studied in the context of that debate favored the unanticipated money version. For instance, Barro and Rush, 1980, found that unanticipated money growth mattered for economic activity (while anticipated did not). Using the difference between preliminary and revised monetary data to model misperceptions, Barro and Hercowitz, 1981, and Boschen and Grossman, 1982, found that misperceived money did not matter for the business cycle.

The sticky price, unanticipated money, Keynesian model of the 70s gradually morphed into the New Keynesian (NK) model¹, which has become the leading monetary model of our days. Like its predecessor, it relies on unanticipated monetary shocks and sticky prices in order to generate monetary non-neutrality. But in spite of its general success, this model has a crucial weakness regarding its implied dynamics of inflation following monetary policy shocks. The standard version cannot generate inflation inertia. The extended version (see Christiano et al., 2005) can, but it relies on pricing assumptions (dynamic indexation) that are not supported by existing micro evidence.

The primary objective of the present paper is to examine whether the embedding of misperceptions a la Lucas into the NK model can improve its dynamic properties. Recent macroeconomic work involving imperfect information about aggregate variables suggests that such an endeavor may represent a promising venue. For instance, Bullard and Eusepi, 2003, and Dellas, 2006 find that imperfect information and signal extraction can serve as an endogenous mechanism of macroeconomic persistence². Mankiw and Reis', 2002, sticky information model as well as

¹Christiano et al, 2005, –henceforth CEE– Clarida, Gali and Gertler, 1999, Goodfriend and King, 1997, Woodford, 2003.

²A similar point regarding inertia can be found in a small, related literature that examines the effects of measurement error (data revisions) on macroeconomic activity in the context of RBC models(Bomfim, 2001,

work based on Sim's, 2003, rational inattention theory also suggest that imperfect information can be a source of inflation inertia. More to the point, Dupor, Kitamura, and Tsuruga, 2006, show that the addition of sticky information to the standard NK model (what they term the dual stickiness model) can improve its dynamic behavior. A related point is made by Coibion and Gorodnichenko, 2008.

A second objective of the paper is to revisit the role of monetary misperceptions for business cycles, especially in the context of the earlier literature on the relative contribution of anticipated vs unanticipated and perceived vs misperceived money. This is an issue that has been left dormant for almost three decades, mostly because the results of the earlier literature seemed to indicate that misperceived money did not matter.

Using information from the Real Time Data Set of the Philadelphia FED we establish several points: a) First, misperceived money (constructed as the difference between the initial release and subsequent revisions of the money stock) shocks are as large and volatile as unanticipated money (constructed from VAR innovations) shocks, b) Both shocks matter significantly for economic activity in the pre-1982 but not in the post-1982³ period, and, c) the presence of monetary misperceptions enables the standard version of the NK model to exhibit inflation inertia. Hence, imperfect information can serve as an alternative to popular but controversial schemes such as dynamic indexation or rule of thumb agents.

The remaining of the paper is organized as follows. Section 1 presents the empirical evidence. Section 2 presents the model while Section 3 discusses its empirical properties.

1 Misperceived and Unanticipated Money

The early vintage of the imperfect information, rational expectations, flexible price models required that the agents did not observe any of the nominal aggregates. But it was quickly realized that it strained credibility to assume that monetary aggregates were not observable at all, or, that they were so but only with substantial time lags. King, 1981, offered a plausible alternative, under which information on monetary aggregates was assumed to be readily available but observations of the current or recent monetary data (the preliminary figures) were ridden

Aruoba, 2008).

³Amato and Swanson, 2001, also use real time data to examine the role of money for economic activity. But unlike the works referenced above and our work which are concerned with misperceived and unanticipated money in the spirit of Lucas as well as of the NK model, Amato and Swanson examine the predictive power of *perceived* money growth for subsequent economic activity. This represents a rather a-theoretical exercise. Nonetheless, to the extent that there are nominal wage rigidities and perceived –real time– money growth contains a significant unanticipated component, their test may be interpreted as a test of the effects of unanticipated money shocks. Interestingly, their main finding of the absence of any significant effects over the period 1978-1997 is quite similar to our finding that unanticipated money shocks do not matter over the 1982-2002 period.

with measurement error. This error was only gradually corrected through subsequent data revisions. Barro and Hercowitz, 1981, and Boschen and Grossman, 1982, tested this interpretation of the model but were unable to find any role for data revisions in the monetary transmission mechanism.

In this section we revisit this issue for the period 1966Q1–2002Q4. The starting period is dictated by the availability of the real data series constructed by the Philadelphia FED. 2002Q4 is chosen as the final period in order to leave room for the computation of subsequent revisions. We work with both the full sample and two sub-samples: 1966–1982Q3 and⁴ 1982Q4–2002Q4. We first examine whether the revisions in monetary aggregates—the difference between preliminary and revised data—represent a measurement error. That, it whether they have the properties required by the imperfect information rational expectations theory in order to represent a legitimate measure of misperceived money. We find that these revisions are not predictable on the basis of macroeconomic information available at the time of the initial release. And that, the hypothesis of errors in variables cannot be rejected, in particular in the second sub-sample.

We also find that variation in misperceived money is of the same order of magnitude as that in unanticipated money. Moreover, variation in both types of money matters for economic activity in the pre- but not post 1982 periods.

We use the real time data set of the Philadelphia FED⁵ to compute the measurement error in successive data releases. In particular, let $M_{t|t}$ be the initial release of M1 of period t (first reported in vintage $t+1$) and $g_{t|t} = \log M_{t|t} - \log M_{t-1|t}$ its growth rate. Let $M_{t|t+i}$ (resp. $g_{t|t+i} = \log M_{t|t+i} - \log M_{t-1|t+i}$) be the revised figure for period t that is available in period $t+i$, $i > 1$. We use $t+i = T$ to represent the “final” release.⁶ We define “unperceived” money growth in period t as $\mu_{t|T} = g_{t|T} - g_{t|t}$.

The initial release of money plays a key role in the construction of misperceived money. As the vintages are collected quarterly, the existence of monthly observations for some variables (such as the money supply) complicates the identification of the initial release for those variables in the Real Time Data Set. This is because a vintage contains the data that would have been available at the vintage date and thus, to the extent that some of the initial monthly numbers from the previous quarter have already been revised within that quarter, the reported numbers may not correspond to the initial releases. For example, consider the rate of money growth

⁴This choice of the splitting point follows standard practice. Nonetheless, we obtained similar results when we used 1979 as the cut off point.

⁵The data can be downloaded from <http://www.phil.frb.org/research-and-data/real-time-center/real-time-data/>.

⁶All data is seasonally unadjusted. Kavajecz and Collins, 1995, have argued that the finding of Mankiw et al., 1984, that the Federal Reserve’s preliminary estimates of growth rates of the money stock are not unbiased predictors of the growth rates of finally-revised data may arise from the specific seasonal adjustment procedure used by the Federal Reserve.

during the first quarter. This is defined as the log difference between the end of March and end of December figures. These figures are reported in the Q2 vintage. But this release does not contain the M1 figure initially released in December but rather the M1 number for December that is available in May (and which is likely to represent a revised version of the initial December figure). Whether this procedure leads to the underestimation of the degree of misperception present in real time data or not depends very much on the frequency and timing of economic decisions as well as the type of data (levels vs growth rates, etc.) used by economic agents. We know very little about these issues. As a robustness check we have also carried out the analysis using an alternative way of constructing the initial release growth rate which combines information from two successive vintages. For instance, in order to construct the initial release of Q1 money growth, we used the December announcement of M1 reported in the Q1 vintage and the March announcement reported in the Q2 vintage. These observations are likely to represent true initial announcements. Nevertheless, this method of constructing the initial release is likely to overestimate the degree of misperception in real time as it ignores the information that the agents have acquired during the course of the quarter and which may have been relevant for the economics decisions made during the quarter. It is encouraging that the results are robust to the choice of the procedure for computing the initial release.⁷

Another issue regarding the identification of the misperception from information on the measurement error in real time data money is that much of the difference between initial releases and final data arises from changes in the definition of M1. Such redefinitions can contaminate the computation of misperceptions. Fortunately, as argued by Kavajesz and Collins, 1995, and Mankiw et al. 1984, this may be of limited concern if the effect of redefinition is mainly on levels and one relies –as they and we do– on growth rates rather than on levels. Nevertheless, we have also examined the robustness of the main findings by using intermediate rather than final data. In particular, we have also used $\mu_{t,t+8}$ in place of $\mu_{t,T}$. The results are not affected.⁸

Table 1 reports the properties (standard deviation and autocorrelation) of unperceived money growth shocks as well as those of unanticipated money shocks, ε_t . The unanticipated money shocks have been obtained from a VAR model suggested by CEE and which features output growth, inflation, commodity prices, the federal funds rate and money supply growth, in that specific order. The monetary shock is identified using a Choleski decomposition of the covariance matrix of the residuals. The corresponding money supply shock is computed as the change in

⁷All robustness check are reported in a companion technical appendix available from <http://fabcol.free.fr/index.php?page=research>.

⁸Redefinition is not the only problem afflicting the measurement of the money stock. “Sweep” programs have led to a severe underestimation of true (that is, transactions related measure of money) growth rate of M1 in the second part of the 90s. But because both $g_{t|T}$ and $g_{t|t}$ are affected symmetrically by such practices, it is unlikely that our measure of misperception is compromised by such programs.

the money supply that is induced by the shock to the interest rate equation.⁹ Note that the unanticipated shocks are computed based on final data¹⁰. In order to gain some idea about the quantitative significance of successive revisions of the preliminary data we also report the properties of $\mu_{t|t+i} = g_{t|t+i} - g_{t|t}$ for $i = 1, 2, 4, 8$.

Table 1: Properties of misperceived and unanticipated money growth shocks

	1966Q1–2002Q4			1966Q1–1982Q3			1982Q4–2002Q4		
	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$
$\mu_{t,t+1}$	0.13	-0.11	0.03	0.10	-0.08	0.11	0.14	-0.12	0.05
$\mu_{t,t+2}$	0.20	0.06	0.08	0.19	-0.10	0.08	0.20	0.16	0.14
$\mu_{t,t+4}$	0.33	0.11	-0.03	0.38	0.23	0.04	0.28	-0.11	0.04
$\mu_{t,t+8}$	0.37	-0.05	0.03	0.38	-0.03	0.04	0.36	-0.09	0.05
$\mu_{t,T}$	0.45	-0.06	0.06	0.57	-0.05	0.11	0.36	-0.11	0.07
ε_t	0.35	-0.00	1.00	0.25	0.13	1.00	0.38	-0.03	1.00

Note: σ , ρ and $\rho(\cdot, \varepsilon_t)$ are the standard deviation, 1st order autocorrelation and correlation between unperceived and unanticipated money respectively.

As can be seen the measurement error is substantial. Its standard deviation is higher than that of unanticipated money in the full sample; more than twice as big in the first sub-sample; and about the same in the second sub-sample. Overall the volatility of unanticipated shocks has increased over time, while that of measurement error has declined.

We view these findings as establishing that measurement error is quantitatively important. It remains to show that the measure of misperceived money constructed from this measurement error corresponds indeed to the concept of monetary misperceptions in the model of Lucas. We do so by relying on the approach pioneered by Mankiw et al., 1984. First, we establish that the preliminary announcements of the money stock are best characterized as measured with classical errors-in-variables. This justifies the signal extraction specification employed in the model of the following section. And second, we show that the difference between the initial and the revised announcements *cannot* be predicted on the basis of information that is available at the time of

⁹This way of computing the monetary shock is standard. But it would only contain a subset of monetary shocks if there were also variation in the money supply that was not related to the interest rate rule. Leeper and Rouch's, 2003, work suggests that this may be the case. Consequently, we have used alternative procedures for identifying the unanticipated money shock. For instance, we have used a standard VAR containing output growth, inflation, the federal funds rate and money supply growth and computed the unanticipated monetary shock as the shock to the last equation using a Choleski decomposition of the covariance matrix of the residuals. Note, though, that to the extent that there are also money demand shocks, the identified shock will be a mixture of money supply and money demand shocks. Finally, we also considered an AR(1) specification. The results are robust across all of these specifications.

¹⁰Croushore and Evans, 2006, show that the use of real time or intermediate vintage data instead of final data matters little for the properties of the identified monetary policy shock. In the technical appendix, we report results for unanticipated money that are based on a series of recursive VARs estimated on real-time data. Our findings echo those of Croushore and Evans.

the initial release.¹¹

Following Mankiw et al., 1984 we deal with the errors-in-variables issue by regressing the initial release of money growth on a constant term and the final release

$$g_{t|t} = \alpha_0 + \alpha_1 g_{t|T} + u_t$$

and testing the joint null hypothesis, $E\hat{\alpha}_0 = 0$ and $E\hat{\alpha}_1 = 1$. The results, as reported in Table 2, indicate that in both subs-samples and, in particular, in the second one, the initial release indeed corresponds to an errors-in-variables phenomenon.

Table 2: Errors-in-Variables

	1966Q1–2000Q4	1966Q1–1982Q3	1982Q4–2000Q4
$\mu_{t,t+4}$	3.2867 [0.0402]	2.6817 [0.0761]	0.9207 [0.4025]
$\mu_{t,t+8}$	4.2642 [0.0159]	3.4882 [0.0364]	1.1059 [0.3360]
$\mu_{t,T}$	3.9013 [0.0224]	2.8013 [0.0681]	1.3492 [0.2654]

Note: The table reports the Fisher statistic for the test $\alpha_0 = 0$ and $\alpha_1 = 1$. p-values into brackets.

In Table 3, we address the issue of the predictability of these errors by regressing $\mu_{t|T}$ on¹² values of the federal fund rate (R), changes in the stock market (ΔSP) –as in Mankiw et al., 1984, and Kavajesz and Collins, 1995) as well as output growth and inflation that were available at the time of the release.¹³ As can be seen from the table, measurement errors cannot be predicted.¹⁴ This implies that the conventional measure of misperceived money is appropriate, specially in the post 1982 period.

We now turn to the question of whether these measurements errors matter for macroeconomic activity. We have used two alternative methodologies for assessing this issue. One follows

¹¹Note that the existence of predictability would contaminate the conventional measure of misperceived money with anticipated money and render this variable unsuitable for testing the imperfect information, rational expectations theory of Lucas.

¹²Note that Table 1 already indicates the absence of autocorrelation and hence predictability based on own lagged values in $\mu_{t|T}$. But this is not sufficient to establish the lack of predictability as there may be other variables at the time of the release that could help forecast future unperceived money shocks.

¹³We have also considered additional variables whose values are available at the time of the release. The results remain the same.

¹⁴Both this and the previous result on errors-in-variables are similar to those reported by Kavajesz and Collins. But they differs from those in Mankiw et al. as well as in the small subsequent literature that followed the Mankiw et al. (see, Mork, 1990). Kavajesz and Collins show that non-overlapping observations as well as seasonality correction problems are responsible for this discrepancy. Note also that there is a related literature on the properties of *other* data revisions (such as Faust et al. 2005 who rely on the Mankiw et al. methodology or, Aruoba, 2008, who uses a somewhat different methodology) which find predictability in revisions, but which, however, does not examine revisions in money, the key variable in the present paper.

Table 3: Forecasting regressions

Cst.	R_t	$\Delta S\&P$	ΔY	π	D.W.	F-Test
<i>1966Q1–2002Q4</i>						
0.0802	-1.2338	-0.5514	3.1552	3.6216	2.13	0.88
(0.1151)	(6.5292)	(0.6417)	(5.1810)	(8.5491)		
0.1058	0.0704	–	–	–	2.10	0.99
(0.0943)	(4.9376)					
0.1150	–	-0.5339	–	–	2.11	0.40
(0.0399)		(0.6292)				
0.0943	–	–	1.9867	–	2.11	0.68
(0.0492)			(4.7395)			
0.0896	–	–	–	1.7269	2.10	0.78
(0.0739)				(6.2464)		
<i>1966Q1–1982Q3</i>						
0.1425	-2.1466	-1.0843	6.4376	1.6657	2.10	0.83
(0.2476)	(10.0080)	(1.2289)	(7.8644)	(15.6271)		
0.2113	-2.7983	–	–	–	2.09	0.71
(0.1666)	(7.5611)					
0.1581	–	-0.8217	–	–	2.07	0.48
(0.0700)		(1.1653)				
0.1245	–	–	5.6539	–	2.10	0.39
(0.0784)			(6.5474)			
0.2206	–	–	–	-4.5033	2.09	0.69
(0.1795)				(11.4200)		
<i>1982Q4–2002Q4</i>						
0.1283	8.3652	-0.1247	-10.4438	-17.5432	2.15	0.71
(0.1294)	(10.1053)	(0.6928)	(8.2010)	(17.9009)		
0.0586	0.5166	–	–	–	2.17	0.94
(0.1183)	(7.2883)					
0.0689	–	-0.0988	–	–	2.18	0.88
(0.0443)		(0.6696)				
0.1252	–	–	-8.2805	–	2.16	0.29
(0.0688)			(7.8037)			
0.1073	–	–	–	-6.4111	2.17	0.62
(0.0929)				(13.0637)		

Note: R = Federal fund rate, $\Delta S\&P$ = changes in the $S\&P$ stock market index, ΔY = changes in GDP (Initial Release), π inflation rate (GDP Deflator, Initial Release). Standard deviations in parenthesis. F-Test column reports the p-value associated with the F-Test of global significance.

Table 4: The effects of unperceived and unanticipated money, F-Tests

	Output			Inflation Rate		
	(p, n)	$\mu_{t T}$	ε_t	(p, n)	$\mu_{t T}$	ε_t
<i>1966Q1–2002Q4</i>	(3,2)	4.7934	6.8977	(5,7)	3.4001	1.4246
		[0.0033]	[0.0002]		[0.0015]	[0.1935]
<i>1966Q1–1982Q3</i>	(3,2)	4.8187	5.9241	(2,1)	0.0949	0.8072
		[0.0050]	[0.0015]		[0.9096]	[0.4513]
<i>1982Q4–2002Q4</i>	(3,1)	0.2592	0.1572	(4,8)	2.2657	1.0958
		[0.7724]	[0.8549]		[0.0336]	[0.3841]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . $\mu_{t|T}$ is unperceived and ε_t is the unanticipated money shock.

Boschen and Grossman, 1982 and involves a regression of the growth rate of output in period t on its lagged values as well as on unperceived money growth –as well as unanticipated money– during that and previous periods. The other method relies on a standard VAR approach.

We have estimated equations for HP-filtered output (GDP) and the inflation rate (GDP deflator) according to the specification

$$x_t = \sum_{i=1}^p \rho_i x_{t-i} + \sum_{\ell=0}^n [\alpha_i \mu_{t-\ell} + \beta_i \varepsilon_{t-\ell}] + u_t \quad (1)$$

The unanticipated money shocks $\varepsilon_{t-\ell}$ have been included in the regressions along side the unperceived one¹⁵ to allow us to judge the relative importance of the two sources of monetary non-neutralities: One arising from nominal rigidities (unanticipated shocks), and the other from informational frictions (unperceived shocks). We test for the significance of unperceived and unanticipated shocks using an F-test. The number of lags is selected based on the AIC and SC information criterion but the results are robust to using different lag structures. The results are reported in Table 4.

The main finding is that both sources of errors matter for economic activity in the full sample as well as in the first sub-sample. But they do not seem to matter in the second sub-sample. Moreover, the size of the effect on output is comparable for the two shocks.

How do our results compare to those in the rational expectations literature? Concerning the effects of unanticipated money on economic activity our finding is similar to that reported by Barro and Rush, 1980, and this, in spite of the differences in the way we compute the unanticipated money shock. Concerning the effect of misperceived money, our findings for the

¹⁵We have simplified notation by using $\mu_t = \mu_{t|T}$. Consequently, $\mu_{t-\ell}$ is unperceived money growth during period $t - \ell$.

1966–1982 period differ from those reported by Barro and Hercowitz (B–H), 1980, and Boschen and Grossman (B–G), 1982. In addition to differences in the sample periods (1950–1975 for B–H and 1953–1978 for B–G) as well as the data used (neither B–H nor B–G rely on end of period values, so there is an averaging problem; moreover, B–H use annual data) this seems to be due to the fact that we estimate jointly the effects of misperceived and unanticipated money (B–H only consider the effects of misperceived money while B–G examine the effects of unanticipated and misperceived shocks separately). When we only consider the effects of misperceived money, the p-value of the estimated coefficient increases from 0.005 to 0.082.

In order to gauge the robustness of these results we have carried out a number of additional exercises. In particular, we have repeated the analysis using: a) the alternative measure of misperceptions described above (namely, the one that makes use of two successive vintages in order to compute the growth rate of money for the initial release); b) the measure of unanticipated money that relies on the four equation VAR and computes the money shock from the error term to the money equation; c) the measure of unanticipated money that on forecast errors of money growth in a series of recursive VARs estimated using real-time data; d) the forecasting regressions (Table 3) in order to filter out the effect of the right hand side variables on the money revisions. The results are quite robust to these modifications.

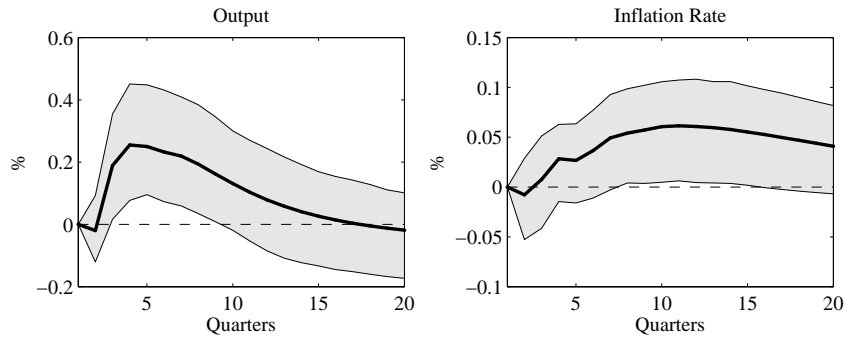
We have also examined and compared the effects of unanticipated and unperceived money in the context of standard VAR specifications. For unanticipated money we used the two VAR specifications described above. For unperceived money we used the μ series described above. In this case, as unperceived money growth ought to be unexplained by any of the other variables in the VAR, we estimate a VARX for output growth, CPI inflation and the federal fund rate where μ_t is introduced as an exogenous variable. Standard likelihood ratio tests and information criteria recommend the use of three lags in the VAR part and the current value and three lags of the unperceived money growth series.

As can be seen, from Figures 1 and 2 the reaction of output and inflation to a shock, whether unanticipated or misperceived is quite similar. Both output and inflation follow a hump shaped pattern. The effects of misperceived shocks are quantitatively larger.

2 The model

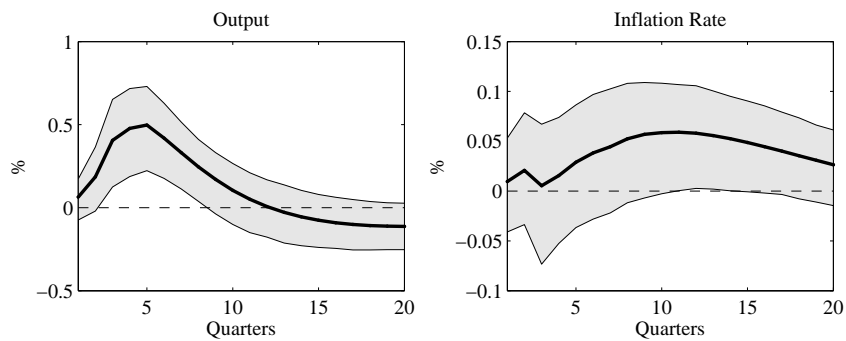
The previous section established empirically that both misperceived and unanticipated money seem to play a significant role in the monetary transmission mechanism. The objective of this section is to examine whether the inclusion of misperceived money can help the NK model exhibit more realistic dynamics of inflation in the absence of other popular but controversial

Figure 1: Response to an unanticipated shock



Note: The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VAR.

Figure 2: Response to a misperceived shock



Note: The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VARX model.

schemes such as myopia (Gali and Gertler, 1999) or backward indexation (Christiano et al., 2005).

The basic set up is the NK model with price rigidities, augmented to include various real rigidities. The production side of the economy consists of two sectors: one producing intermediate goods and the other a final good. The intermediate good is produced with capital and labor and the final good with intermediate goods. The final good is homogeneous and can be used for consumption (private and public) and investment purposes.

2.1 Final sector

The final good, y , is produced by combining intermediate goods, y_i $i \in [0, 1]$, by perfectly competitive firms. The production function is given by

$$y_t = \left(\int_0^1 y_{it}^\theta di \right)^{\frac{1}{\theta}} \quad (2)$$

where $\theta \in (-\infty, 1)$.

The final good may be used for consumption — private or public — and investment purposes.

2.2 Intermediate goods producers

Each firm i , $i \in (0, 1)$, produces an intermediate good using of capital and labor according to a Cobb–Douglas production function

$$y_{it} = a_t (u_{it} k_{it})^\alpha n_{it}^{1-\alpha} \text{ with } \alpha \in (0, 1) \quad (3)$$

where k_{it} and n_{it} are physical capital and labor used by firm i . a_t is an exogenous, stationary, stochastic, technology shock whose properties will be defined later. u_{it} is the rate of capital utilization.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. Following Calvo, we assume that in each and every period, a firm either gets the chance to adjust its price (with probability γ) or it does not. If it does not get the chance, then it sets its price according to

$$P_{it} = \pi_{t-1}^\nu \pi^{\star 1-\nu} P_{it-1} \quad (4)$$

where $\nu \in (0, 1)$ measures the degree with which prices are anchored to past inflation rather than steady state inflation. As a benchmark, we consider two scenarios regarding ν . In the first one, which will be used in the version of the model with the signal extraction formulation, the

price grows with steady state inflation until the firm gets a call that allows it to reset its price optimally. In this case, we have $\nu = 0$. The second scenario has these firms index their prices to the lagged, economy wide rate of inflation (as in CEE). Hence $\nu = 1$.¹⁶ As shown by Collard and Dellas, 2005, this assumption plays a critical role in allowing the NK model to produce satisfactory inflation dynamics.

For a firm i that sets its price optimally in period t , its price, P_t^* , is given by

$$P_t^* = \frac{1}{\theta} \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} (1-\gamma)^\tau \Phi_{t+\tau} P_{t+\tau}^{\frac{2-\theta}{1-\theta}} \Xi_{t,\tau}^{\frac{1}{\theta-1}} \psi_{t+\tau} y_{t+\tau}}{\mathbb{E}_t \sum_{\tau=0}^{\infty} (1-\gamma)^\tau \Phi_{t+\tau} \Xi_{t,\tau}^{\frac{\theta}{\theta-1}} P_{t+\tau}^{\frac{1}{\theta-1}} y_{t+\tau}} \quad (5)$$

where ψ_t is real marginal cost, P_t is the aggregate price index, Φ_t is an appropriate discount factor derived from the household's optimality conditions and

$$\Xi_{t+\tau} = \begin{cases} \prod_{\ell=0}^{\tau-1} \pi_{t-1+\ell}^\nu \pi^{*1-\nu} & \text{for } \tau \geq 1 \\ 1 & \tau = 0 \end{cases}$$

Since the price setting scheme is independent of any firm specific characteristic, all firms that reset their prices will choose the same price.

2.3 The Household

The preferences of the representative household are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\log(c_{t+\tau} - \vartheta c_{t+\tau-1}) + \frac{\nu^m}{1-\sigma_m} \left(\frac{M_{t+\tau}}{P_{t+\tau}} \right)^{1-\sigma_m} - \frac{\nu^h}{1+\sigma_h} h_{t+\tau}^{1+\sigma_h} \right] \quad (6)$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption in period t , M_t/P_t is real balances and h_t is the quantity of labor she supplies. Preferences are characterized by habit persistence governed by the parameter ϑ .

In each period, the representative household faces the budget constraint

$$E_t Q_t B_t + M_t + P_t(c_t + i_t + a(u_t)k_t + \tau_t) = B_{t-1} + M_{t-1} + P_t z_t u_t k_t + P_t w_t h_t + \Omega_t + \Pi_t \quad (7)$$

where B_t is a state contingent claim with corresponding price Q_t . M_t is end of period t money holdings. P_t , the nominal price of goods. c_t and i_t are consumption and investment expenditure respectively; k_t is the amount of physical capital owned by the household and leased to the firms

¹⁶In the companion technical appendix, we investigate the robustness of our results to intermediate values for ν . For sufficiently high levels of ν , the response of inflation to an unanticipated money shock exhibits a hump.

at the real rental rate z_t . Only a fraction u_t of the capital stock is utilized in any period, which involves an increasing and convex cost $a(u_t)$. w_t is the real wage. τ_t is a lump-sum tax paid to the government, Ω_t is a nominal lump-sum transfer received from the monetary authority and Π_t denotes the profits distributed to the household by the firms.

Capital accumulates according to the law of motion

$$k_{t+1} = \Phi(i_t, i_{t-1}) + (1 - \delta)k_t \quad (8)$$

where $\delta \in [0, 1]$ denotes the rate of depreciation. $\Phi(\cdot)$ captures the existence of investment adjustment costs in the capital accumulation process (its properties will be discussed later).

2.4 The monetary authorities

We assume that monetary policy is described by a standard interest rate rule (Henderson-McKibbin-Taylor):

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1 - \rho_r) [\log(\bar{R}) + \kappa_\pi(\log(\pi_t) - \log(\pi^*)) + \kappa_y(\log(y_t) - \log(y^*))] + \varepsilon_{R,t}$$

where the output, y^* and inflation, π^* , targets are the steady state level of output and inflation respectively. $\varepsilon_{R,t} \rightsquigarrow \mathcal{N}(0, \sigma_R^2)$ is a monetary policy shock.

We have also considered the case where monetary policy follows an exogenous money supply rule, with money evolving according to

$$M_t = \exp(\mu_t)M_{t-1} \quad (9)$$

where the growth rate of the money supply μ_t is assumed to follow an exogenous stochastic process.

2.5 The government

The government purchases the domestic final good using lump sum taxes ($g_t = \tau_t$). Government expenditure, g_t , are assumed to follow an exogenous stochastic process whose properties will be defined later.

3 Parametrization

For comparison purposes, the parametrization of the model relies heavily on CEE, 2005. The model is parameterized on US quarterly data for the post WWII period. When necessary, the data are taken from the Federal Reserve Database.¹⁷ The parameters are reported in Table 5.

¹⁷URL:<http://research.stlouisfed.org/fred/>

Table 5: Calibration: Benchmark case

Discount factor	β	0.988
Habit persistence	ϑ	0.650
Inverse labor supply elasticity	σ_h	1.000
Money demand elasticity	σ_m	10.500
Capital elasticity of intermediate output	α	0.281
Parameter of markup	θ	0.850
Depreciation rate	δ	0.025
Adjustment costs parameter	φ	0.650
Probability of price resetting	γ	0.333
Price indexation	ν	0/1
Steady state money supply growth (gross)	μ	1.013
Share of government spending	g/y	0.200
Monetary policy, persistence	ρ_r	0.750
Monetary policy, reaction to inflation	κ_π	1.800
Monetary policy, reaction to output	κ_y	0.100

The capital accumulation function $\Phi(i_t, i_{t-1})$ is assumed to take the following form

$$\Phi(i_t, i_{t-1}) = \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t$$

The function $S(\cdot)$ satisfies $S(1) = S'(1) = 0$ and $S''(1) = \varphi > 0$. The investment adjustment cost parameter φ is then chosen so that the model can match the first order autocorrelation of output (0.88). This implies $\varphi = 0.650$. Note, however, that the same results obtain when we borrow the value of φ used in CEE ($\varphi = 2.5$), instead of calibrating it. The capital utilization function $a(u_t)$ satisfies $a(1) = 0$, $a''(1)/a'(1) = 1/\sigma_a$. We set $\sigma_a = 100$.

The parameters of the interest rate rule are $\rho_r=0.750$, $\kappa_y=0.100$ and $\kappa_\pi=1.800$. The probability that a firm gets a chance to reoptimize prices is set such that the average length of contracts is 3 quarters. This lies in the middle of the range of values usually found in the literature (See e.g. Christiano et al., 2005, or Smets and Wouters, 2007).

Two of the three shocks, namely the technology shock, $a_t = \log(A_t/\bar{A})$ and the fiscal shock, $\log(g_t)$ are assumed to follow independent, AR(1) processes with persistence parameters ρ_a, ρ_g respectively and standard deviation of innovations σ_a, σ_g respectively. These values are given in Table 6. The monetary policy shock is assumed to be iid. The process for government expenditures was estimated on historical data. σ_a and σ_R were selected so that the model matches the volatility of output (1.49) and the volatility of inflation (0.30).¹⁸

¹⁸These volatilities correspond to the the volatility of the cyclical component of output and inflation, as obtained by applying the HP filter.

Table 6: Shocks

	ρ	σ
Technology	0.9500	0.0055
Fiscal	0.9684	0.0104
Money	0.0000	0.0015

3.1 Information

We now specify the structure of information in the case of a signal extraction problem. We assume that while the agents may observe individual specific variables (such as their own consumption, technology shock, capital stock and so on) they can only imperfectly estimate the *true* aggregate state of the economy. Moreover, we assume that the agents learn gradually about the true state using the Kalman filter, based on a set of signals on aggregate variables. Without loss of generality we can assume that some of the aggregate variables may be perfectly observed, some other may not be observed at all and yet some other may be observed with error. For any mis-measured variable x , we assume that

$$x_t^* = x_t^T + \gamma \eta_t$$

where x_t^T denotes the true value of the variable, γ is a vector of endogenously determined coefficients and η_t is a vector of noises¹⁹ that satisfies $E(\eta_t) = 0$ for all t ; $E(\eta_t \varepsilon_{a,t}) = E(\eta_t \varepsilon_{g,t}) = E(\eta_t \varepsilon_{\mu,t}) = 0$; and

$$E(\eta_t \eta_k) = \begin{cases} \Sigma_{\eta\eta} & \text{if } t = k \\ 0 & \text{Otherwise} \end{cases}$$

This specification is consistent with the results reported above regarding the errors-in-variables properties of the money stock announcements.

Knowledge of the *aggregate* state of the economy matters for the agents because individual price setting depends on expectations of future nominal marginal cost and marginal revenue, which in turn depend on future aggregate prices, wages and so on.

An important principle is that the informational constraints are sensible in terms of location, timing and amount of noise. Recall that the objective of our paper is to examine the effects of a monetary policy shock. We cannot allow the *true* value of this shock to be perfectly observable as this does away with the signal extraction problem. But we cannot assume either (without straining credibility) that the agents do not observe monetary aggregates at all (or that they

¹⁹Note that the measurement error in any endogenous variable is a function of the noise in this as well as all other noisy variables. In particular, it is an endogenously determined linear combination of all the noisy processes present in the model. Consequently, the individual $\eta_{i,t}$ cannot be given a structural interpretation.

do so with substantial time lags), a common feature of the early vintage of the flexible price, rational expectations models. Based on the findings of the previous section we assume that while information on monetary aggregates is readily available, observations of the current or recent monetary data (the preliminary figures) are ridden with measurement error.

We assume that the agents receive noisy signals on the variables²⁰, $\{\pi, \mu, y\}$, while they perfectly observe the nominal interest rate, R . In order to capture the process of periodic data revision we include signals on lagged, along side the signals on the current variables. In particular, we specify that in period t the agents receive noisy signals $\{\pi_{t|t}, \pi_{t-1|t}, \pi_{t-2|t}, \mu_{t|t}, \mu_{t-1|t}, \mu_{t-2|t}, y_{t|t}, y_{t-1|t}, y_{t-2|t}\}$. We calibrate the noise in the signals of inflation, money and output so that the variances of the –endogenously determined in the model– measurement error in these variables match those in the Real Time Data Set.²¹ In particular, for $i = 0, 1, 2, x = \pi, \mu, y$ let $\zeta_{t-i|t}^{x,M} = x_{t-i|t} - x_{t-i}$ be the discrepancy (measurement error) between the perception that agents have in period t about the value of variable x in period $t - i$ and the true value of that variable, x_{t-i} , in the model. Let $\zeta_{t-i|t}^{x,D}$ be the discrepancy between the vintage t release of $x_{t-i|t}$ and the final release, $x_{t-i|T}$ as found in the data. We select the values of σ_η^2 in order to match the variances of the vectors ζ^M and ζ^D . Table 7 reports the standard deviation of these revisions in the data. Note that in the

Table 7: Volatility of revisions

	$\zeta_{t t}^D$	$\zeta_{t-1 t}^D$	$\zeta_{t-2 t}^D$
Output Growth	0.5419	0.5319	0.5189
Inflation	0.2211	0.2335	0.2301
Money Growth	0.4546	0.4367	0.3906

model, the money demand equation imposes a restriction on the relationship of the variances of revisions in inflation and money growth (given R and the noise on output). It turns out that in the model the two variances are about the same size, while they differ in the data, with the variance of inflation measurement errors being about half the size of that of money growth. Hence, it is impossible for the model, as it stands, to match both moments. We have opted to make things harder for our model by matching the smaller of the measurement error variances in the data (that of inflation) so that the model has on average less noise than real time data²²

²⁰Note that the choice of the noisy variables is not restrictive. We could add any other variables to the list without affecting materially the results as long as we satisfied the requirement that signal extraction remained meaningful in the model.

²¹In the case of output, we actually match the volatility of revisions in output growth.

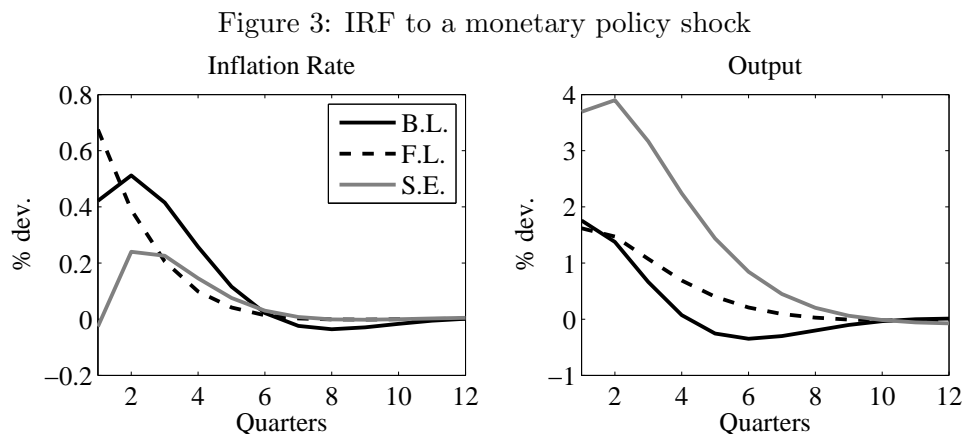
²²As Dellas, 2006, has shown, increasing the noise increases the amount of inertia in the model. Actually, the exercise here represents a critical departure from Dellas, 2006. Dellas demonstrates that the NK model with a signal extraction problem may generate persistence in inflation and output. But the key question is not whether such a model can generate inflation inertia (this is already hinted in other related work, such as Svensson and Woodford, 2003) but whether it can do so under plausible informational assumptions. Dellas' example requires a

and imposed the same amount of noise on inflation and money growth.

4 The results

The model is log-linearized around its deterministic steady state and then solved. The solution method for the case in which the agents solve a signal extraction problem is to be found in a technical appendix available at the authors' web page.

Figure 3 presents the response of inflation, output, the nominal and the real interest rate to a 1% shock to the interest rate policy rule under three model specifications: (i) The standard, forward looking, NK model (forward looking); (ii) the version with dynamic indexation (backward looking); and (iii) the forward looking version with signal extraction. In *all three cases*, the model includes three real rigidities, namely, habit persistence, variable capital utilization and investment adjustment costs.²³



Note: Three model specifications: a) B.L.: Backward looking, b) F.L.: Forward looking, c) S.E.: Signal extraction.

The three versions differ considerably regarding the dynamics of output, and in particular, inflation. The version with forward looking agents cannot generate inflation (or output) inertia. This finding confirms the well known fact (see Collard and Dellas, 2005) that price staggering does not suffice to produce plausible dynamics. It also demonstrates that real rigidities alone cannot help the NK model deliver the hump either²⁴. For instance, there is a widely held view

large amount of noise on all nominal variables, including R.

²³Using capital in place of investment adjustment costs makes no difference for the behavior of the model with signal extraction.

²⁴Notice that the degree of inflation inertia could be increased further by increasing the amount of real rigidities, by introducing additional, commonly used inertial features such as wage stickiness, expenditure lags and so on. We have not done so because it would not do affect the main point of this paper.

that habit persistence is sufficient to generate inertial behavior. As Figure 3 shows (see also Collard and Dellas, 2005) this is not the case. On the other hand, it must be emphasized that real rigidities are important in order to generate sufficient inertia under either backward indexation or signal extraction. This is illustrated in the technical Appendix (available at the authors' web pages) which shuts all real rigidities down.

The model with dynamic indexation does generate a hump in inflation as does the model with signal extraction. This finding confirms the speculation made in the introduction that imperfect information could serve as a substitute for dynamic indexation in helping the NK exhibit inflation inertia (a hump) following a monetary policy shock. Note that the former model also generates greater inertia.

The imperfect information model also exhibits more inertia in output than the model with dynamic indexation²⁵. This confirms the importance of signal extraction as an inertial mechanism. But at the same time, relative to the model with dynamic indexation, the imperfect information model overstates the response of output while understating that of inflation.

Before concluding this section let us briefly report on the role played by price rigidity. Is it possible that a pure Lucas version of the paper (namely a version with flexible prices) could give rise to similar dynamics? We have found that under flexible prices, the model cannot produce plausible inflation dynamics unless one is willing to accept levels of informational frictions (very large noise on monetary aggregates, including the nominal interest rate) that do not seem plausible.

5 Conclusions

The introduction of the imperfect information, rational expectations paradigm was followed by intensive debate regarding the role of misperceived money (the key ingredient of the flexible price version) relative to that of unanticipated money (the key ingredient of the fixed price version). This debate was settled at the time conclusively in favor of the unanticipated money version. In this paper we have attempted to revisit this issue, both empirically and theoretically. At the empirical front, we have updated the work of Kavajesz and Collins, 1995 and established that the difference between initial and subsequent releases of the money stock seems to correspond well to the theoretically correct measure of misperceived money. And that misperceptions are as sizeable –quantitatively– as unanticipated movements in money, and also have statistically significant and similar quantitative effects on economics activity during the full sample as well as in the pre 1982 period.

²⁵Adding also nominal wage rigidity would allow the latter model to exhibit a hump, as in CEE, 2005.

At the theoretical front, we have established that adding imperfect information about monetary shocks to the standard version of the New Keynesian model (that is, to that without dynamic indexation) allows the model to exhibit inflation inertia. Consequently, imperfect information can serve as a substitute for other schemes (such as dynamic indexation or myopic agents) that have been introduced in order to improve the dynamic properties of the NK model.

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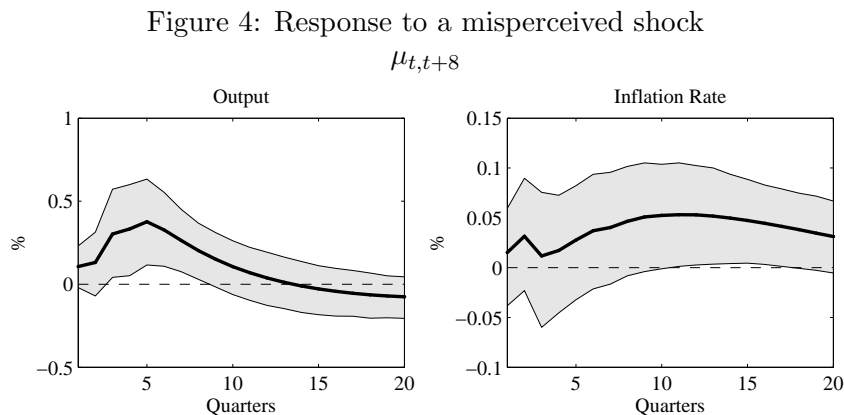
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6 NOT-FOR-PUBLICATION APPENDIX

6.1 Robustness Analysis

6.1.1 Using intermediate revisions



Note: The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VARX model.

Table 8: The effects of unperceived and unanticipated money, F-Tests

	(p, n)	Output		(p, n)	Inflation Rate	
		$\mu_{t,t+4}$	ε_t		$\mu_{t,t+4}$	ε_t
<i>1966Q1-2002Q4</i>	(5,7)	1.7625 [0.0916]	2.3611 [0.0217]	(3,8)	4.5848 [0.0000]	1.9374 [0.0534]
<i>1966Q1-1982Q3</i>	(2,2)	1.8151 [0.1559]	1.2929 [0.2867]	(1,8)	2.4231 [0.0294]	1.9230 [0.0808]
<i>1982Q4-2002Q4</i>	(3,1)	0.8327 [0.4393]	0.0682 [0.9342]	(4,8)	2.1174 [0.0467]	1.7076 [0.1139]
<i>1966Q1-2002Q4</i>	(5,7)	1.7770 [0.0878]	–	(3,8)	4.2870 [0.0001]	–
<i>1966Q1-1982Q3</i>	(2,2)	1.8185 [0.1545]	–	(1,8)	2.0073 [0.0612]	–
<i>1982Q4-2002Q4</i>	(3,1)	1.0347 [0.3608]	–	(4,8)	1.5380 [0.1574]	–
<i>1966Q1-2002Q4</i>	(5,7)	–	2.4132 [0.0187]	(3,8)	–	1.5319 [0.1439]
<i>1966Q1-1982Q3</i>	(2,2)	–	1.2689 [0.2941]	(1,8)	–	1.4609 [0.1927]
<i>1982Q4-2002Q4</i>	(3,1)	–	0.2434 [0.7846]	(4,8)	–	1.1046 [0.3747]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

Table 9: The effects of unperceived and unanticipated money, F-Tests (VAR-CEE, Δ)

	(p, n)	Output		(p, n)	Inflation Rate	
		$\mu_{t,t+8}$	ε_t		$\mu_{t,t+8}$	ε_t
<i>1966Q1-2002Q4</i>	(1,2)	2.3756 [0.0728]	7.2442 [0.0002]	(5,1)	0.9971 [0.3717]	0.6351 [0.5315]
<i>1966Q1-1982Q3</i>	(1,2)	1.6972 [0.1785]	4.5005 [0.0069]	(2,1)	0.6714 [0.5151]	0.9622 [0.3884]
<i>1982Q4-2002Q4</i>	(3,1)	1.8803 [0.1605]	0.1242 [0.8834]	(4,8)	2.9029 [0.0082]	1.6253 [0.1355]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

6.1.2 Using the unforecastable component of misperceptions

Table 10: The effects of unperceived and unanticipated money

	Output			Inflation Rate		
	(p, n)	$\mu_{t,t+8}$	ε_t	(p, n)	$\mu_{t,t+8}$	ε_t
<i>1966Q1–2002Q4</i>	(3,2)	1.5034 [0.2167]	5.3580 [0.0016]	(5,1)	0.3699 [0.6915]	0.6260 [0.5363]
<i>1966Q1–1982Q3</i>	(2,2)	1.9133 [0.1386]	4.5265 [0.0067]	(2,1)	0.6596 [0.5211]	0.9915 [0.3775]
<i>1982Q4–2002Q4</i>	(3,1)	0.9801 [0.3807]	0.0669 [0.9353]	(4,8)	3.4918 [0.0023]	1.9805 [0.0636]
	(p, n)	$\mu_{t,T}$	ε_t	(p, n)	$\mu_{t,T}$	ε_t
<i>1966Q1–2002Q4</i>	(3,2)	5.1373 [0.0022]	6.8477 [0.0003]	(5,7)	2.9175 [0.0053]	1.3708 [0.2166]
<i>1966Q1–1982Q3</i>	(3,2)	4.4059 [0.0079]	5.5018 [0.0024]	(2,1)	0.0699 [0.9325]	0.8144 [0.4482]
<i>1982Q4–2002Q4</i>	(3,1)	0.1810 [0.8348]	0.1483 [0.8625]	(8,2)	3.1157 [0.0336]	0.8265 [0.4850]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock. μ_t is the residual from the forecasting regressions reported in Table 3

6.2 Alternative VAR representation: Money Supply Shocks

Table 11: Properties of misperceived and unanticipated money growth shocks

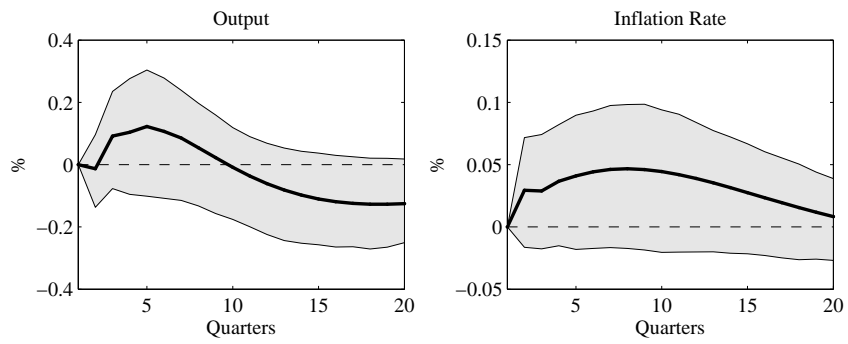
	1966Q1–2002Q4			1966Q1–1982Q3			1982Q4–2002Q4		
	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$
$\mu_{t,t+1}$	0.13	-0.11	0.05	0.10	-0.08	0.08	0.14	-0.12	0.04
$\mu_{t,t+2}$	0.20	0.06	-0.09	0.19	-0.10	0.08	0.20	0.16	-0.19
$\mu_{t,t+4}$	0.33	0.11	-0.07	0.38	0.23	-0.01	0.28	-0.11	-0.16
$\mu_{t,t+8}$	0.37	-0.05	-0.02	0.38	-0.03	-0.04	0.36	-0.09	-0.13
$\mu_{t,T}$	0.47	-0.06	-0.02	0.57	-0.05	0.05	0.36	-0.11	-0.21
ε_t	1.02	-0.03	1.00	0.59	-0.05	1.00	1.06	-0.03	1.00

Table 12: The effects of unperceived and unanticipated money, F-Tests

	(p, n)	Output		Inflation Rate		
		$\mu_{t,t+8}$	ε_t	$\mu_{t,t+8}$	ε_t	
<i>1966Q1–2002Q4</i>	(3,2)	1.2918	1.0259	(5,1)	1.0315	1.2083
		[0.2800]	[0.3834]		[0.3594]	[0.3020]
<i>1966Q1–1982Q3</i>	(2,2)	3.3839	6.3672	(1,1)	0.6086	1.0103
		[0.0247]	[0.0009]		[0.5476]	[0.3705]
<i>1982Q4–2002Q4</i>	(3,1)	1.9844	0.4449	(8,2)	4.2348	1.9693
		[0.1455]	[0.6427]		[0.0092]	[0.1293]
	(p, n)	$\mu_{t,T}$	ε_t	(p, n)	$\mu_{t,T}$	ε_t
<i>1966Q1–2002Q4</i>	(3,2)	2.7596	1.1293	(5,1)	0.6040	1.2035
		[0.0448]	[0.3397]		[0.5482]	[0.3035]
<i>1966Q1–1982Q3</i>	(3,2)	2.6330	5.4037	(1,1)	0.0254	0.8974
		[0.0598]	[0.0026]		[0.9750]	[0.4133]
<i>1982Q4–2002Q4</i>	(3,1)	0.5065	0.2992	(3,8)	3.0944	1.5677
		[0.6049]	[0.7424]		[0.0052]	[0.1522]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

Figure 5: Response to an unanticipated shock



Note: The shaded area depicts the 95% confidence interval obtained from 1000 bootstraps of the VAR model.

6.3 Use of Successive Vintages in the Computation of the Initial Release

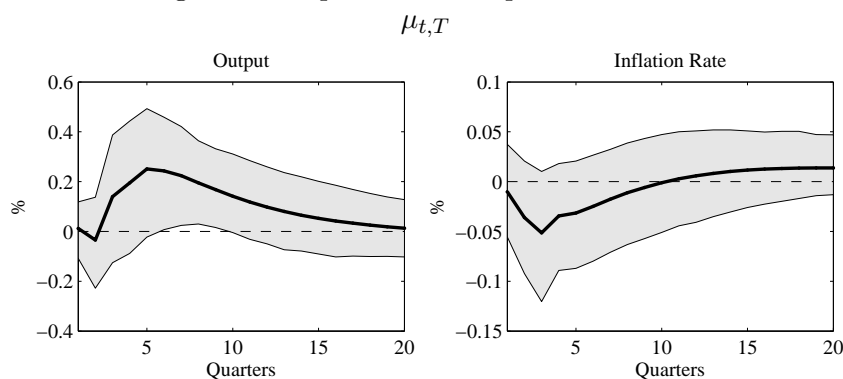
Table 13: Properties of misperceived and unanticipated money growth shocks

	1966Q1–2002Q4			1966Q1–1982Q3			1982Q4–2002Q4		
	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$
$\mu_{t,t+1}$	0.64	-0.51	0.14	0.92	-0.50	0.13	0.25	-0.57	-0.11
$\mu_{t,t+2}$	0.65	-0.03	-0.00	0.92	0.01	-0.05	0.26	-0.37	-0.13
$\mu_{t,t+4}$	0.65	-0.05	0.04	0.92	-0.01	-0.05	0.26	-0.27	-0.12
$\mu_{t,t+8}$	0.54	-0.11	0.08	0.69	-0.09	0.08	0.39	-0.16	-0.13
$\mu_{t,T}$	0.64	-0.14	0.09	0.85	-0.14	0.06	0.39	-0.15	-0.06
ε_t	0.35	-0.00	1.00	0.25	0.13	1.00	0.38	-0.03	1.00

Table 14: Errors-in-Variables

	1966Q1–2002Q4	1966Q1–1982Q3	1982Q4–2002Q4
$\mu_{t,t+4}$	0.1359 [0.8730]	0.8628 [0.4267]	0.1445 [0.8657]
$\mu_{t,t+8}$	0.1818 [0.8340]	0.2563 [0.7747]	0.0468 [0.9543]
$\mu_{t,T}$	0.0120 [0.9881]	0.0684 [0.9340]	0.0910 [0.9131]

Figure 6: Response to a misperceived shock



Note: The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VARX model.

Table 15: Forecasting regressions ($\mu_{t,T}$)

Cst.	R_t	$\Delta S\&P$	ΔY	π	D.W.	F-Test
<i>1966Q1–2002Q4</i>						
0.0365 (0.1568)	-3.2451 (8.8939)	-1.3176 (0.8741)	3.8235 (7.0573)	1.8466 (11.6453)	2.27	0.63
0.0462 (0.1290)	-2.4740 (6.7568)	–	–	–	2.28	0.71
0.0219 (0.0544)	–	-1.2426 (0.8574)	–	–	2.28	0.15
-0.0163 (0.0673)	–	–	3.0679 (6.4877)	–	2.27	0.64
0.0140 (0.1012)	–	–	–	-1.0729 (8.5536)	2.28	0.90
<i>1966Q1–1982Q3</i>						
-0.1090 (0.3639)	-8.7252 (14.7057)	-3.2485 (1.8058)	10.4133 (11.5559)	15.3164 (22.9623)	2.23	0.42
0.0290 (0.2496)	-2.2164 (11.3295)	–	–	–	2.28	0.85
-0.0063 (0.1031)	–	-2.7988 (1.7166)	–	–	2.26	0.11
-0.0475 (0.1178)	–	–	5.8873 (9.8320)	–	2.26	0.55
-0.0518 (0.2690)	–	–	–	2.5256 (17.1161)	2.27	0.88
<i>1982Q4–2002Q4</i>						
0.0904 (0.1412)	2.0987 (11.0291)	-0.1484 (0.7562)	-8.2422 (8.9507)	-6.4981 (19.5374)	2.29	0.92
0.0421 (0.1281)	-1.5471 (7.8900)	–	–	–	2.29	0.85
0.0225 (0.0480)	–	-0.1550 (0.7249)	–	–	2.29	0.83
0.0722 (0.0746)	–	–	-7.5534 (8.4675)	–	2.28	0.38
0.0388 (0.1007)	–	–	–	-3.1695 (14.1626)	2.29	0.82

Note: R = Federal fund rate, $\Delta S\&P$ = changes in the $S\&P$ stock market index, ΔY = changes in GDP, π inflation rate (GDP Deflator). Standard deviations in parenthesis. F-Test column reports the p-value associated with the F-Test of global significance.

Table 16: The Effects of Unperceived and Unanticipated Money, F-Tests

	Output			Inflation Rate		
	(p, n)	$\mu_{t,T}$	ε_t	(p, n)	$\mu_{t,T}$	ε_t
<i>1966Q1-2002Q4</i>	(5,2)	2.2975	4.1524	(5,1)	0.3044	0.7582
		[0.0807]	[0.0076]		[0.7381]	[0.4706]
<i>1966Q1-1982Q3</i>	(2,2)	1.7890	3.3471	(2,1)	0.3134	0.7728
		[0.1605]	[0.0258]		[0.7322]	[0.4667]
<i>1982Q4-2002Q4</i>	(3,1)	0.2243	0.1205	(5,3)	1.7119	1.1000
		[0.7997]	[0.8867]		[0.1594]	[0.3651]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

6.4 Alternative Measures of Unanticipated Money Shocks

6.4.1 A four equation VAR with exogenous money shocks

Table 17: Properties of misperceived and unanticipated money growth shocks

	1966Q1–2002Q4			1966Q1–1982Q3			1982Q4–2002Q4		
	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$
$\mu_{t,t+1}$	0.13	-0.11	0.05	0.10	-0.08	0.08	0.14	-0.12	0.04
$\mu_{t,t+2}$	0.20	0.06	-0.09	0.19	-0.10	0.08	0.20	0.16	-0.19
$\mu_{t,t+4}$	0.33	0.11	-0.07	0.38	0.23	-0.01	0.28	-0.11	-0.16
$\mu_{t,t+8}$	0.37	-0.05	-0.02	0.38	-0.03	-0.04	0.36	-0.09	-0.13
$\mu_{t,T}$	0.47	-0.06	-0.02	0.57	-0.05	0.05	0.36	-0.11	-0.21
ε_t	1.02	-0.03	1.00	0.59	-0.05	1.00	1.06	-0.03	1.00

Table 18: The effects of unperceived and unanticipated money, F-Tests

	(p, n)	Output		Inflation Rate		
		$\mu_{t,t+8}$	ε_t	$\mu_{t,t+8}$	ε_t	
<i>1966Q1–2002Q4</i>	(3,2)	1.2918	1.0259	(5,1)	1.0315	1.2083
		[0.2800]	[0.3834]		[0.3594]	[0.3020]
<i>1966Q1–1982Q3</i>	(2,2)	3.3839	6.3672	(1,1)	0.6086	1.0103
		[0.0247]	[0.0009]		[0.5476]	[0.3705]
<i>1982Q4–2002Q4</i>	(3,1)	1.9844	0.4449	(8,2)	4.2348	1.9693
		[0.1455]	[0.6427]		[0.0092]	[0.1293]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

6.4.2 A four equation VAR estimated with real time data

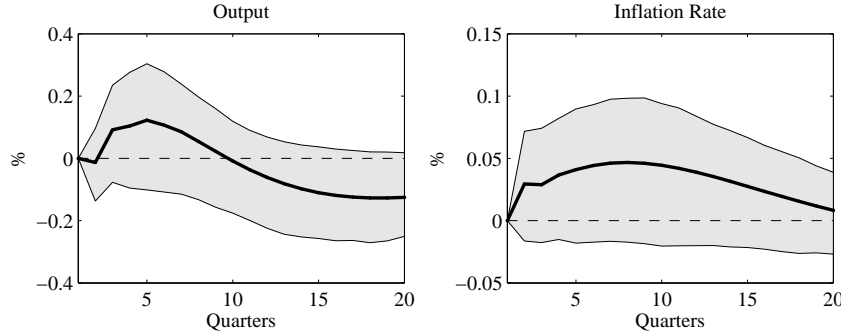
In this section, the unanticipated money shock is constructed as follows. We estimate a sequence of real-time VARs for the output, inflation, the federal fund rate and money growth for a sample running from 1960:Q1 until the quarter of the vintage under consideration. We then use the data from the last vintage to compute the first release of money supply growth for the next quarter. For instance, we estimate the VAR using data released in 1982:Q3 and build the forecast of money growth for 1982:Q4 using this VAR. We then compute the difference between this forecast and the first release of money growth as reported in the Real Time Data Set to obtain the unanticipated component.

Table 19: The effects of unperceived and unanticipated money, F-Tests

	Output			Inflation Rate		
	(p, n)	$\mu_{t,T}$	ε_t	(p, n)	$\mu_{t,T}$	ε_t
1966Q1–2002Q4	(3,2)	2.7596	1.1293	(5,1)	0.6040	1.2035
		[0.0448]	[0.3397]		[0.5482]	[0.3035]
1966Q1–1982Q3	(3,2)	2.6330	5.4037	(1,1)	0.0254	0.8974
		[0.0598]	[0.0026]		[0.9750]	[0.4133]
1982Q4–2002Q4	(3,1)	0.5065	0.2992	(3,8)	3.0944	1.5677
		[0.6049]	[0.7424]		[0.0052]	[0.1522]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

Figure 7: Response to an unanticipated shock



Note: The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VAR.

Table 20: Properties of misperceived and unanticipated money growth shocks (Recursive RT-VAR)

	1966Q1–2002Q4			1966Q1–1982Q3			1982Q4–2002Q4		
	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$	σ	ρ	$\rho(\cdot, \varepsilon_t)$
$\mu_{t,t+1}$	0.13	-0.11	0.04	0.10	-0.08	0.15	0.14	-0.12	-0.02
$\mu_{t,t+2}$	0.20	0.06	0.12	0.19	-0.10	0.08	0.20	0.16	0.16
$\mu_{t,t+4}$	0.33	0.11	0.00	0.38	0.23	0.10	0.28	-0.11	-0.09
$\mu_{t,t+8}$	0.37	-0.05	0.09	0.38	-0.03	0.12	0.36	-0.09	0.07
$\mu_{t,T}$	0.47	-0.06	-0.02	0.57	-0.05	-0.08	0.36	-0.11	0.04
ε_t	1.39	-0.02	1.00	1.22	-0.17	1.00	1.52	0.06	1.00

Table 21: The effects of unperceived and unanticipated money, F-Tests (Recursive RT-VAR)

	Output			Inflation Rate		
	(p, n)	$\mu_{t,t+8}$	ε_t	(p, n)	$\mu_{t,t+8}$	ε_t
<i>1966Q1-2002Q4</i>	(3,2)	2.4656 [0.0650]	1.6888 [0.1724]	(5,1)	1.2798 [0.2815]	0.2869 [0.7510]
<i>1966Q1-1982Q3</i>	(2,2)	6.1884 [0.0010]	5.4264 [0.0024]	(1,1)	0.4382 [0.6472]	0.3376 [0.7148]
<i>1982Q4-2002Q4</i>	(3,1)	1.5892 [0.2114]	0.3339 [0.7172]	(6,2)	2.6311 [0.0579]	0.2717 [0.8456]
	(p, n)	$\mu_{t,T}$	ε_t	(p, n)	$\mu_{t,T}$	ε_t
<i>1966Q1-2002Q4</i>	(3,2)	4.9390 [0.0028]	2.7609 [0.0446]	(5,7)	2.9408 [0.0049]	0.3171 [0.9583]
<i>1966Q1-1982Q3</i>	(1,2)	9.7858 [0.0000]	8.8322 [0.0001]	(1,1)	0.0054 [0.9946]	0.4267 [0.6546]
<i>1982Q4-2002Q4</i>	(3,1)	0.2825 [0.7547]	0.2384 [0.7885]	(6,2)	1.7350 [0.1691]	0.1767 [0.9118]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

6.5 Splitting the sample in 1979

Table 22: The effects of unperceived and unanticipated money, F-Tests

	Output			Inflation Rate		
	(p, n)	$\mu_{t,t+8}$	ε_t	(p, n)	$\mu_{t,t+8}$	ε_t
<i>1966Q1–2002Q4</i>	(3,2)	2.6082 [0.0543]	6.0790 [0.0007]	(5,1)	0.9971 [0.3717]	0.6351 [0.5315]
<i>1966Q1–1982Q3</i>	(2,2)	3.8031 [0.0153]	5.2938 [0.0029]	(2,1)	0.6714 [0.5151]	0.9622 [0.3884]
<i>1982Q4–2002Q4</i>	(3,1)	1.4962 [0.2314]	0.0813 [0.9220]	(4,8)	2.9029 [0.0082]	1.6253 [0.1355]
	(p, n)	$\mu_{t,T}$	ε_t	(p, n)	$\mu_{t,T}$	ε_t
<i>1966Q1–2002Q4</i>	(3,2)	4.7934 [0.0033]	6.8977 [0.0002]	(5,7)	3.4001 [0.0015]	1.4246 [0.1935]
<i>1966Q1–1982Q3</i>	(3,2)	4.8187 [0.0050]	5.9241 [0.0015]	(2,1)	0.0949 [0.9096]	0.8072 [0.4513]
<i>1982Q4–2002Q4</i>	(3,1)	0.2592 [0.7724]	0.1572 [0.8549]	(4,8)	2.2657 [0.0336]	1.0958 [0.3841]

Note: p-values in brackets (they correspond to the F-test of the significance of each type of shock). (p, n) refers to the number of lags of the endogenous variable, p , and of the monetary shocks, n . μ_t is unperceived and ε_t is the unanticipated money shock.

7 Model: Robustness Check

Figure 8: IRF to a monetary Shock: No real rigidities

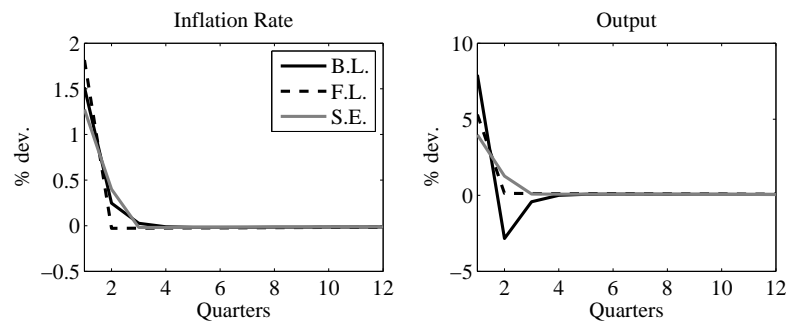
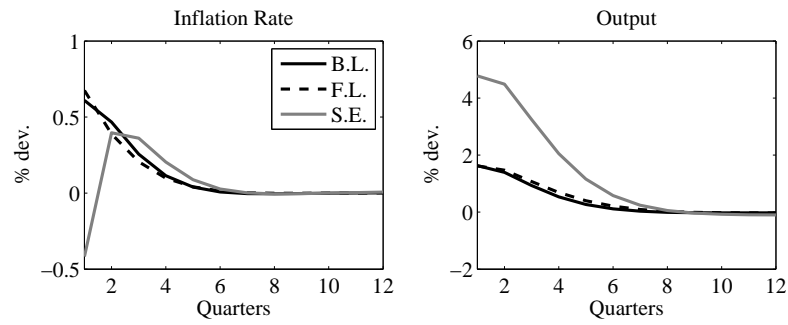
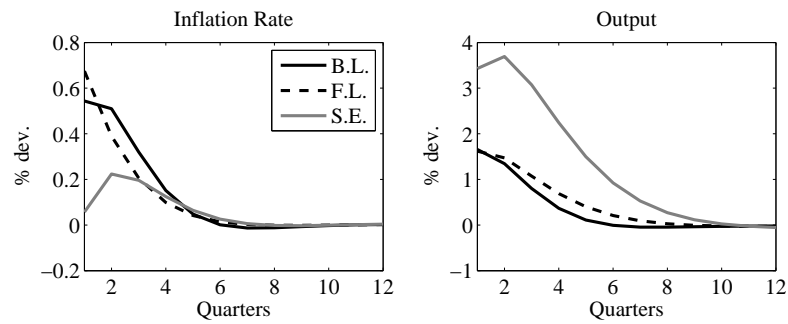


Figure 9: IRF to a monetary Shock: Degree of Indexation

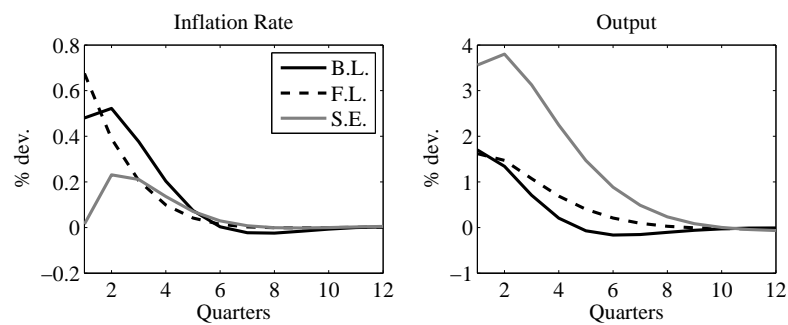
(a) Degree of Indexation: $\nu = 0.25$



(b) Degree of Indexation: $\nu = 0.50$



(c) Degree of Indexation: $\nu = 0.75$



7.1 Solution Method

Let's consider the following system

$$M_{cc}Y_t = M_{cs} \begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + M_{ce} \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \quad (10)$$

$$M_{ss0} \begin{pmatrix} X_{t+1}^b \\ X_{t+1|t}^f \end{pmatrix} + M_{ss1} \begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + M_{se1} \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} = M_{sc0}Y_{t+1|t} + M_{sc1}Y_t + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \quad (11)$$

$$S_t = C^0 \begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + C^1 \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} + v_t \quad (12)$$

Y is a vector of n_y control variables, S is a vector of n_s signals used by the agents to form expectations, X^b is a vector of n_b predetermined (backward looking) state variables (including shocks to fundamentals), X^f is a vector of n_f forward looking state variables, finally u and v are two Gaussian white noise processes with variance-covariance matrices Σ_{uu} and Σ_{vv} respectively and $E(uv') = 0$.

$X_{t+i|t} = E(X_{t+i}|\mathcal{I}_t)$ for $i \geq 0$ and where \mathcal{I}_t denotes the information set available to the agents at the beginning of period t . The information set available to the agents consists of *i*) the structure of the model and *ii*) the history of the observable signals they are given in each period:

$$\mathcal{I}_t = \{S_{t-j}, j \geq 0, M_{cc}, M_{cs}, M_{ce}, M_{ss0}, M_{ss1}, M_{sc0}, M_{sc1}, M_{se1}, M_e, C^0, C^1, \Sigma_{uu}, \Sigma_{vv}\}$$

Therefore, it is when we specify the signals that we may impose the information structure of the agents.

Before solving the system, note that, from (10), we have

$$Y_t = B^0 \begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + B^1 \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \quad (13)$$

where $B^0 = M_{cc}^{-1}M_{cs}$ and $B^1 = M_{cc}^{-1}M_{ce}$, such that

$$Y_{t|t} = B \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \quad (14)$$

with $B = B^0 + B^1$.

Step 1: We first solve for the expected system:

$$M_{ss0} \begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} + (M_{ss1} + M_{se1}) \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} = M_{sc0} Y_{t+1|t} + M_{sc1} Y_{t|t} \quad (15)$$

Plugging (14) in (15), we get

$$\begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} = W \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \quad (16)$$

where

$$W = -(M_{ss0} - M_{sc0}B)^{-1} (M_{ss1} + M_{se1} - M_{sc1}B)$$

After getting the Jordan form associated to (16) and applying standard methods for eliminating bubbles, we get

$$X_{t|t}^f = GX_{t|t}^b$$

From which we get

$$X_{t+1|t}^b = (W_{bb} + W_{bf}G)X_{t|t}^b = W^b X_{t|t}^b \quad (17)$$

$$X_{t+1|t}^f = (W_{fb} + W_{ff}G)X_{t|t}^b = W^f X_{t|t}^b \quad (18)$$

Step 2: We go back to the initial system to get and write

Then, (11) rewrites

$$\begin{aligned} M_{ss0} \begin{pmatrix} X_{t+1}^b \\ X_{t+1|t}^f \end{pmatrix} + M_{ss1} \begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} + M_{se1} \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} &= M_{sc0}B \begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} + M_{sc1}B^0 \begin{pmatrix} X_t^b \\ X_t^f \end{pmatrix} \\ &+ M_{sc1}B^1 \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \end{aligned}$$

Taking expectations, we have

$$\begin{aligned} M_{ss0} \begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} + M_{ss1} \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} + M_{se1} \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} &= M_{sc0}B \begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} + M_{sc1}B^0 \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \\ &+ M_{sc1}B^1 \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \end{aligned}$$

Subtracting, we get

$$M_{ss0} \begin{pmatrix} X_{t+1}^b - X_{t+1|t}^b \\ 0 \end{pmatrix} + M_{ss1} \begin{pmatrix} X_t^b - X_{t|t}^b \\ X_t^f - X_{t|t}^f \end{pmatrix} = M_{sc1}B^0 \begin{pmatrix} X_t^b - X_{t|t}^b \\ X_t^f - X_{t|t}^f \end{pmatrix} + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \quad (19)$$

which rewrites

$$\begin{pmatrix} X_{t+1}^b - X_{t+1|t}^b \\ 0 \end{pmatrix} = W^c \begin{pmatrix} X_t^b - X_{t|t}^b \\ X_t^f - X_{t|t}^f \end{pmatrix} + M_{ss0}^{-1} \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \quad (20)$$

where, $W^c = -M_{ss0}^{-1}(M_{ss1} - M_{sc1}B^0)$. Hence, considering the second block of the above matrix equation, we get

$$W_{fb}^c(X_t^b - X_{t|t}^b) + W_{ff}^c(X_t^f - X_{t|t}^f) = 0$$

which gives

$$X_t^f = F^0 X_t^b + F^1 X_{t|t}^b$$

with $F^0 = -W_{ff}^c^{-1}W_{fb}^c$ and $F^1 = G - F^0$.

Now considering the first block we have

$$X_{t+1}^b = X_{t+1|t}^b + W_{bb}^c(X_t^b - X_{t|t}^b) + W_{bf}^c(X_t^f - X_{t|t}^f) + M^2 u_{t+1}$$

from which we get, using (17)

$$X_{t+1}^b = M^0 X_t^b + M^1 X_{t|t}^b + M^2 u_{t+1}$$

with $M^0 = W_{bb}^c + W_{bf}^c F^0$, $M^1 = W^b - M^0$ and $M^2 = M_{ss0}^{-1} M_e$.

We also have

$$S_t = C_b^0 X_t^b + C_t^0 X_t^f + C_b^1 X_{t|t}^b + C_f^1 X_{t|t}^f + v_t$$

from which we get

$$S_t = S^0 X_t^b + S^1 X_{t|t}^b + v_t$$

where $S^0 = C_b^0 + C_f^0 F^0$ and $S^1 = C_b^1 + C_f^1 F^1 + C_f^1 G$

Finally, we get

$$Y_t = B_b^0 X_t^b + B_t^0 X_t^f + B_b^1 X_{t|t}^b + B_f^1 X_{t|t}^f$$

from which we get

$$Y_t = \Pi^0 X_t^b + \Pi^1 X_{t|t}^b$$

where $\Pi^0 = B_b^0 + B_f^0 F^0$ and $\Pi^1 = B_b^1 + B_f^1 F^1 + B_f^1 G$

Filtering: Since our solution involves terms in $X_{t|t}^b$, we would like to compute this quantity. However, the only information we can exploit is a signal S_t that we described previously. We therefore use a Kalman filter approach to compute the optimal prediction of $X_{t|t}^b$.

In order to recover the Kalman filter, it is a good idea to think in terms of expectation errors. Therefore, let us define

$$\hat{X}_t^b = X_t^b - X_{t|t-1}^b$$

and

$$\widehat{S}_t = S_t - S_{t|t-1}$$

Note that since S_t depends on $X_{t|t}^b$, only the signal relying on $\widetilde{S}_t = S_t - S^1 X_{t|t}^b$ can be used to infer anything on $X_{t|t}^b$. Therefore, the policy maker revises its expectations using a linear rule depending on $\widetilde{S}_t^e = S_t - S^1 X_{t|t}^b$. The filtering equation then writes

$$X_{t|t}^b = X_{t|t-1}^b + K(\widetilde{S}_t^e - \widetilde{S}_{t|t-1}^e) = X_{t|t-1}^b + K(S^0 \widehat{X}_t^b + v_t)$$

where K is the filter gain matrix, that we would like to compute.

The first thing we have to do is to rewrite the system in terms of state–space representation. Since $S_{t|t-1} = (S^0 + S^1)X_{t|t-1}^b$, we have

$$\begin{aligned} \widehat{S}_t &= S^0(X_t^b - X_{t|t}^b) + S^1(X_{t|t}^b - X_{t|t-1}^b) + v_t \\ &= S^0 \widehat{X}_t^b + S^1 K(S^0 \widehat{X}_t^b + v_t) + v_t \\ &= S^* \widehat{X}_t^b + \nu_t \end{aligned}$$

where $S^* = (I + S^1 K)S^0$ and $\nu_t = (I + S^1 K)v_t$.

Now, consider the law of motion of backward state variables, we get

$$\begin{aligned} \widehat{X}_{t+1}^b &= M^0(X_t^b - X_{t|t}^b) + M^2 u_{t+1} \\ &= M^0(X_t^b - X_{t|t-1}^b - X_{t|t}^b + X_{t|t-1}^b) + M^2 u_{t+1} \\ &= M^0 \widehat{X}_t^b - M^0(X_{t|t}^b + X_{t|t-1}^b) + M^2 u_{t+1} \\ &= M^0 \widehat{X}_t^b - M^0 K(S^0 \widehat{X}_t^b + v_t) + M^2 u_{t+1} \\ &= M^* \widehat{X}_t^b + \omega_{t+1} \end{aligned}$$

where $M^* = M^0(I - KS^0)$ and $\omega_{t+1} = M^2 u_{t+1} - M^0 K v_t$.

We therefore end–up with the following state–space representation

$$\widehat{X}_{t+1}^b = M^* \widehat{X}_t^b + \omega_{t+1} \tag{21}$$

$$\widehat{S}_t = S^* \widehat{X}_t^b + \nu_t \tag{22}$$

For which the Kalman filter is given by

$$\widehat{X}_{t|t}^b = \widehat{X}_{t|t-1}^b + PS^{*'}(S^* PS^{*'} + \Sigma_{\nu\nu})^{-1}(S^* \widehat{X}_t^b + \nu_t)$$

But since $\widehat{X}_{t|t}^b$ is an expectation error, it is not correlated with the information set in $t - 1$, such that $\widehat{X}_{t|t-1}^b = 0$. The prediction formula for $\widehat{X}_{t|t}^b$ therefore reduces to

$$\widehat{X}_{t|t}^b = PS^{*'}(S^* PS^{*'} + \Sigma_{\nu\nu})^{-1}(S^* \widehat{X}_t^b + \nu_t) \tag{23}$$

where P solves

$$P = M^*PM^{*\prime} + \Sigma_{\omega\omega}$$

and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime}$

Note however that the above solution is obtained for a given K matrix that remains to be computed. We can do that by using the basic equation of the Kalman filter:

$$\begin{aligned} X_{t|t}^b &= X_{t|t-1}^b + K(\tilde{S}_t^e - \tilde{S}_{t|t-1}^e) \\ &= X_{t|t-1}^b + K(S_t - S^1X_{t|t}^b - (S_{t|t-1} - S^1X_{t|t-1}^b)) \\ &= X_{t|t-1}^b + K(S_t - S^1X_{t|t}^b - S^0X_{t|t-1}^b) \end{aligned}$$

Solving for $X_{t|t}^b$, we get

$$\begin{aligned} X_{t|t}^b &= (I + KS^1)^{-1}(X_{t|t-1}^b + K(S_t - S^0X_{t|t-1}^b)) \\ &= (I + KS^1)^{-1}(X_{t|t-1}^b + KS^1X_{t|t-1}^b - KS^1X_{t|t-1}^b + K(S_t - S^0X_{t|t-1}^b)) \\ &= (I + KS^1)^{-1}(I + KS^1)X_{t|t-1}^b + (I + KS^1)^{-1}K(S_t - (S^0 + S^1)X_{t|t-1}^b) \\ &= X_{t|t-1}^b + (I + KS^1)^{-1}K\hat{S}_t \\ &= X_{t|t-1}^b + K(I + S^1K)^{-1}\hat{S}_t \\ &= X_{t|t-1}^b + K(I + S^1K)^{-1}(S^*\hat{X}_t^b + \nu_t) \end{aligned}$$

where we made use of the identity $(I + KS^1)^{-1}K \equiv K(I + S^1K)^{-1}$. Hence, identifying to (23), we have

$$K(I + S^1K)^{-1} = PS^{*\prime}(S^*PS^{*\prime} + \Sigma_{\nu\nu})^{-1}$$

remembering that $S^* = (I + S^1K)S^0$ and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$, we have

$$K(I + S^1K)^{-1} = PS^{0\prime}(I + S^1K)'((I + S^1K)S^0PS^{0\prime}(I + S^1K)' + (I + S^1K)\Sigma_{vv}(I + S^1K)')^{-1}(I + S^1K)S^0$$

which rewrites as

$$\begin{aligned} K(I + S^1K)^{-1} &= PS^{0\prime}(I + S^1K)' \left[(I + S^1K)(S^0PS^{0\prime} + \Sigma_{vv})(I + S^1K)' \right]^{-1} \\ K(I + S^1K)^{-1} &= PS^{0\prime}(I + S^1K)'(I + S^1K)^{\prime-1}(S^0PS^{0\prime} + \Sigma_{vv})^{-1}(I + S^1K)^{-1} \end{aligned}$$

Hence, we obtain

$$K = PS^{0\prime}(S^0PS^{0\prime} + \Sigma_{vv})^{-1} \tag{24}$$

Now, recall that

$$P = M^*PM^{*\prime} + \Sigma_{\omega\omega}$$

Remembering that $M^* = M^0(I + KS^0)$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0'} + M^2\Sigma_{uu}M^{2'}$, we have

$$\begin{aligned} P &= M^0(I - KS^0)P [M^0(I - KS^0)]' + M^0K\Sigma_{vv}K'M^{0'} + M^2\Sigma_{uu}M^{2'} \\ &= M^0 \left[(I - KS^0)P(I - S^{0'}K') + K\Sigma_{vv}K' \right] M^{0'} + M^2\Sigma_{uu}M^{2'} \end{aligned}$$

Plugging the definition of K in the latter equation, we obtain

$$P = M^0 \left[P - PS^{0'}(S^0PS^{0'} + \Sigma_{vv})^{-1}S^0P \right] M^{0'} + M^2\Sigma_{uu}M^{2'} \quad (25)$$

Summary: We finally end-up with the system of equations:

$$X_{t+1}^b = M^0X_t^b + M^1X_{t|t}^b + M^2u_{t+1} \quad (26)$$

$$S_t = S_b^0X_t^b + S_b^1X_{t|t}^b + v_t \quad (27)$$

$$Y_t = \Pi_b^0X_t^b + \Pi_b^1X_{t|t}^b \quad (28)$$

$$X_t^f = F^0X_t^b + F^1X_{t|t}^b \quad (29)$$

$$X_{t|t}^b = X_{t|t-1}^b + K(S^0(X_t^b - X_{t|t-1}^b) + v_t) \quad (30)$$

$$X_{t+1|t}^b = (M^0 + M^1)X_{t|t}^b \quad (31)$$

to describe the dynamics of our economy.

This may be recasted as a standard state-space problem as

$$\begin{aligned} X_{t+1|t+1}^b &= X_{t+1|t}^b + K(S^0(X_{t+1}^b - X_{t+1|t}^b) + v_{t+1}) \\ &= (M^0 + M^1)X_{t|t}^b + K(S^0(M^0X_t^b + M^1X_{t|t}^b + M^2u_{t+1} - (M^0 + M^1)X_{t|t}^b) + v_{t+1}) \\ &= KS^0M^0X_t^b + ((I - KS^0)M^0 + M^1)X_{t|t}^b + KS^0M^2u_{t+1} + Kv_{t+1} \end{aligned}$$

Then

$$\begin{pmatrix} X_{t+1}^b \\ X_{t+1|t+1}^b \end{pmatrix} = M_X \begin{pmatrix} X_t^b \\ X_{t|t}^b \end{pmatrix} + M_E \begin{pmatrix} u_{t+1} \\ v_{t+1} \end{pmatrix}$$

where

$$M_X = \begin{pmatrix} M^0 & M^1 \\ KS^0M^0 & ((I - KS^0)M^0 + M^1) \end{pmatrix} \text{ and } M_E = \begin{pmatrix} M^2 & 0 \\ KS^0M^2 & K \end{pmatrix}$$

and

$$Y_t = M_Y \begin{pmatrix} X_t^b \\ X_{t|t}^b \end{pmatrix}$$

where

$$M_Y = \begin{pmatrix} \Pi_b^0 & \Pi_b^1 \end{pmatrix}$$