

# Optimal Monetary and Prudential Policies\*

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January 20, 2016

## Abstract

The recent financial crisis has highlighted the interconnectedness between macroeconomic and financial stability, raising questions about how to combine monetary and prudential policies. This paper offers a characterization of the jointly optimal monetary and prudential policies, setting the interest rate and bank-capital requirements. The source of financial fragility is the socially excessive risk taking by banks due to limited liability and deposit insurance. We characterize the conditions under which locally optimal (Ramsey) policy dedicates the prudential instrument to preventing inefficient risk-taking by banks; and the monetary instrument to dealing with the business cycle, with the two instruments co-varying negatively. Our analysis thus identifies circumstances that can validate the prevailing view among central bankers that standard interest-rate policy cannot serve as the first line of defense against financial instability. In addition, we provide conditions under which the two instruments might optimally co-move positively and counter-cyclically.

**JEL Class:** E32, E44, E52

**Keywords:** Prudential policy, Capital requirements, Monetary policy, Ramsey-optimal policies

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\*We are grateful to Toni Ahnert, Pierpaolo Benigno, Xavier Freixas, Nobu Kiyotaki, Franck Smets, Javier Suarez, Skander Van den Heuvel, and Mike Woodford, as well as to our discussants Ivan Jaccard, Lóránt Kaszab, Kebin Ma, Enrico Perotti, Margarita Rubio, Carlos Thomas, Cédric Tille, and Karl Walentin, for valuable suggestions. The editor and our referees made particularly helpful comments.

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# 1 Introduction

Monetary and prudential policies have traditionally been designed and analyzed in isolation from one another. The 2007-2009 financial crisis, however, has aroused interest in analyzing the interactions between these policies. Policymakers [e.g., Bernanke (2010), Yellen (2014)] have commented on the extent to which monetary policy can or should address concerns about financial stability. And policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012), Committee on International Economic Policy and Reform (2011)] have summarized alternative views about the potential substitutability or complementarity of these policies and the need for policy coordination. There is a general presumption that both policies will be counter-cyclical most of the time, as reflected in the “counter-cyclical capital buffer” [Basel Committee on Banking Supervision (2010)]. But policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)] have also envisioned scenarios that may put the two policies at odds with each other over the business cycle.

In this paper, we develop a New Keynesian model with banks and use it to study the optimal interactions between monetary and prudential policies. We focus on a prudential policy that sets a state-contingent capital requirement for banks. We first articulate a benchmark model in which the Tinbergen separation principle applies and prescribes a particular assignment of goals to the policy instruments: it is optimal to relegate the goal of financial stability to prudential policy and assign a mandate of macroeconomic stabilization to interest-rate policy.<sup>1</sup> In this model, the bank-capital requirement is optimally used to deter excessive risk taking by banks (countering the risk-taking temptations that arise from limited liability and deposit insurance). Monetary policy cannot deter risk-taking at all and optimally focuses on macroeconomic stabilization, by adjusting the policy rate in response to changes in macroeconomic conditions, including those that reflect optimal changes in prudential policy.<sup>2</sup> In this sense, our benchmark model is a stark rendition of what Smets (2013) calls “the modified Jackson Hole consensus.” Although our main goal is to fully articulate a model in which monetary policy has no effect on financial stability, we also consider a simple extension in which monetary policy does affect risk-taking incentives; and we highlight how this changes the key features of optimal policy interactions.

We depart in two main ways from other recent contributions that study the interactions between monetary and prudential policies from a normative perspective. First, in our model excessive risk taking arises from limited liability and involves the *type* (not necessarily the *volume*) of credit

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<sup>1</sup>In this context, the Tinbergen separation principle, as articulated by the Committee on International Economic Policy and Reform (2011) among others, refers to the idea that each goal should be pursued with a separate and dedicated instrument.

<sup>2</sup>To be clear, our paper is about optimal assignment of policy instruments, or optimal interactions between instruments, when both policies have the common (Ramsey) objective of maximizing welfare. De Paoli and Paustian (2013) study policy coordination in a different setting that involves separate prudential and monetary authorities with potentially different objectives. They consider optimal policy interactions under discretion as well as commitment.

extended by banks. By contrast, the literature typically views excessive risk taking in terms of the aggregate volume of credit, as we elaborate below. Second, our focus is on jointly Ramsey-optimal policies, i.e. on the state-contingent path for the two policy instruments that maximizes the representative household’s expected utility.<sup>3</sup> By contrast, the existing literature usually compares simple monetary and prudential policy rules with each other by computing welfare numerically, and does not address the issue of the optimal capital requirement in the steady state.<sup>4</sup>

Recent work on monetary policy and financial stability emphasizes the credit cycle and the “risk-taking channel” of monetary policy [as discussed, for example, in Borio and Zhu (2008)]. It typically views excessive risk taking in terms of the aggregate volume of credit. Angeloni and Faia (2013), for example, consider a link between the bank leverage ratio and the risk of bank runs; Christensen, Meh and Moran (2011) postulate an externality that links the riskiness of bank projects to the ratio of aggregate credit to GDP. While abstracting from monetary policy, a number of other contributions [e.g., Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010)] similarly view financial instability as the result of excessive borrowing. In these contributions, a pecuniary externality associated with a collateral constraint plays a central role: it makes an economic expansion increase the value of borrowers’ collateral and leads to excessive borrowing. A tax on debt can then make borrowers internalize the externality.<sup>5</sup> Benigno et al. (2011) add monetary policy to this setting and examine how it may pursue financial stability in addition to its conventional goals. They also consider the role of a tax on debt, but do not characterize optimal policy. In all these models, economic expansions – following, for example, a favorable productivity shock or a period of low interest rates – lead to excessive risk taking and/or excessive borrowing and call for a policy response that may be either monetary or prudential.

We find these insights about the recent crisis persuasive.<sup>6</sup> Nonetheless, we can also envision other ways in which monetary and prudential policies may interact with each other, and think that these alternative perspectives can also serve to inform the design of future regulatory frameworks. To make our point, we start with a benchmark model that deliberately abstracts from any connection between risk taking and the volume of credit, and focuses instead on the type of credit, i.e. the

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<sup>3</sup>We characterize analytically the capital requirement under jointly optimal policies, and this enables us to determine numerically the associated optimal interest rate.

<sup>4</sup>Loisel (2014) summarizes the main features of a number of contributions that consider simple monetary and prudential policy rules [e.g., Angeloni and Faia (2013), Benes and Kumhof (2012), Christensen, Meh, and Moran (2011)]. Two exceptions on this front are De Paoli and Paustian (2013), who also study Ramsey-optimal policies but motivate prudential policy differently, and Du and Miles (2014), who study output-maximizing policies in a model with limited liability but without aggregate shocks.

<sup>5</sup>Bianchi (2011) discusses how this tax on debt may be a model proxy for prudential policies (like capital requirements) that work through the banking system.

<sup>6</sup>There is now compelling empirical evidence in support of the risk-taking channel of monetary policy [e.g., Altunbas et al. (2010), Ioannidou et al. (2009), Jimenez et al. (2012)]. Schularick and Taylor (2012, p. 1032) claim that banking crises are “credit booms gone wrong.” And Kashyap, Berner and Goodhart (2011) emphasize the relevance of the downside of pecuniary externalities (contractions accompanied by fire sales of assets) for the design of prudential policies.

composition of banks' loan portfolios. Our model follows a branch of the micro-banking literature [surveyed by Freixas and Rochet (2008)] in which the need for capital requirements arises from limited liability and deposit insurance. These institutional features truncate the distribution of risky returns facing investors, the banks lending to these investors, and the depositors funding the banks; this is the externality that leads to excessive risk taking. In our model, excessive risk taking involves the type of projects that banks may be tempted to finance because limited liability protects them from incurring large losses, and deposit insurance decouples their funding costs from their risk taking.

More specifically, we develop a variant of Van den Heuvel's (2008) model of optimal capital requirements, and embed it in a DSGE framework with aggregate shocks, sticky prices, and monetary policy.<sup>7</sup> Sufficiently high capital requirements can always force banks to internalize the riskiness of their loans and thus tame risk-taking behavior. But monetary policy may not be suited to this task as it works primarily through the volume rather than the composition of credit. In our benchmark model, due to the assumption of perfectly competitive banks operating under constant returns to scale, the interest rate has no effect on risk-taking incentives as it affects the cost of funding *all* (safe or risky) projects equally. From this vantage point, capital requirements and the interest rate are sharply distinct policy tools that do not affect the same margins: monetary policy affects the volume but not the type of credit, while prudential policy affects both the type and the volume of credit. This makes monetary policy ineffective in ensuring financial stability. As such, our framework accords with the standard view among policymakers [expressed, for instance, in Bernanke (2011) and Yellen (2014)] that standard interest-rate policy cannot serve as the first line of defense against financial instability.

Our normative analysis highlights the desirability of a policy that sets the capital requirement to the minimum level that prevents inefficient risk taking by banks. First, we show that this policy is locally Ramsey-optimal. Indeed, setting the capital requirement *just below* this threshold level is not optimal because it triggers a discontinuous increase in the amount of inefficient risk taken by banks. This discontinuity is due to banks' limited liability, which makes their expected excess return convex in the amount of risk that they take, so that they take either the minimum or the maximum amount of risk. And setting the capital requirement *just above* this threshold level is not optimal because it has a negative first-order effect on welfare that cannot be offset by any change in the interest rate around its optimal value (as this change would have a zero first-order effect on welfare). This negative first-order effect on welfare, in turn, is due to the fact that taxes on banks' profits make equity finance more expensive than debt finance for the banks. This tax distortion implies that raising the capital requirement above the threshold level increases banks'

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<sup>7</sup>Martinez-Miera and Suarez (2012) examine capital requirements from a perspective similar to ours; but their model incorporates systemic risk and abstracts from aggregate shocks and monetary policy.

funding costs and decreases the (bank-loan-financed) capital stock, which is already inefficiently low due to monopolistic competition and the tax distortion itself.<sup>8</sup>

Next, we show that this locally optimal policy may be globally optimal within a class of policies. Specifically, we show that this policy maximizes steady-state welfare under some condition on parameters (met in our calibration exercise). Therefore, for sufficiently small fluctuations, this locally optimal policy is globally optimal in the sense of dominating any other policy that keeps the capital requirement in the neighborhood of any steady-state value. Our calibration exercise suggests that prudential policy has a large effect on the steady-state component of welfare and a small effect on its fluctuations component; this makes a case for our state-dependent locally optimal policy.

This state-dependent policy raises the capital requirement in response to shocks that increase banks' incentives to fund risky projects. In our benchmark model, the interest rate and the capital requirement do not affect the same margins, so there is a clear-cut optimal division of tasks between monetary and prudential policies: in response to shocks that do not affect banks' risk-taking incentives, prudential policy should leave the capital requirement constant, and monetary policy should move the interest rate in a standard way. In response to shocks that increase (decrease) banks' risk-taking incentives, prudential policy should raise (cut) the capital requirement, and monetary policy should cut (raise) the interest rate in order to mitigate the effects of prudential policy on bank lending and output. In the latter case, optimal prudential policy is pro-cyclical (as it is the proximate cause of output fluctuations), while optimal monetary policy is counter-cyclical. So, with this chain of causality, the two policies move in opposite directions over the cycle – a situation that has been envisaged by some policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)].

In this benchmark model, risk taking is exclusively related to the type of credit extended by banks. We can, however, modify our setup to consider situations in which both the type and the volume of credit matter. To illustrate this, we develop an extension that incorporates a risk-taking channel of monetary policy. In this extension, the cost of originating and monitoring safe loans is an increasing function of the aggregate volume of such loans.<sup>9</sup> Consequently, all the shocks that affect the volume of safe loans also affect the cost of such loans and thus banks' risk-taking incentives. Although the particular extension that we consider is motivated by tractability, we

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<sup>8</sup>An alternative to our model with the tax distortion would be to follow Van den Heuvel (2008) and model the cost of raising capital requirements as foregone liquidity from holding bank deposits. In his model, liquid deposits and equity are the only sources of funding for bank loans. So, when capital requirements are higher, banks don't issue as much liquid deposits, and households suffer a loss of utility. We don't pursue this track because commercial paper (rather than liquid deposits) is a more likely marginal source of funding for US banks, as Cúrdia and Woodford (2009) point out. For the same reason, following Cúrdia and Woodford (2009) and others, our modeling of optimal monetary policy will abstract from the transactions frictions that motivate the Friedman Rule.

<sup>9</sup>We use this ad-hoc assumption about costs of banking to keep the extension brief. Hachem (2010) develops a full model of this type of externality in banking costs. In her model, banks ignore the effect of their own lending decision on the pool of borrowers, with heterogeneous levels of risk, that is available to other banks.

think it highlights the main features of optimal policy interactions in other environments that link higher output levels and/or lower interest rates to higher risk-taking incentives. Compared to our benchmark model, the main novelty here is that both policies optimally take a counter-cyclical stance in response to some shocks. A favorable productivity shock, for instance, raises the volume and hence the cost of safe loans, which in turn increases banks' risk-taking incentives. Following this shock, optimal prudential policy raises the capital requirement, and optimal monetary policy raises the interest rate.<sup>10</sup> But the optimal interest-rate hike is smaller than it would be in our benchmark model, because optimal monetary policy mitigates the effects of the rise in the capital requirement on bank lending and output. Optimal policy responses to other shocks (shocks that directly increase risk-taking incentives) are also attenuated when we allow risk-taking incentives to rise with the volume of credit. Nonetheless, the qualitative aspects of the optimal policy responses to these shocks do not change: tighter prudential policy tames the risk taking incentives, and easier monetary policy alleviates some of the contractionary consequences.

The rest of the paper is organized as follows. Section 2 presents our benchmark model. Section 3 derives some implications of banks' optimization problem. Section 4 studies prudential policies ruling out risk taking. Section 5 deals with jointly optimal monetary and prudential policies. Section 6 presents our calibration and Section 7 reports our numerical results. Section 8 presents two extensions (one with an externality in the cost of banking, the other with correlated shocks) that seem relevant for policy concerns. Section 9 contains concluding remarks.

## 2 Benchmark Model

To motivate the role of banks in our model, we assume that the capital stock has to be refurbished at the end of each period by capital producers who need to borrow the necessary funds. The capital producers have access to two alternative technologies to refurbish capital: one is safe and the other risky. The latter technology is less efficient on average, but limited liability tempts the capital producers to use it. Banks are needed to monitor the producers who claim to use the safe technology, to ensure that they do so. Banks themselves, however, may have adverse incentives due to limited liability and deposit insurance, and these adverse incentives create a role for prudential policy.

Each period is divided into two subperiods. At the beginning of the first subperiod, all aggregate shocks are realized; households, production firms, and banks observe these shocks and make their optimization decisions. In the second subperiod capital producers borrow from banks and buy the unfurnished capital from households. Capital producers using the risky technology (if there are any) will be subject to a failure shock that is identically and independently distributed across these

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<sup>10</sup>There are also other ways to make both policies optimally counter-cyclical in our setup. As an example, we will present a case with correlated shocks.

producers. The probability of failure (which is equal to the fraction of risky producers who will fail) is known up-front, but the identity of failing producers is only discovered in the second subperiod.

## 2.1 Households

Preferences are defined by the discount factor  $\beta \in (0,1)$  and the period utility  $u(c_t, h_t)$  over consumption  $c_t$  and hours of work  $h_t$ . Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where the function  $u$  has the usual properties. All household decisions are taken in the first subperiod of each period  $t$ . We assume that, during this subperiod, households own the furbished capital stock  $k_t$  and rent it, at the rental price  $z_t$ , to intermediate goods producers. At the end of the subperiod, after production has taken place, households get back  $(1 - \delta)k_t$  worn-out capital from intermediate goods producers, where  $0 < \delta < 1$ , and invest  $i_t$  in new capital. Unfurbished capital  $x_t$ , made of both worn-out capital and new capital, has to be furbished before it can be used for production next period. So, at this stage, households sell their unfurbished capital

$$x_t = (1 - \delta)k_t + i_t, \tag{1}$$

at the price  $q_t^x$ , to capital goods producers, who can furbish it in the second subperiod of period  $t$ . At the beginning of the next period, households buy furbished capital  $k_{t+1}$ , at a price  $q_{t+1}$ , from capital goods producers.

Households also acquire  $s_t$  shares in banks at a price  $q_t^b$ . These banks are perfectly competitive and last for only one period. Households face the budget constraint

$$c_t + d_t + q_t^b s_t + q_t k_t + i_t = w_t h_t + \frac{1 + R_{t-1}^D}{\Pi_t} d_{t-1} + s_{t-1} \omega_t^b + z_t k_t + q_t^x x_t + (\omega_t^k + \omega_t^f - \tau_t^h), \tag{2}$$

where  $d_t$  represents the real value of bank deposits with a nominal return  $R_t^D$ ,  $\Pi_t = \frac{P_t}{P_{t-1}}$  is the gross inflation rate in the price index for consumption,  $w_t$  is the real wage,  $\omega_t^k$  and  $\omega_t^f$  represent the profits of capital producers and firms producing intermediate goods,  $\omega_t^b$  stands for dividends paid by banks, and  $\tau_t^h$  is a lump-sum tax paid by households.<sup>11</sup>

Households choose  $(c_t, h_t, d_t, s_t, k_t, i_t, x_t)_{t \geq 0}$  to maximize utility subject to (1) and (2). The

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<sup>11</sup>We do not need to model equity stakes in firms as we assume that the representative household owns these firms forever.

first-order conditions for optimality are:

$$\begin{aligned}
u_c(c_t, h_t) &= \lambda_t, \\
\lambda_t &= \beta \left(1 + R_t^D\right) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}, \\
-u_h(c_t, h_t) &= \lambda_t w_t, \\
\lambda_t q_t^x &= \lambda_t^k, \\
\lambda_t &= \lambda_t^k, \\
\lambda_t (q_t - z_t) &= \lambda_t^k (1 - \delta), \\
\lambda_t q_t^b &= \beta \mathbb{E}_t \left\{ \lambda_{t+1} \omega_{t+1}^b \right\},
\end{aligned} \tag{3}$$

where  $\mathbb{E}_t \{ \cdot \}$  denotes the expectation operator conditional on the information available in the first subperiod of period  $t$ , which includes the realization of all the aggregate shocks. The optimality conditions imply in particular

$$q_t^x = 1, \tag{4}$$

$$q_t = 1 - \delta + z_t. \tag{5}$$

## 2.2 Intermediate Goods Producers

There is a unit mass of monopolistically competitive firms producing intermediate goods. Firm  $j$  operates the production function:

$$y_t(j) = h_t(j)^{1-\nu} k_t(j)^\nu \exp\left(\eta_t^f\right),$$

where  $0 < \nu < 1$ ,  $k_t(j)$  is capital rented by firm  $j$ ,  $h_t(j)$  hours of work used by firm  $j$ , and  $\eta_t^f$  is an exogenous productivity shock. We assume that firms set their prices facing a Calvo-type price rigidity (with no indexation). Since their optimization problem is standard, we don't present the details. We let  $\alpha$  denote the probability that a firm does not get to set a new price at a given date.

The firms' cost minimization problem implies

$$\frac{z_t}{w_t} = \left( \frac{\nu}{1-\nu} \right) \left[ \frac{h_t(j)}{k_t(j)} \right].$$

## 2.3 Final Goods Producers

Producers of the final good are perfectly competitive and combine the intermediate goods  $y_t(j)$  to form the final good  $y_t$ . The production function is given by

$$y_t = \left( \int_0^1 y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \tag{6}$$



where  $\sigma > 1$ . Profit maximization leads to the demand for good  $j$

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\sigma} y_t, \quad (7)$$

and free entry leads to the price index

$$P_t = \left( \int_0^1 P_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (8)$$

The final good may be used for consumption, investment, and the monitoring of firms.

## 2.4 Capital Goods Producers

The capital producing firms are owned by households and are perfectly competitive. They buy unfurnished capital  $x_t$  during the second subperiod of period  $t$  to produce furnished capital  $k_{t+1}$  that they sell to households at price  $q_{t+1}$  in the first subperiod of period  $t+1$ . Each capital producer chooses to operate either a safe technology (S for “safe” or “storage”) or a risky technology (R for “risky”). A producer  $i$  choosing technology S uses  $x_t(i)$  units of unfurnished capital to produce  $k_{t+1}(i)$  units of furnished capital with

$$k_{t+1}(i) = x_t(i). \quad (9)$$

Producers choosing technology R are subject to a failure shock  $\theta_t$  that is independently and identically distributed across risky producers. When  $\theta_t(i) = 0$ , producer  $i$  does not produce anything. More specifically, risky producer  $i$  uses  $x_t(i)$  units of unfurnished capital to produce

$$k_{t+1}(i) = \theta_t(i) \exp\left(\eta_t^R\right) x_t(i) \quad (10)$$

units of furnished capital, with

$$\begin{aligned} \theta_t(i) &= 0 \text{ with probability } \phi, \\ \theta_t(i) &= 1 \text{ with probability } 1 - \phi, \end{aligned}$$

where  $\phi$  is the exogenous probability of failure and  $\eta_t^R$  is the exogenous stochastic productivity (common to all risky producers) if the project is successful. We assume that the realization of  $\eta_t^R$  is always positive ( $\eta_t^R > 0$ ), so that in the absence of failure, the risky technology is more productive than the safe one. Producer  $i$  chooses whether to use technology S or technology R after observing the realization of  $\eta_t^R$  but before observing the realization of  $\theta_t(i)$ .

Our setup with two technologies serves to highlight a familiar connection between limited liability and excessive risk taking: if capital producers are not monitored properly, they may take on more risk than a hypothetical social planner would. For exposition purposes, and without affecting much

our main points, we assume that using the risky technology to any degree is always inefficient from a planner's perspective, as we elaborate below.<sup>12</sup> However, capital producers may have an incentive to use the risky technology, to the extent that they can hide the fact that they do so, because they have limited liability. There is therefore a need to monitor capital producers who claim to use the safe technology, and we assume that only banks have the appropriate monitoring skills. This motivates a setup with capital producers getting funds from banks to buy unfurnished capital.

More specifically, the risky technology is assumed to be inefficient in the sense that, for all realizations of  $\eta_t^R$ ,

$$(1 - \phi) \exp(\eta_t^R) \leq 1 - \Psi, \quad (11)$$

where  $\Psi > 0$  is the exogenous marginal resource cost of monitoring a capital producer who claims to use the safe technology.<sup>13</sup> The left-hand side of (11) represents the expected benefit of allocating one unit of unfurnished capital to the risky technology. The right-hand side is the opportunity cost, which is the output of the safe technology net of the monitoring cost. This inefficiency condition is stronger than what we actually need for the risky technology to be socially undesirable; but we use it because the necessary and sufficient condition involves the degree of risk aversion and we prefer to define inefficiency only in terms of technology parameters.

Our model simplifies (we think in a harmless way) the relationship between capital goods producers, their owners, and the creditor banks. In reality non-bank firms prefer debt finance because they get a tax deduction. They also need some equity, presumably because of the agency problem associated with debt. Their owners absorb losses up to their equity stake. In our model, for simplicity, we abstract from this agency problem and capital goods producers have no equity. So this translates into a framework in which their funding consists entirely of loans and they pay no tax; and any profits or losses arising from stochastic disturbances in the absence of failure of the risky technology accrue to households.<sup>14</sup> Thus, a capital producer  $i$  choosing technology  $j \in \{S, R\}$  borrows

$$q_t^x x_t(i) = l_t(i) \quad (12)$$

at a nominal interest rate  $R_t^j$ .<sup>15</sup> Since capital producers have limited liability, those using the risky technology will default on their loans in the event of failure.<sup>16</sup>

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<sup>12</sup>One way to extend our model to incorporate efficient risk taking would involve adding a third technology that is risky but can be efficiently combined with the safe technology. This would make the model more realistic by adding some desirable risk, but it would require solving a portfolio problem that does not seem directly relevant for our purposes.

<sup>13</sup>In Section 8, we will consider an extension of the model in which this cost is time-varying and endogenous.

<sup>14</sup>Our results however would be qualitatively unchanged if capital goods producers were allowed to borrow only a fraction of the funds they need.

<sup>15</sup>There is no need to work with nominal loan contracts in our model. However, since we will assume that monetary policy sets a nominal interest rate, and for the sake of realism, we make loan contracts nominal.

<sup>16</sup>A capital producer, in our setup, will have no incentive to diversify across multiple risky projects (or to combine risky and safe technologies). As we will elaborate in Section 3, the benefits of limited liability are maximized by

A producer  $i$  using technology S chooses  $x_t(i)$  and  $l_t(i)$  to maximize

$$\beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1} k_{t+1}(i) - \frac{1 + R_t^S}{\Pi_{t+1}} l_t(i) \right] \right\}$$

subject to (9) and (12). The optimality condition implies

$$\mathbb{E}_t \{ \lambda_{t+1} q_{t+1} \} = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^S) q_t^x. \quad (13)$$

A producer  $i$  using technology R chooses  $x_t(i)$  and  $l_t(i)$  to maximize

$$(1 - \phi) \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ q_{t+1} k_{t+1}(i) - \frac{1 + R_t^R}{\Pi_{t+1}} l_t(i) \right] \right\}$$

subject to (10) and (12). The optimality condition implies

$$\mathbb{E}_t \{ \lambda_{t+1} q_{t+1} \} \exp(\eta_t^R) = \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^R) q_t^x. \quad (14)$$

Since our model allows for two distinct interest rates, banks need to monitor the capital producers that borrow at the lower rate to ensure that they use the associated technology. Our model has no equilibrium with  $R_t^R < R_t^S$ .<sup>17</sup> Therefore, there is no need for banks to monitor capital producers that claim to use the risky technology. Accordingly, we will associate a cost with monitoring capital producers that claim to use the safe technology.

When both (13) and (14) hold, capital producers are indifferent between the two technologies and

$$\frac{1 + R_t^R}{1 + R_t^S} = \exp(\eta_t^R). \quad (15)$$

If the interest-rate ratio on the left-hand side is strictly higher than the critical value on the right-hand side, then capital producers use only technology S.

## 2.5 Banks

Banks are owned by households. They are perfectly competitive. They can make safe and risky loans ( $l_t^S$  and  $l_t^R$ ). They incur a cost  $\Psi l_t^S$  of monitoring safe loans, where  $\Psi$  satisfies (11). They can fund their loans by raising equity ( $e_t$ ) or issuing deposits ( $d_t$ ), so that their balance-sheet identity is

$$l_t^S + l_t^R = e_t + d_t, \quad (16)$$

as  $e_t$  is defined net of monitoring costs.

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undertaking a single risky project.

<sup>17</sup>Indeed, if we had  $R_t^R < R_t^S$ , then funding the safe projects would strictly dominate funding the risky projects because it would pay more in every state (whatever the realization of the failure shocks) and incur no monitoring cost.

We will show, in Section 3 below, that each bank in our model will extend risky loans to at most one capital producer employing the risky technology (because the benefits of limited liability are maximized by concentrating the risk in a single project). For now, we just assume this and use  $\theta_t$  to refer to the failure shock associated with the project that a particular bank may fund.

Given our inefficiency assumption (11), risky projects reduce welfare. So, if the regulators could detect any risky project, they would devise a sufficient penalty to prevent it. We need an information friction to rule out a trivial and unrealistic solution in which the regulators directly forbid risk taking. Following Van den Heuvel (2008), we assume that banks can hide some risky loans in their portfolio from regulators. More specifically, we assume that regulators observe the total amount of loans made by each bank but cannot detect its risky loans up to a given fraction  $\gamma$  of its safe loans. The prudential authority imposes risk-weighted capital requirements on risky loans above this fraction. We specify the capital requirement as

$$e_t \geq \kappa_t \left( l_t^S + l_t^R \right) + \bar{\kappa} \max \left\{ 0, l_t^R - \gamma l_t^S \right\}. \quad (17)$$

The higher the capital requirement, the more banks internalize the social cost of risk, as they have more “skin in the game.” The prudential authority will optimally choose a sufficiently high  $\bar{\kappa}$  to ensure that  $l_t^R \leq \gamma l_t^S$  in equilibrium. Therefore, this is equivalent to rewriting the capital requirement as a minimum ratio of equity to loans:

$$e_t \geq \kappa_t \left( l_t^S + l_t^R \right), \quad (18)$$

and imposing the following constraint on banks:

$$l_t^R \leq \gamma l_t^S. \quad (19)$$

In the first subperiod of period  $t + 1$ , regulators close the banks that cannot meet their deposit obligations, i.e. the banks with

$$(1 + R_t^S)l_t^S + \theta_t(1 + R_t^R)l_t^R - (1 + R_t^D)d_t < 0,$$

or equivalently, using (16), the banks whose equity satisfies

$$e_t < \left[ 1 - \theta_t \left( \frac{1 + R_t^R}{1 + R_t^D} \right) \right] l_t^R - \left( \frac{R_t^S - R_t^D}{1 + R_t^D} \right) l_t^S.$$

We want our model to capture the fact that banks find equity finance more costly than debt finance in reality. We attribute this to a tax distortion (tax deduction for debt finance), although this interpretation is not essential for our analysis. We take this distortionary tax to be a feature of the environment: the model does not explain why this tax is in place, and the policymakers in our

model (the monetary and prudential authorities) cannot set this tax optimally.<sup>18</sup> Specifically, we assume that gross revenues from loans are taxed at the constant rate  $\tau$  after deductions for gross payments on deposits and monitoring costs.<sup>19</sup> The amount of bank equity, net of monitoring costs, is therefore  $e_t = q_t^b s_t - (1 - \tau)\Psi l_t^S$ .

Banks choose  $e_t$ ,  $d_t$ ,  $l_t^R$  and  $l_t^S$  to maximize

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b}{\lambda_t} \right\} - e_t - (1 - \tau) \Psi l_t^S,$$

where

$$\omega_{t+1}^b = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right\}, \quad (20)$$

subject to (16), (18) and (19).

## 2.6 Government and Market-Clearing Conditions

The government guarantees bank deposits. The lump-sum tax on households balances the budget.<sup>20</sup> The losses imposed by bank  $j$  on the deposit-insurance fund amount to

$$\zeta_t(j) = \max \left\{ 0, \frac{1 + R_{t-1}^D}{\Pi_t} d_{t-1}(j) - \frac{1 + R_{t-1}^S}{\Pi_t} l_{t-1}^S(j) - \theta_{t-1}(j) \frac{1 + R_{t-1}^R}{\Pi_t} l_{t-1}^R(j) \right\},$$

and the lump-sum tax paid by households is

$$\tau_t^h = \int_0^1 \left\{ \zeta_t(j) - \tau [\omega_t^b(j) + \Psi l_t^S(j)] \right\} dj.$$

We consider two policy instruments: the deposit rate  $R_t^D$  for monetary policy and the capital requirement  $\kappa_t$  for prudential policy. We will discuss our specifications of prudential policy in Sections 4 and 7. For each specification, our monetary policy will be the Ramsey-optimal policy.

Firms producing intermediate goods rent their capital from the representative household; in equilibrium, their choices must satisfy

$$\int_0^1 k_t(j) dj = k_t.$$

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<sup>18</sup>This feature of the tax code seems to be one of the primary reasons for banks to lobby against higher capital requirements, at least in the US and the euro area. It is commonly invoked in models with both debt and equity finance [e.g. Jermann and Quadrini (2009, 2012)], to break the Modigliani-Miller theorem about irrelevance of financial structure.

<sup>19</sup>Our specification is motivated by analytical tractability. In our model, gross revenues from loans are received at date  $t + 1$ , gross payments on deposits are made at date  $t + 1$ , but monitoring costs are paid at date  $t$ . Therefore, banks in effect receive a subsidy at date  $t$  and pay a tax at date  $t + 1$ . This has the (admittedly awkward) implication that a bank in our model may collect the subsidy at  $t$  and subsequently fail, paying no tax at  $t + 1$ .

<sup>20</sup>It is harmless to abstract from deposit insurance fees paid by banks and include these in the lump-sum tax paid by households who own the banks.

Similarly obvious market-clearing conditions must be satisfied in the markets for labor, loans, and unfurnished capital. The market-clearing condition for goods is

$$c_t + i_t + \Psi l_t^S = y_t. \quad (21)$$

### 3 Implications of Banks' Optimization

This section presents four implications of our banks' optimization problem: (1) each bank funds at most one risky project; (2) either all the banks take no risk ( $l_t^R = 0$ ), or they take the maximum undetected risk ( $l_t^R = \gamma l_t^S$ ); (3) the capital constraint is binding; and (4) there is a financial wedge (a non-zero spread between lending and deposit rates) that depends on capital requirements and exogenous variables.

We first show that a bank in our setup (and, by a similar reasoning, a capital goods producer) has no incentive to fund more than one risky project. Clearly, a bank would not fund a continuum of risky projects: a fraction  $\phi$  of these projects would fail each period, and the overall return would be below the safe rate. Suppose a bank funds the risky projects of a number of capital producers  $i$  in some finite set  $I$ . The bank chooses the set  $I$  and the loan amounts  $l_t(i)$  for  $i \in I$  subject to

$$\sum_{i \in I} l_t^R(i) = l_t^R.$$

We define

$$r_{t+1} \equiv \left(1 + R_t^R\right) \sum_{i \in I} \theta_t(i) l_t^R(i)$$

as the gross nominal return on the bank's portfolio of risky loans (the subscript  $t + 1$  highlights the fact that this is a random variable with respect to the information set at date  $t$ ). The bank's objective amounts to maximizing

$$\mathbb{E}_t \{ \max [0, r_{t+1} - b_t] \},$$

where  $b_t \equiv \left(1 + R_t^D\right) d_t - \left(1 + R_t^S\right) l_t^S$ , for given values of  $l_t^R$ ,  $l_t^S$ , and  $d_t$ . Since we have

$$\mathbb{E}_t r_{t+1} = (1 - \phi) \left(1 + R_t^R\right) l_t^R$$

regardless of the choices of  $I$  and  $l_t^R(i)$  for  $i \in I$ , we get

$$\begin{aligned} \mathbb{E}_t \{ \max [0, r_{t+1} - b_t] \} &= \Pr \{ r_{t+1} > b_t \} \mathbb{E} \{ r_{t+1} - b_t \mid r_{t+1} > b_t \} \\ &= \mathbb{E}_t \{ r_{t+1} - b_t \} - \Pr \{ r_{t+1} \leq b_t \} \mathbb{E} \{ r_{t+1} - b_t \mid r_{t+1} \leq b_t \} \\ &= (1 - \phi) \left(1 + R_t^R\right) l_t^R - b_t - \Pr \{ r_{t+1} \leq b_t \} \mathbb{E} \{ r_{t+1} - b_t \mid r_{t+1} \leq b_t \}. \end{aligned}$$

So, the bank's objective amounts to minimizing the negative random variable  $\Pr \{ r_{t+1} \leq b_t \} \mathbb{E}_t \{ r_{t+1} - b_t \mid r_{t+1} \leq b_t \}$ . Loosely stated, maximizing the gains to the bank when it does not default is equivalent

to maximizing the losses of the deposit-insurance fund when the bank defaults. These losses are maximized when the bank does not diversify at all and puts  $l_t^R$  in a single risky project (since this maximizes the probability of the worst outcome,  $r_{t+1} = 0$ ). We formalize this argument and prove the following proposition in the Appendix.

**Proposition 1:** *Each bank funds at most one risky project.*

Next, we show that our model only admits two types of equilibria: one with no bank taking any risk ( $l_t^R = 0$ ), the other with all banks taking the maximum amount of risk ( $l_t^R = \gamma l_t^S$ ). The basic insight follows Van den Heuvel (2008), but since we have made changes to his model, we prove the following proposition in the Appendix.

**Proposition 2:** *There are no equilibria with  $0 < l_t^R < \gamma l_t^S$ .*

The intuition is as follows. If, given the loan portfolio, bank equity is sufficiently small to be wiped out when risky projects fail, then banks do not internalize the cost of additional risk taking. Additional losses from increasing  $l_t^R$ , if risky projects fail, are truncated by deposit insurance and limited liability. Consequently, the only candidate for an equilibrium with the possibility of bank failure involves the corner solution  $l_t^R = \gamma l_t^S$ . Alternatively, if bank equity is sufficiently large for banks to remain solvent even when risky projects fail, then banks internalize the cost of additional risk taking. In that case, since we assume that the risky technology is inefficient, banks can increase their market value by reducing  $l_t^R$ . Accordingly, the only candidate for an equilibrium without the possibility of bank failure involves the corner solution  $l_t^R = 0$ . In particular, if bank equity is large enough to make banks residual claimants on their risky loans when  $l_t^R = \gamma l_t^S$ , then there does not exist an equilibrium with  $l_t^R = \gamma l_t^S$ .

Next, we show that there are no equilibria in which the capital constraint is lax:

**Proposition 3:** *In equilibrium, the capital constraint is binding:*

$$e_t = \kappa_t (l_t^S + l_t^R). \tag{22}$$

This proposition follows almost directly from our assumption about the tax advantage of debt finance over equity finance, but we provide a proof in the Appendix.

Finally, we derive the spread between the lending and deposit rates at each of the two candidate equilibria. Consider first the candidate equilibrium with  $l_t^R = 0$ . Using (16) to eliminate  $d_t$  and (22)

to eliminate  $e_t$ , the representative bank's objective can be rewritten as

$$(1 - \tau) \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b}{\lambda_t} \right\} - [\kappa_t + (1 - \tau) \Psi] l_t^S,$$

where

$$\omega_{t+1}^b = \left[ \frac{R_t^S - R_t^D}{\Pi_{t+1}} + \frac{1 + R_t^D}{\Pi_{t+1}} \kappa_t \right] l_t^S.$$

The representative bank chooses  $l_t^S$  so as to maximize its expected excess return. Using (3), the first-order condition of this program can be written as

$$\frac{1 + R_t^S}{1 + R_t^D} = 1 + \Psi + \frac{\tau \kappa_t}{1 - \tau}. \quad (23)$$

The financial wedge is exogenous because banks live for only one period in our model. The wedge reflects monitoring costs and the higher cost of equity funding that arises from the interaction between the tax distortion and capital requirements. A more stringent prudential policy (an increase in  $\kappa_t$ ) increases the wedge by forcing banks to rely more heavily on equity finance.

At the maximum-risk corner with  $l_t^R = \gamma l_t^S$ , the balance-sheet identity gives  $(1 + \gamma) l_t^S = e_t + d_t$ , and the binding capital requirement implies  $e_t = \kappa_t (1 + \gamma) l_t^S$ . Using these, the representative bank's optimization problem amounts to choosing  $l_t^S$  to maximize

$$(1 - \tau)(1 - \phi) \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}} \right\} \left[ (1 + R_t^S) + \gamma(1 + R_t^R) - (1 + \gamma)(1 - \kappa_t)(1 + R_t^D) \right] l_t^S \\ - [\kappa_t(1 + \gamma) + (1 - \tau) \Psi] l_t^S.$$

The optimality condition, in conjunction with (3) and (15), implies

$$\frac{1 + R_t^S}{1 + R_t^D} = \frac{1 + \gamma}{1 + \gamma \exp(\eta_t^R)} \left[ 1 + \frac{\tau \kappa_t}{1 - \tau} + \frac{\phi \kappa_t}{(1 - \tau)(1 - \phi)} + \frac{\Psi}{(1 + \gamma)(1 - \phi)} \right]. \quad (24)$$

The financial wedge now depends on parameters ( $\gamma$  and  $\phi$ ) and shocks ( $\eta_t^R$ ) related to the risky technology, because making safe loans enables banks to make risky loans (given that hiding the risk is subject to the constraint  $l_t^R \leq \gamma l_t^S$  in our model). In fact, banks at the maximum-risk corner incur losses on their safe loans and make profits on their hidden risky loans that compensate for these losses. If they did not incur losses on their safe loans, then the maximum-risk corner would not be an equilibrium. The fact that they do incur losses on their safe loans can be easily verified by showing that the right-hand side of (24) is lower than the right-hand side of (23) for any value of  $\kappa_t$  consistent with the maximum-risk corner, as (23) is the condition for zero profits on safe loans at that corner.<sup>21</sup>

<sup>21</sup>More specifically, for any value of  $\kappa_t$  such that  $\kappa_t < \kappa_t^*$ , where  $\kappa_t^*$  is defined in the next section, the condition for the right-hand side of (24) to be lower than the right-hand side of (23) can be written as  $\tau < \frac{(1 + \gamma)\phi}{\gamma(1 - \phi)[\exp(\eta_t^R) - 1]}$ . This condition is met because  $\tau < 1 < \frac{1 + \gamma}{\gamma} < \frac{(1 + \gamma)\phi}{\gamma(1 - \phi)[\exp(\eta_t^R) - 1]}$ , where the last inequality follows from the inefficiency condition (11).



## 4 Prudential Policy

A prudential policy that is sufficient to rule out risk taking is one that makes banks residual claimants to any losses they may incur, so that they internalize the externality arising from limited liability. This section first characterizes this prudential policy, and then derives the least stringent prudential policy that rules out risk taking.

Consider a bank that takes the maximum amount of risk by setting  $l_t^R = \gamma l_t^S$ . Using (16) and (22) to eliminate  $d_t$  from (20), it is straightforward to show that this bank remains solvent ( $\omega_{t+1}^b \geq 0$ ) when its risky project fails ( $\theta_t = 0$ ) if and only if

$$\kappa_t \geq \tilde{\kappa}(R_t^D, R_t^S) \equiv 1 - \frac{1}{1 + \gamma} \frac{1 + R_t^S}{1 + R_t^D}. \quad (25)$$

The solution for  $\tilde{\kappa}(R_t^D, R_t^S)$ , at the candidate equilibrium with  $l_t^R = 0$ , follows from (23) and (25). This leads to the following proposition:

**Proposition 4:** (a) A sufficient condition for  $l_t^R = 0$  in equilibrium is

$$\kappa_t \geq \tilde{\kappa} \equiv \frac{(1 - \tau)(\gamma - \Psi)}{\tau + (1 - \tau)(1 + \gamma)}; \quad (26)$$

(b)  $\tilde{\kappa}$  is increasing in  $\gamma$  and decreasing in  $\Psi$ .

We assume  $\gamma > \Psi$ , which implies  $\tilde{\kappa} > 0$ , so that condition (25) may or may not be met depending on the value of  $\kappa_t$ . Without this restriction, banks would never be tempted to take risk even in the absence of (positive) capital requirements. The threshold  $\tilde{\kappa}$  is increasing in  $\gamma$ : the higher the fraction of risky loans that a bank deviating from the safe corner can hide, the riskier this bank, and the higher the capital requirement needed to make it remain solvent in case of failure. And  $\tilde{\kappa}$  is decreasing in  $\Psi$ : the higher the cost of monitoring safe loans, the higher the spread between the interest rate on safe loans and that on deposits; thus, the larger the cash flow from safe loans that is available to redeem the deposits, and the lower the capital requirement needed to make a deviating bank remain solvent if its risky project fails.

Although this policy is sufficient to rule out risk taking, it is more stringent than necessary. The following proposition characterizes the least stringent prudential policy that rules out risk taking:

**Proposition 5:** (a) A necessary and sufficient condition for  $l_t^R = 0$  in equilibrium is  $\kappa_t \geq \kappa_t^*$ , where

$$\kappa_t^* \equiv (1 - \tau) \frac{(1 - \phi) \gamma [\exp(\eta_t^R) - 1] + \Psi [(1 - \phi) \gamma \exp(\eta_t^R) - \phi]}{\phi(1 + \gamma) - \gamma\tau(1 - \phi) [\exp(\eta_t^R) - 1]}; \quad (27)$$

(b)  $\kappa_t^* < \tilde{\kappa}$ ; (c)  $\kappa_t^*$  is increasing in the probability of success of the risky technology  $1 - \phi$ , the productivity of the risky technology conditionally on its success  $\eta_t^R$ , and the maximum ratio of risky to safe loans  $\gamma$ .

We prove this proposition in the Appendix by considering a bank that deviates from a candidate equilibrium with  $l_t^R = 0$ . The same intuition we gave for Proposition 2 applies: if there are profitable deviations, the most profitable one is at the corner with maximum risk ( $l_t^R = \gamma l_t^S$ ). To derive the value of  $\kappa_t^*$ , we make this bank indifferent between staying at the safe corner and moving to the maximum-risk corner. The bank turns indifferent with less equity at stake than what would make it residual claimant (i.e., we have  $\kappa_t^* < \tilde{\kappa}$ ) because risk is inefficient in our model. Indeed, when  $\kappa_t$  is just below  $\tilde{\kappa}$ , the deviating bank benefits little from limited liability, as it incurs most of the loss on its risky loans when the risk materializes. This small benefit is dominated by the cost of risky loans due to their inefficiency, as expressed in (11), so that the bank prefers to stay at the safe corner. It is only when  $\kappa_t$  falls below  $\kappa_t^*$  that the limited-liability benefit starts to outweigh the inefficiency cost, and the deviation to be profitable.

The preceding reasoning also helps us understand the nature of the state dependence, in our model, of the constraint  $\kappa_t \geq \kappa_t^*$ . Macro-prudential policy must be tight enough to prevent risk taking in equilibrium. The threshold  $\kappa_t^*$  depends positively on the productivity of the risky technology conditionally on its success  $\eta_t^R$  because an increase in this productivity raises risk-taking incentives for banks. Similarly,  $\kappa_t^*$  depends negatively on the probability of failure of the risky technology  $\phi$  and positively on the maximum ratio of risky to safe loans  $\gamma$ .<sup>22</sup>

Finally,  $\kappa_t^*$  may depend positively or negatively on  $\Psi$  because of conflicting effects. To understand these effects, consider a deviating risky bank making one unit of safe loans and  $\gamma$  units of risky loans, and rewrite the coefficient of  $\Psi$  in the numerator of the fraction in (27) as the sum of three terms:  $-1 + (1 - \phi) + (1 - \phi)\gamma \exp(\eta_t^R)$ . The first term,  $-1$ , is negative and reflects a franchise-value effect: the deviating risky bank has incurred monitoring costs on its one unit of safe loans and has a vested interest in remaining solvent to recoup these costs. In a way, monitoring costs in our model work like giving the banks some charter value that they would like to preserve by avoiding bankruptcy. The second term,  $1 - \phi$ , is positive and reflects the fact that with probability  $1 - \phi$ , the bank gets at date  $t + 1$  gross interest payments on its one unit of safe loans, whose discounted value at date  $t$  is  $\frac{1 + R_t^S}{1 + R_t^D}$ , which increases one-to-one with monitoring costs (as reflected in (23)). Similarly, the last term,  $(1 - \phi)\gamma \exp(\eta_t^R)$ , is also positive and reflects the fact that with probability  $1 - \phi$ , the bank gets

<sup>22</sup>Our model could be extended to allow the prudential authority to choose a state-dependent level  $\gamma_t$  by incurring some supervision cost, following Van den Heuvel (2008). The only change in (27) that this would entail is that the parameter  $\gamma$  on the right-hand side should then be replaced by the endogenous variable  $\gamma_t$ . In this case, the prudential authority would optimally respond to shocks that increase risk-taking incentives by devoting more resources to supervision (i.e. lowering  $\gamma_t$ ) and raising the capital requirement  $\kappa_t$  by less.

at date  $t + 1$  gross interest payments on its  $\gamma$  units of risky loans, whose discounted value at date  $t$  is  $\gamma \frac{1+R_t^R}{1+R_t^D} = \gamma \exp(\eta_t^R) \frac{1+R_t^S}{1+R_t^D}$ , where the spread  $\frac{1+R_t^S}{1+R_t^D}$  increases one-to-one with monitoring costs. When the sum of these three terms is positive, an increase in monitoring costs leads to an increase in the expected excess return of the deviating risky bank, without affecting the expected excess return of the representative safe bank (which remains zero in equilibrium). Therefore, risk-taking incentives increase, and so does  $\kappa_t^*$ .<sup>23</sup>

Perhaps a more surprising feature of (27) is that  $\kappa_t^*$  does not depend on the monetary policy instrument  $R_t^D$ . This is because, in our model with perfect competition and constant returns, the deposit rate  $R_t^D$  does not affect the spread between the interest rate on risky loans  $R_t^R$  and the interest rate on safe loans  $R_t^S$ , and therefore does not affect banks' incentives for risk taking. This implication of our model is in contrast to arguments that periods of economic boom or low interest rates raise risk-taking incentives. We will revisit this contrast in Subsection 8.1 and in the concluding section.

We assume that parameter values and shock processes are such that  $\kappa_t^*$  is positive.<sup>24</sup> Its steady-state value has to be positive for our analysis to be relevant. To allow for realizations of  $\eta_t^R$  that make  $\kappa_t^*$  negative, we would just need to replace  $\kappa_t^*$  with  $\max(0, \kappa_t^*)$ .

## 5 Optimal Policies

This section presents our analytical results about jointly optimal monetary and prudential policies. We first show that the policy setting  $\kappa_t$  to  $\kappa_t^*$  and optimizing over  $R_t^D$  is locally optimal. Then, we study steady-state welfare and use the results to extend our local-optimality result to a global-optimality result under suitable conditions.

### 5.1 Locally Optimal Policies

We define locally Ramsey-optimal policies as follows: a policy  $(\widehat{R}_\tau^D, \widehat{\kappa}_\tau)_{\tau \geq 0}$  is locally Ramsey-optimal if there exists a neighborhood of  $(\widehat{R}_\tau^D, \widehat{\kappa}_\tau)_{\tau \geq 0}$  such that no other policy in this neighborhood gives a higher value for the representative household's expected utility than  $(\widehat{R}_\tau^D, \widehat{\kappa}_\tau)_{\tau \geq 0}$  does. Let  $(R_\tau^{D*})_{\tau \geq 0}$  denote the monetary policy that is (globally) Ramsey-optimal when the prudential policy is  $(\kappa_\tau^*)_{\tau \geq 0}$ . The following proposition states that, under a certain condition, setting jointly  $(R_\tau^D)_{\tau \geq 0}$  to  $(R_\tau^{D*})_{\tau \geq 0}$  and  $(\kappa_\tau)_{\tau \geq 0}$  to  $(\kappa_\tau^*)_{\tau \geq 0}$  is locally Ramsey-optimal:

<sup>23</sup>Monitoring costs may thus provide an additional source of risk-taking incentives in our benchmark model. They are not, however, a necessary ingredient in the sense that all our results are still valid when  $\Psi$  is equal to zero. It is only in our extended model (considered in Subsection 8.1), with an externality in the cost of banking, that they play a key role.

<sup>24</sup>This implies that  $\tilde{\kappa} > 0$ , given Point (b) of Proposition 5, and therefore that  $\gamma > \Psi$ .

**Proposition 6:** *If the right derivative of welfare with respect to  $\kappa_t$  at  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D^*}, \kappa_\tau^*)_{\tau \geq 0}$  is strictly negative for all  $t \geq 0$ , then the policy  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D^*}, \kappa_\tau^*)_{\tau \geq 0}$  is locally Ramsey-optimal.*

We prove this proposition in the Appendix. The basic idea is the following. First, for any  $R_t^D$  in the neighborhood of  $R_t^{D^*}$ , setting  $\kappa_t$  just below  $\kappa_t^*$  is not optimal, because it moves the economy from the safe to the maximum-risk corner.<sup>25</sup> Under our inefficiency condition (11), this triggers a discontinuous drop in the average productivity of capital goods producers and therefore in welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that  $(R_t^D, \kappa_t)$  is close enough to  $(R_t^{D^*}, \kappa_t^*)$ . Second, if the right derivative of welfare with respect to  $\kappa_t$  at  $(R_t^{D^*}, \kappa_t^*)$  is strictly negative, then setting  $\kappa_t$  just above  $\kappa_t^*$  is not optimal either, because it has a negative first-order effect on welfare that cannot be offset by any change in  $R_t^D$  around its optimal value  $R_t^{D^*}$  (as this change would have a zero first-order effect on welfare).

The right derivative of welfare with respect to  $\kappa_t$  at  $(R_t^{D^*}, \kappa_t^*)$  can be expected to be strictly negative because raising  $\kappa_t$  from  $\kappa_t^*$  increases banks' funding costs and therefore decreases the capital stock, which is already inefficiently low due to the monopoly and tax distortions, without reducing the use of the risky technology, which is already zero. We will check numerically in Section 7 that this condition is met. We now show analytically that it is met at the steady state.

## 5.2 Steady-State Welfare

Since our model has no monetary distortion, the optimal steady-state rate of inflation is zero regardless of the prudential policy in place. Accordingly, we focus on zero-inflation steady states in the following. The Appendix presents the solution for the steady-state values of some key endogenous variables in our model, for a given prudential policy. We assume iso-elastic consumption-utility and labor-disutility functions.

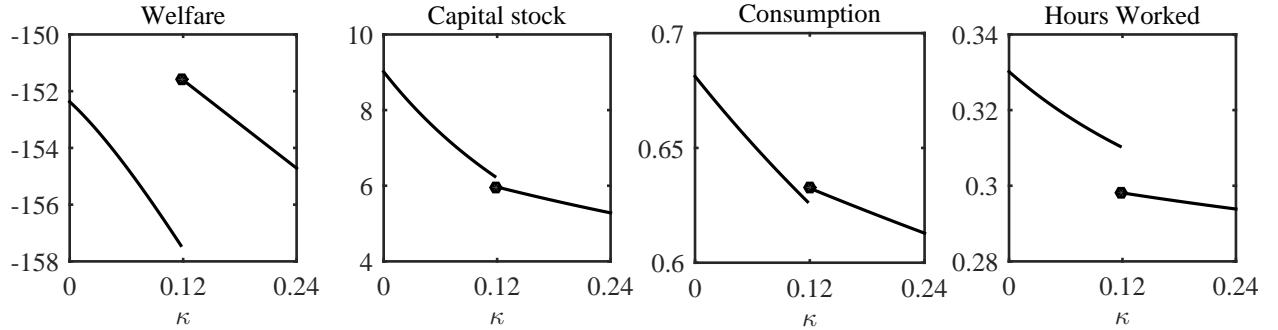
At the safe corner (i.e. for  $\kappa \geq \kappa^*$ , where variables without time subscript denote steady-state values), the capital stock is inefficiently low, all the more so as the capital requirement  $\kappa$  is high. To understand why, write the marginal product of capital ( $MPK$ ) as the product of the rental price of capital ( $z$ ) times the monopoly markup, and use the steady-state versions of (3), (4), (5), (13), and (23) to rewrite it as

$$MPK = \left( \frac{\sigma}{\sigma - 1} \right) z = \left( \frac{\sigma}{\sigma - 1} \right) \left[ \frac{1}{\beta} - (1 - \delta) + \frac{\frac{\tau \kappa}{1 - \tau} + \Psi}{\beta} \right].$$

<sup>25</sup>Our model has a unique equilibrium for any given value of  $\kappa_t$ , and this equilibrium is symmetric across banks. That is, the threshold value of  $\kappa_t$  that makes a bank indifferent between deviating or not deviating from a candidate equilibrium with  $l_t^R = \gamma l_t^S$  is also  $\kappa_t^*$ .

As apparent from the latter expression, there are three forces that make  $MPK$  too high relative to the first-best benchmark: the monopoly distortion ( $\frac{\sigma}{\sigma-1} > 1$ ), the interaction between the tax distortion and the capital requirement ( $\tau\kappa$ ), and the monitoring cost ( $\Psi$ ).<sup>26</sup> As  $\kappa$  decreases (while remaining above  $\kappa^*$ ), banks' cost of financing loans declines (as banks can finance these loans with a larger fraction of deposits, which are cheaper than equity because of the tax distortion); therefore, capital goods producers' cost of borrowing declines too, so that the capital stock rises. As we show in the Appendix, not only the capital stock, but also welfare rises as  $\kappa$  decreases to  $\kappa^*$ . The first two panels of Figure 1 illustrate this result using the calibration that will be presented in the next section (in which  $\kappa^* = 0.12$ ). The last two panels show that, under this calibration, consumption and hours worked both rise too as  $\kappa$  decreases to  $\kappa^*$ .

Figure 1: Steady-State Effects of Capital Requirements



As  $\kappa$  crosses the threshold  $\kappa^*$  from above, banks move from the safe corner to the maximum-risk corner. There is a discontinuous increase in the use of the inefficient technology R and, therefore, in the resource cost of maintaining a given capital stock, or, equivalently, in the effective depreciation rate. Indeed, at the safe corner, this cost (steady-state investment and monitoring cost) is  $(\delta + \Psi)k$ , so that the effective depreciation rate is  $\delta^S \equiv \delta + \Psi$ . Using the steady-state versions of (1), (4), (9), (10), (12), (19) with equality, and (21), it is easy to show that, at the maximum-risk corner, the effective depreciation rate is instead

$$\begin{aligned}
\delta^R &\equiv \frac{1 + \gamma + \Psi}{1 + \gamma(1 - \phi) \exp(\eta^R)} - (1 - \delta) \\
&= \delta^S + \frac{\gamma [1 - (1 + \Psi)(1 - \phi) \exp(\eta^R)]}{1 + \gamma(1 - \phi) \exp(\eta^R)} \\
&> \delta^S + \frac{\gamma(1 + \Psi) [1 - \Psi - (1 - \phi) \exp(\eta^R)]}{1 + \gamma(1 - \phi) \exp(\eta^R)} \\
&> \delta^S,
\end{aligned}$$

where the first inequality follows from  $\frac{1}{1+\Psi} > 1 - \Psi$  and the second one from the inefficiency

<sup>26</sup>In addition to distorting  $MPK$ , the monitoring cost also lowers welfare through the goods-market-clearing condition.

condition (11). Thus, as  $\kappa$  crosses the threshold  $\kappa^*$  from above, there is a discontinuous increase in the effective depreciation rate. The only other effect of  $\kappa$  crossing the threshold  $\kappa^*$  is a change in the *MPK* distortion. Unlike the former one, however, the latter effect is continuous in  $\kappa$ : it is easy to show that the steady-state versions of (23) and (24) give the same value for the financial wedge at  $\kappa = \kappa^*$ . This explains why, in Figure 1, consumption decreases, hours worked increase, and welfare decreases discontinuously as  $\kappa$  crosses the threshold  $\kappa^*$  from above. Given the discontinuous increase in hours worked, there has to be a discontinuous increase in the capital stock as well in order to maintain the marginal product of capital constant, as illustrated in the second panel of Figure 1.

Finally, at the maximum-risk corner (i.e. for  $\kappa < \kappa^*$ ), the capital stock rises as  $\kappa$  decreases to 0, as we show in the Appendix and as is illustrated in the second panel of Figure 1. This happens for the same reason as above, namely because of a decline in banks' cost of financing loans. We also show in the Appendix that, under a certain condition on the parameters (which is met by our calibration, as apparent from the first panel of Figure 1), welfare rises too as  $\kappa$  decreases to 0.<sup>27</sup> This rise in welfare is associated with increases in both consumption and hours worked, as shown in the last two panels of Figure 1.

### 5.3 Globally Optimal Policies

Our discussion above implies that steady-state welfare is maximized either at  $\kappa = \kappa^*$ , or at some value for  $\kappa$  between zero and  $\kappa^*$  (the latter value being zero when welfare is decreasing in  $\kappa$  for  $0 \leq \kappa < \kappa^*$ , as is the case under the calibration that we consider). Which of these dominates will notably depend on the size of the *MPK* distortion and the effective depreciation rates. The condition on the structural parameters for steady-state welfare to be maximized at  $\kappa = \kappa^*$  can easily be obtained from the values of  $c$  and  $h$  (as functions of  $\kappa$ ) given in the Appendix. In the rest of this subsection, we restrict the analysis to the set of structural-parameter values such that this condition is met. Note that it is met in particular under the calibration that we consider, as apparent from the first panel of Figure 1.

Turning to the dynamic model, consider alternative prudential policies in conjunction with their respective Ramsey-optimal monetary policies. The welfare ranking of these policies involves comparison of both the steady-state and the fluctuations components of welfare across these policies. For sufficiently small fluctuations, this welfare ranking coincides with the ranking of steady-state welfare levels, so that policies that keep  $\kappa_t$  in the neighborhood of  $\kappa^*$  dominate policies that keep

<sup>27</sup>When this condition is not met, welfare first increases and then decreases as  $\kappa$  falls from  $\kappa^*$  to 0. The latter decrease is due to the fact that the capital stock is then inefficiently large because of banks' limited liability. This can be seen by using the steady-state versions of (3), (4), (5), (13), and (24) to express *MPK* as a function of  $\kappa$  and show that, for sufficiently low values of  $\kappa$ , *MPK* can be lower than its optimal value conditionally on the effective depreciation rate, i.e. lower than the value  $\frac{1}{\beta} - (1 - \delta^R)$ .

$\kappa_t$  in the neighborhood of any other steady-state value. Now, our locally-optimal-policy result (Proposition 6) states that the policy  $(\kappa_t, R_t^D) = (\kappa_t^*, R_t^{D*})$  is optimal among all policies keeping  $\kappa_t$  in the neighborhood of  $\kappa^*$ . Therefore, this policy is globally optimal in the sense of dominating any policy that keeps  $\kappa_t$  in the neighborhood of any steady-state value.

We turn next to a calibration exercise to confirm that the policy  $(\kappa_t, R_t^D) = (\kappa_t^*, R_t^{D*})$  can be optimal under plausible parameter values and magnitudes of shocks.

## 6 Calibration

Our calibration is reported in Table 1. Our parameter specifications for households and firms are fairly standard. The period of time is a quarter. We let

$$u(c_t, h_t) = \log(c_t) - \frac{\Xi}{1 + \chi} h_t^{1+\chi}$$

and set  $\Xi = 3.409$  and  $\chi = 0.276$  following Gertler and Karadi (2011).<sup>28</sup> We also follow Gertler and Karadi (2011) in setting the Calvo parameter  $\alpha$  to 0.779, the capital elasticity in the intermediate-good technology  $\nu$  to 0.330, and the depreciation rate  $\delta$  to 0.025. The discount factor  $\beta$  is such that the household discounts the future at the deposit rate, 2.76% per year (see Van den Heuvel, 2008). The value of the elasticity of substitution between intermediate goods  $\sigma$  is related to the degree of monopoly power firms have. Estimates of markups fall in the 10–20 percent range, implying that the elasticity of substitution lies in the 6–11 range. We follow Golosov and Lucas (2007) and set the elasticity of substitution to 7, implying a firms’ markup of about 16 percent. We fit an AR(1) process on the detrended logarithm of the TFP series corrected for utilization, as reported by Fernald (2014), for the period 1993Q1-2007Q4. This leads to a persistence parameter of 0.966 for the technology shock  $\eta_t^f$  and a standard error of 0.0068 for its innovations.

The parameters pertaining to the banking system are set as follows. The tax rate on bank profits is set to 2.29%. This value is chosen to equate the after-tax return on bank equity in our model to the after-tax return in US data.<sup>29</sup> Other parameters and shock processes are set according to the following interpretation of the 2007-2009 crisis. We assume that, before the corresponding NBER recession (2007Q4-2009Q2), the U.S. economy was at the safe corner with a constant value of  $\kappa_t$  equal to 10% (taken from Van den Heuvel, 2008). We set the average spread between the safe loan rate and the deposit rate to 2.26% per annum during that period (using Van den Heuvel’s

<sup>28</sup>Gertler and Karadi (2011) also allow for consumption habits and investment-adjustment costs to get empirically plausible impulse-response functions (IRFs). Our model does not incorporate these features, nor any other features added to medium-scale New Keynesian models to match empirical IRFs.

<sup>29</sup>In the data, the after-tax return on equity is given by  $(1 - \tau^c)\pi/e$  where  $\tau^c$ ,  $\pi$  and  $e$  respectively denote corporate tax rate, profits and equity. In our model, this quantity is given by  $(1 - \tau)(\pi + e)/e - 1$  where  $\tau$  denotes the proper tax rate that applies in our model. By equating these two quantities, and using the fact that the average return on equity is 7% and the tax rate on corporate profits is 35%, we obtain the number reported in Table 1.

calculations for total loans between 1993 and 2004). We then use (23) and obtain  $\Psi = 0.003$  per quarter.<sup>30</sup>

We think of  $R_t^R$  as the rate of return on risky assets that are always traded in the economy (although not necessarily by banks). We use Gilchrist and Zakrajšek’s (2012) spread as our proxy of  $R_t^R - R_t^S$ , and use (15) to compute  $\eta_t^R$  from 1993Q1 to 2012Q4. For our benchmark calibration, we estimate an AR(1) process for  $\eta_t^R$  over the 1993Q1-2007Q3 period, and obtain the persistence parameter  $\rho_R = 0.905$  and the innovation standard deviation  $sd_R=0.6e-03$ . We will also consider an alternative calibration estimating the AR(1) process over 1993Q1-2012Q3. This does not change our estimate of the persistence parameter, but more than doubles our estimate of the standard deviation (to 1.5e-03). We will report the results under the alternative calibration when the difference is noteworthy.

Table 1: Calibration

Parameter	Description	Value
Preferences		
$\beta$	Discount factor	0.993
$\Xi$	Relative utility weight of labor	3.409
$\chi$	Inverse of labor supply elasticity	0.276
Technology		
$\nu$	Capital elasticity	0.330
$\sigma$	Elasticity of substitution	7.000
$\delta$	Depreciation rate	0.025
Nominal rigidities		
$\alpha$	Price stickiness	0.779
Banking		
$\tau$	Tax rate	0.023
$\kappa^*$	Steady-state optimal capital requirement	0.120
$\Psi$	Marginal monitoring cost	0.003
$\phi$	Failure probability	0.034
$\gamma$	Maximal risky/safe loans ratio	0.385
$\eta^R$	Steady-state productivity of the risky technology	0.012
Shock processes		
$\rho_f$	Persistence of $\eta_t^f$	0.966
$\rho_R$	Persistence of $\eta_t^R$	0.905
$sd_f$	Standard deviation of innovations to $\eta_t^f$	6.8e-03
$sd_R$	Standard deviation of innovations to $\eta_t^R$	0.6e-03

Van den Heuvel (2008) finds that a constant capital requirement of 10% is sufficient to deter

<sup>30</sup>We also considered a shock on the monitoring cost,  $\Psi_t$ , and calibrated its stochastic process using (23) (with  $\Psi$  replaced by  $\Psi_t$ ) and Federal Reserve Bank of St. Louis data on the bank prime loan rate and the deposit rate. We found this shock to be quantitatively insignificant and chose therefore not to keep it.



inefficient risk taking, and our interpretation of the pre-crisis period is consistent with this finding. More recent contributions suggest that higher capital requirements are necessary to deter risk taking in the aftermath of the crisis. For example, Begenau (2015) obtains an optimal capital requirement of 14%. For our calibration, we assume that the steady-state value of  $\kappa_t^*$  is currently 12%; given the estimated shock process for  $\eta_t^R$  and the rest of our calibration, this implies that  $\kappa_t^*$  fluctuates between 10% and 14% with probability 0.927.<sup>31</sup>

We assume that changes in the steady-state value  $\eta^R$  increased  $\kappa^*$  and triggered the crisis. Setting  $\kappa^*$  to 12% and  $\eta^R$  to 0.012 (the average value of Gilchrist and Zakrajšek’s spread during the crisis, 2007M12-2009M6) in the steady-state version of (27) leads to one relationship between  $\gamma$  and  $\phi$ . Moreover, in our model, when the economy switches from the safe to the maximum-risk corner, the charge-off rate increases from 0 to  $\frac{\gamma\phi}{1+\gamma}$ . Setting  $\frac{\gamma\phi}{1+\gamma}$  to the observed increase in the charge-off rate during the crisis leads to another relationship between  $\gamma$  and  $\phi$ .<sup>32</sup> Solving the two-equation system leads to  $\gamma = 0.385$  and  $\phi = 0.034$ .<sup>33</sup>

It is easy to check that this calibration satisfies the two conditions imposed on the parameters: the inefficiency condition (11) and, by construction, the condition  $\kappa^* > 0$ .

## 7 Numerical Results

We consider alternative specifications of prudential policy. One specification sets  $\kappa_t = \kappa_t^*$ , which ensures that the risky technology is not used. The other specifications set constant values of  $\kappa_t$ , either below 10% or above 14%. We treat these as low (or high) enough for the risky technology to be always (or never) used, given the low probability that  $\kappa_t^*$  falls outside the 10%–14% range under our baseline calibration. For each specification of prudential policy, we consider the Ramsey-optimal monetary policy. We use the program Get Ramsey [developed by Levin and López-Salido (2004) and used in Levin, Onatski, Williams and Williams (2005)] to get the optimality conditions of the Ramsey monetary-policy problem. We then use Dynare to solve numerically, at the first or second order, the resulting system.<sup>34</sup>

The condition stated in Proposition 6 is that the right derivative of welfare with respect to  $\kappa_t$  at  $(R_t^{D^*}, \kappa_t^*)$  is strictly negative. We check numerically that this derivative is indeed strictly negative. We can infer this derivative from the Lagrange multiplier associated to the constraint  $\kappa_t = \kappa_t^*$  in

<sup>31</sup>In our alternative calibration with a more volatile  $\eta_t^R$ ,  $\kappa_t^*$  fluctuates between 8% and 16% with probability 0.865.

<sup>32</sup>More specifically, using Federal Reserve data on the charge-off rate on total loans and leases, we subtract the average charge-off rate before the crisis (1993Q1-2007Q3) from the average charge-off rate during the crisis (2007Q4-2009Q2) and get  $\frac{\gamma\phi}{1+\gamma} = 1.61 - 0.67 = 0.94\%$ .

<sup>33</sup>This two-equation system leads to a quadratic equation in  $\gamma$ , which has a unique positive solution.

<sup>34</sup>Following Benigno and Woodford (2006, 2012), the welfare computation is performed using a second-order perturbation method.

the optimization problem that determines  $R_\tau^{D*}$ . This Lagrange multiplier is negative in the steady state, and remains negative (at the first order) in the presence of shocks.<sup>35</sup> Therefore, the policy  $(R_t^D, \kappa_t) = (R_t^{D*}, \kappa_t^*)$  is locally Ramsey-optimal under our calibration.

We express the welfare cost of the constant- $\kappa_t$  regimes relatively to the  $\kappa_t = \kappa_t^*$  regime in units of consumption. We distinguish between a steady-state component and a fluctuations component of these welfare costs. The steady-state component  $x^{ss}$  solves

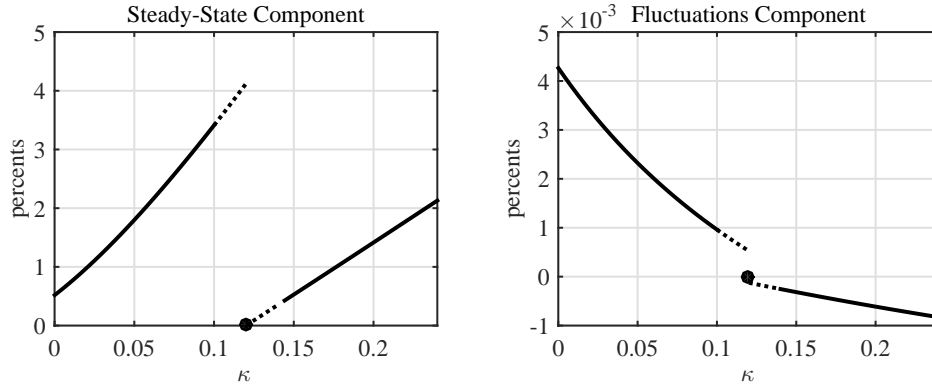
$$\log[(1 + x^{ss})c] - \Xi \frac{h^{1+\chi}}{1+\chi} = \log(c^*) - \Xi \frac{h^{*1+\chi}}{1+\chi},$$

where  $c$  and  $h$  (respectively  $c^*$  and  $h^*$ ) denote steady-state consumption and hours worked in the constant- $\kappa_t$  regime considered (respectively in the  $\kappa_t = \kappa_t^*$  regime). Likewise, the fluctuations component  $x^f$  solves

$$\mathbb{E} \left[ \sum_{t=0}^{+\infty} \beta^t \left( \log(1 + x^f) - (1 + \chi) \Xi h^{1+\chi} \frac{\widehat{h}_t^2}{2} \right) \right] = \mathbb{E} \left[ \sum_{t=0}^{+\infty} \beta^t \left( - (1 + \chi) \Xi h^{*1+\chi} \frac{\widehat{h}_t^{*2}}{2} \right) \right],$$

where  $\widehat{h}_t$  (respectively  $\widehat{h}_t^*$ ) denotes the log-deviation of hours worked relatively to their steady-state value in the constant- $\kappa_t$  regime considered (respectively in the  $\kappa_t = \kappa_t^*$  regime). Figure 2 reports these two components in percentages ( $100 \times x^{ss}$  and  $100 \times x^f$ ), as a function of the constant value  $\kappa$  considered for the capital requirement  $\kappa_t$ . The lines are dashed for  $\kappa$  within the 10%-14% range to indicate that the corresponding figures should be taken cautiously (as the assumption that the risky technology is always or never used cannot reasonably be considered to be satisfied for  $\kappa$  within that range).

Figure 2: Welfare Cost



The left panel of Figure 2 shows large steady-state welfare costs.<sup>36</sup> Under our baseline calibration, setting  $\kappa_t = \kappa_t^*$  dominates the next best policy, which sets  $\kappa_t = 0$ , as we noted in Subsection 5.3.

<sup>35</sup>The mean of this Lagrange multiplier is  $-0.110$  and its standard deviation is  $0.002$ .

<sup>36</sup>These welfare costs reflect differences across steady-state welfare levels, ignoring any welfare effects arising from transitional dynamics in our model. Alternatively, we could consider, for example, an economy that is in a steady state with a 14% capital requirement and ask what is the welfare cost of raising the capital requirement to 16%. The

The welfare cost of the latter policy is worth about half a percent of consumption per period. However, this particular comparison ( $\kappa_t = \kappa_t^*$  versus  $\kappa_t = 0$ ) is somewhat sensitive to our calibration assuming a monopoly markup of about 16%; if we raise the monopoly markup to about 30%, we can reverse the result. One result that is quite robust across plausible parameter values is the high steady-state welfare cost associated with capital requirements that are well above zero but still fall short of taming risk taking. For example, the cost of a policy that sets  $\kappa_t = 0.08$  instead of  $\kappa_t = \kappa_t^*$  is over 2.7% of consumption. These high costs arise from the effective depreciation rate and the MPK distortion, as we discussed in Subsection 5.2. The steady-state welfare costs are lower but still sizeable for policies that set a constant capital requirement and make it high enough to deter risk taking. For example, the cost of setting  $\kappa_t = 0.16$  is over 0.7% of consumption. These costs arise from aggravating the MPK distortion, as we discussed in Subsection 5.2.

The right panel of Figure 2 shows that the fluctuations component of welfare costs is very small under optimal monetary policy. For  $\kappa > \kappa^*$ , this component is negative (i.e., corresponds to a welfare gain) because fluctuations are smaller in the  $\kappa_t = \kappa$  regime than in the  $\kappa_t = \kappa_t^*$  regime, as the financial shock  $\eta_t^R$  is not transmitted to the economy in the former regime. The cost of setting  $\kappa_t = \kappa_t^*$  compared to setting  $\kappa_t = 0.16$ , for example, only amounts to 0.0004% of consumption.<sup>37</sup>

We now turn to the impulse-response functions, focusing on three alternative specifications:  $\kappa_t = \kappa_t^*$ ,  $\kappa_t = 0.14$ , and  $\kappa_t = 0.10$ , and expressing the responses of output, hours worked, consumption, investment, and the capital stock as percentage deviations from each steady state. Figure 3 reports the responses to a favorable technology shock (a one standard-deviation innovation to  $\eta_t^f$ ). Since a productivity shock does not affect risk-taking incentives in our benchmark model, optimal prudential policy does not respond to this shock, and the responses of optimal monetary policy are essentially the same across our three specifications of prudential policy. Optimal policy raises the deposit rate to keep inflation at zero; output, consumption, investment, and hours rise. The patterns are familiar from standard New Keynesian models with capital.<sup>38</sup>

Figure 4 reports the responses to a favorable shock to the risky technology for producing capital (a one standard-deviation innovation to  $\eta_t^R$ ). Such a shock increases the temptation for banks to finance risky projects. Optimal prudential policy raises the capital requirement to avert risk

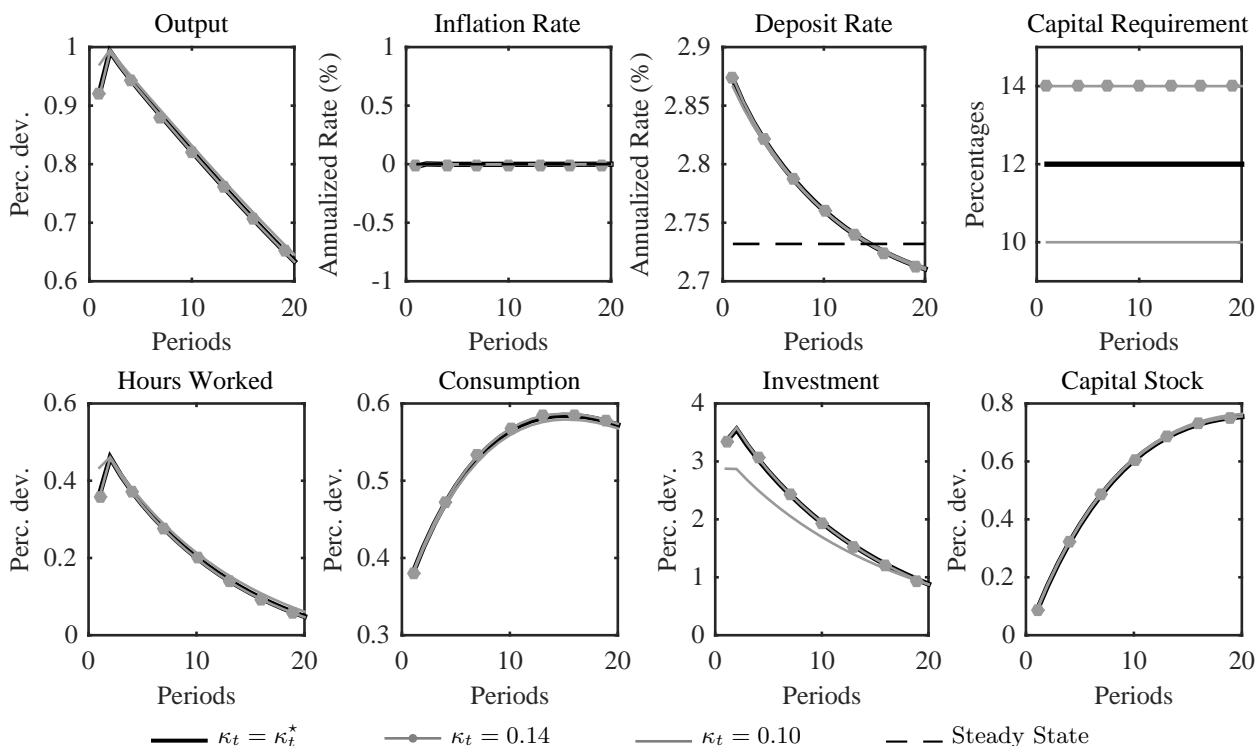
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welfare cost will be smaller than the difference across the two steady states because the capital stock will be falling during the transition, and consumption can be higher during the transition to a higher capital requirement. The welfare cost taking account of the transition is 0.24% in our model, which is smaller than, but of the same order of magnitude as the welfare difference of 0.40% across the two steady states depicted in Figure 2.

<sup>37</sup>The results for the fluctuations component of welfare costs are similar under our alternative calibration (which uses the GZ-spread data until 2012). Under the latter calibration, however,  $\kappa_t^*$  fluctuates substantially more, so that the assumption that  $\kappa_t = 0.14$  always deters risk taking and  $\kappa_t = 0.10$  always entails the use of the risky technology seems no longer reasonable.

<sup>38</sup>In particular, the deposit rate is raised under optimal policy because both the favorable productivity shock and the resulting increase in employment increase the marginal product of capital and therefore the natural real interest rate.

Figure 3: Response to a Favorable Technology Shock ( $\eta_t^f$ )



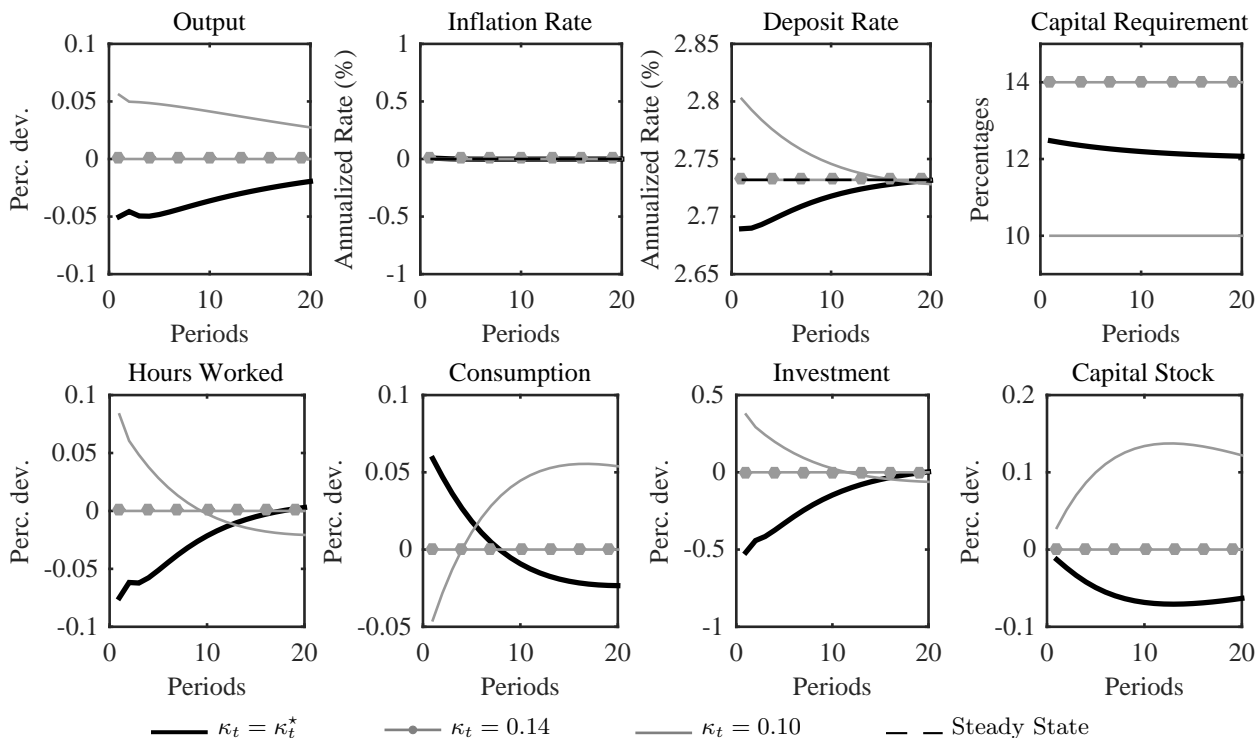
taking. Because the risky technology is not used, the only effect of the shock (in the absence of a monetary-policy reaction) goes through the increase in the financial wedge entailed by the rise in the optimal capital requirement. This effect is contractionary, because it increases the cost of banking. In response, optimal monetary policy cuts the deposit rate. The optimal monetary response is analogous to how optimal monetary policy would respond to an exogenous shock to the financial wedge. As Equation (23) shows, changes in the capital requirement in our model essentially amount to exogenous shocks to the financial wedge, as far as monetary policy is concerned. In the end, the contractionary effect is very small; output only falls by about 0.05% and inflation essentially remains at zero. The higher capital requirement reduces investment and, with the lower deposit rate, the composition of aggregate demand tilts slightly towards consumption.<sup>39</sup>

The  $\eta_t^R$  shock has no effects (in Figure 4) when we set  $\kappa_t = 0.14$  because this policy is sufficient to deter risk taking, and there is no reason for monetary policy to change. When we set  $\kappa_t = 0.10$  and allow the the risky technology to be used, the  $\eta_t^R$  shock lowers the effective depreciation rate of capital for a while (as the shock is persistent). This encourages investment, hours and output

<sup>39</sup>The same observations would apply (at least qualitatively) to optimal responses to hypothetical shocks to the probability of failure of the risky technology ( $\phi_t$ ) and the maximal risky/safe loans ratio ( $\gamma_t$ ). These shocks would affect the economy only through their effect on the optimal capital requirement  $\kappa_t^*$  (defined by (27) with  $\phi$  and  $\gamma$  being replaced by  $\phi_t$  and  $\gamma_t$  respectively), which in turn would call for a monetary-policy response to mitigate the macroeconomic effects.

increase, while consumption falls slightly upon impact. The optimal monetary response is a small increase in the deposit rate to moderate the expansion and keep inflation at zero. Overall, the effects of  $\eta_t^R$  when  $\kappa_t = 0.10$  are similar to those of an investment-specific shock in standard models.

Figure 4: Response to an Increase in the Productivity of the Risky Technology ( $\eta_t^R$ )



We find this thought experiment quite useful in the context of policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012), Macklem (2011), Wolf (2012), Yellen (2010)] of how monetary and prudential policies may be substitutes for each other or move to offset each other's effects. In our benchmark model, under jointly optimal policies, one policy is contractionary and the other expansionary in order to manage risk-taking incentives with the smallest possible adverse effects on investment. Thus, our model highlights a distinction across policy instruments that we think deserves more emphasis than it gets in the existing literature: changes in the capital requirement can directly manage risk-taking incentives, while changes in the policy interest rate cannot. When the capital requirement rises to curb risk taking, a contraction ensues, and the policy interest rate is cut. With this chain of causality, optimal prudential policy is pro-cyclical, and optimal monetary policy is counter-cyclical.

Nonetheless, our model also provides a framework for thinking about some scenarios (or extensions) that can make optimal prudential policy counter-cyclical, as we discuss below.

## 8 Extensions and Policy Concerns

Our benchmark model, while stylized, provides several useful insights. For example, as Angeloni and Faia (2011) elaborate, the leading argument for Basel III-type counter-cyclical capital requirements is the observation that default risk rises during recessions; and risk-weighted (Basel II-type) capital requirements automatically tighten policy in recessions, unless the regulatory rate is lowered.<sup>40</sup> Our model suggests a reason for the latter to happen, that is, for cutting capital requirements when default risk is high: when the banks have enough skin in the game, the increased default risk makes banks less inclined to fund risky projects, allowing prudential policy to set lower requirements without undermining the stability of the banking system.

In this section, we illustrate how (admittedly ad hoc) extensions can provide additional insights. We consider two extensions: externalities in bank lending, and correlation between shocks affecting the incentives to take risks and shocks to the business cycle. We show that each of these two extensions can make both policy instruments counter-cyclical under optimal policy. We also show that, although the first extension gives rise to a risk-taking channel of monetary policy, it does not qualitatively affect the optimal policy responses to shocks that directly affect risk-taking incentives.

### 8.1 An Externality

Our model assumes perfect competition and constant returns in the banking sector. As we noted earlier, these assumptions imply that shocks that directly affect the optimal policy interest rate (like standard productivity or fiscal shocks) do not affect the optimal bank-capital requirement. We now consider a simple (ad-hoc) extension that links the cost of banking to the aggregate volume of safe loans and thus allows such shocks to affect both policy margins. Hachem (2010) develops a model with an externality in banking costs. In her model, banks ignore the effect of their own lending decision on the pool of borrowers, with heterogeneous levels of risk, that is available to other banks.<sup>41</sup> Here, we only consider a simple example of such an externality – in order to preserve our earlier derivations that treated  $\Psi$  as exogenous to the banks' decisions – but we think this example highlights the main features of policy interactions that arise when an economic boom increases risk-taking incentives. Specifically, we assume that the monitoring cost is now

$$\log(\Psi_t) = \log(\Psi) + \varrho \left[ \log(l_t^S) - \log(l^S) \right] \quad (28)$$

where the term  $\log(l_t^S) - \log(l^S)$  is the log-deviation of the aggregate volume of safe loans from its steady-state value, and  $\varrho \geq 0$  ( $\varrho = 0$  corresponding to our benchmark model). We report the impulse

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<sup>40</sup>See Covas and Fujita (2010) for a quantitative assessment of the procyclical effects of bank-capital requirements under Basel II.

<sup>41</sup>Gete and Tiernan (2011) consider the role of capital requirements in Hachem's (2010) model, but abstract from monetary policy.

responses under optimal policy for  $\varrho = 0, 1,$  and  $5$ . Figure 5 shows the responses to a favorable productivity shock. Following this shock, the volume of safe loans increases, and therefore so do the monitoring cost and risk-taking incentives. Optimal prudential policy raises the capital requirement in order to discourage risk taking. This makes optimal prudential policy counter-cyclical, which leads optimal monetary policy to be less restrictive (raises the deposit rate by less, and later on cuts it by more) than in the benchmark model.<sup>42</sup>

Figure 5: Response to a Favorable Technology Shock ( $\eta_t^f$ )

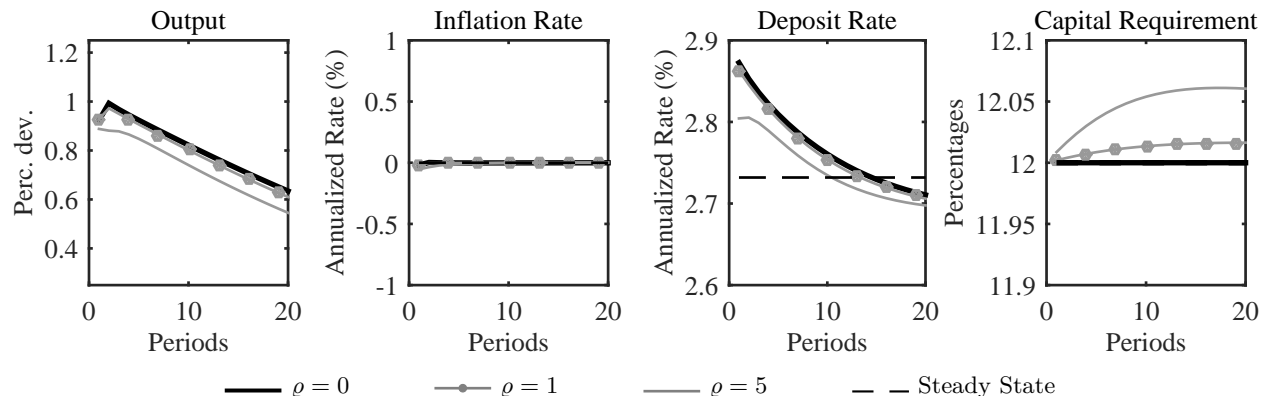


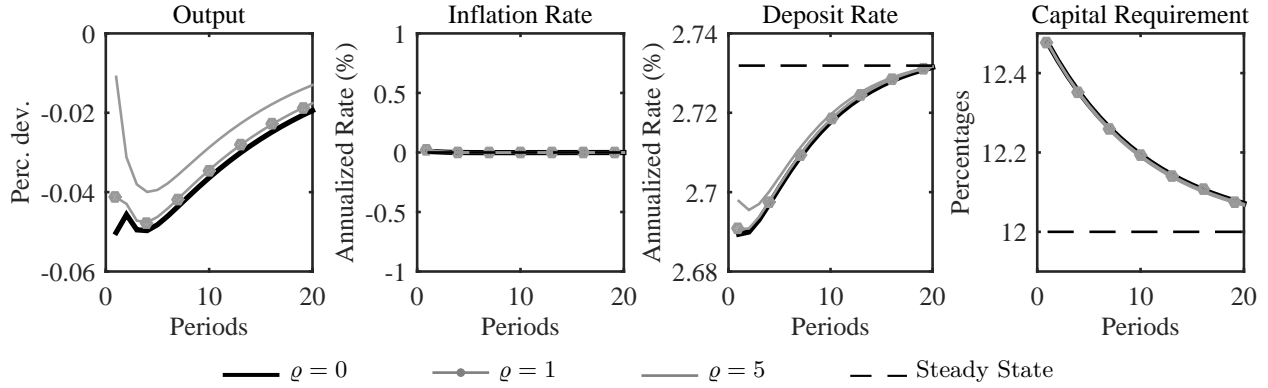
Figure 6 shows the optimal responses to an increase in  $\eta_t^R$ , the productivity of the risky technology conditionally on its success. Absent the externality (looking at the solid black lines in the Figure), optimal prudential policy raises the capital requirement because banks are more tempted to take risk, while optimal monetary policy cuts the deposit rate to curb the contractionary effects of prudential policy. With the externality, the contraction creates a temptation to take less risk (as the cost of making safe loans decreases). So, optimal prudential policy raises the capital requirement by less, and optimal monetary policy cuts the deposit rate by less.<sup>43</sup> In terms of optimal output fluctuations in Figures 5 and 6, the externality always dampens the optimal response (expansion or contraction) of output.

Thus, some key normative implications of the benchmark model are, qualitatively speaking, robust to the introduction of a risk-taking channel of monetary policy (via a lending externality). Optimal policy still uses capital requirements to counter risk-taking incentives, i.e. still raises (respectively cuts) capital requirements in response to shocks that increase (respectively decrease) these incentives. In principle, the deposit rate could have been used for this purpose, since the risk-taking channel of monetary policy implies that it now affects risk-taking incentives. But optimal policy does not use the deposit rate this way in response to shocks that *directly* affect

<sup>42</sup>Optimal monetary policy actually strikes a balance between this effect and another, smaller effect stemming from the externality (which is that banks have a tendency to lend too much as they ignore the effect of their own lending decision on monitoring costs).

<sup>43</sup>The dampening effect of the externality on the optimal capital requirement is quantitatively very small and therefore little apparent in Figure 6.

Figure 6: Response to an Increase in the Productivity of the Risky Technology ( $\eta_t^R$ )



risk-taking incentives (as in Figure 6). Instead, in response to these shocks, it still uses the deposit rate to mitigate the macroeconomic effects of capital requirements, i.e. still cuts (respectively raises) the deposit rate when capital requirements are raised (respectively cut). Moreover, as the strength of the risk-taking channel of monetary policy varies, optimal monetary policy becomes more accommodative (or less restrictive) when optimal prudential policy becomes more restrictive (or less accommodative) in response to any given shock.

In terms of optimal institutional arrangement, the implications of the benchmark model are also robust to the introduction of a risk-taking channel of monetary policy. In the absence of this channel, only prudential policy can affect risk-taking incentives, so that the prudential authority should be assigned a financial-stability mandate (to be fulfilled with minimal damage in terms of increased banking costs) and the monetary authority a macroeconomic-stability mandate. This separation principle remains optimal in the presence of that channel, in the sense that optimal policy can still be implemented by assigning the same respective mandates to the prudential and monetary authorities.<sup>44</sup>

## 8.2 Correlated Shocks

Correlations across shocks may also link risk-taking incentives to shocks that have direct business-cycle effects and may make both optimal policies counter-cyclical. As an example, we replace (9) by

$$k_{t+1}(i) = \exp\left(\eta_t^S\right) x_t(i),$$

thus adding a shock to the safe technology for producing capital goods, and we allow for the possibility that  $\eta_t^S$  is correlated with  $\eta_t^R$  (the shock to the risky technology). This modification

<sup>44</sup>It should be noted, however, that within the realm of the model the optimal institutional arrangement is not unique in the presence of a risk-taking channel of monetary policy. For instance, swapping the two mandates would deliver the same allocation.



changes our inefficiency condition (11) to

$$(1 - \phi) \exp(\eta_t^R) \leq \exp(\eta_t^S) - \Psi,$$

the optimality condition (13) to

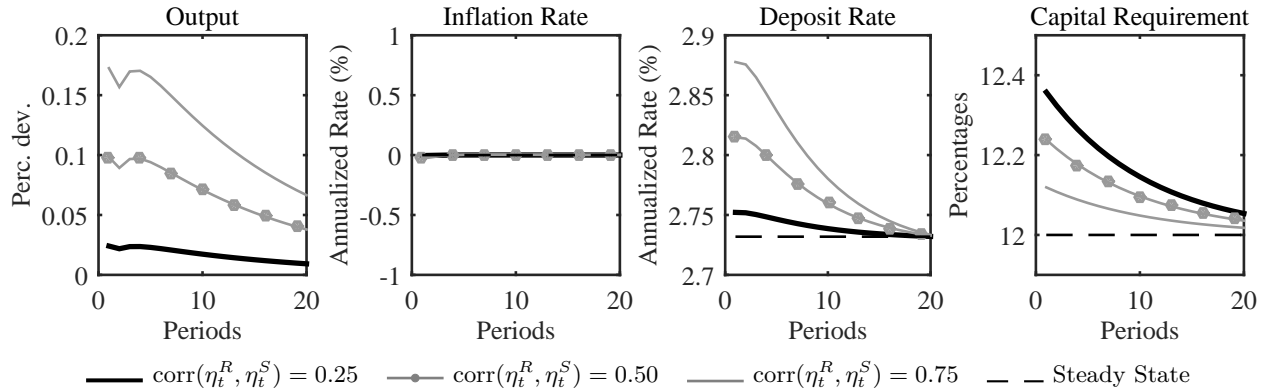
$$E_t \{ \lambda_{t+1} q_{t+1} \} = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^S) \exp(-\eta_t^S) q_t^x,$$

and our optimal capital requirement to

$$\kappa_t^* = (1 - \tau) \frac{(1 - \phi) \gamma [\exp(\eta_t^R - \eta_t^S) - 1] + \Psi [(1 - \phi) \gamma \exp(\eta_t^R - \eta_t^S) - \phi]}{\phi (1 + \gamma) - \gamma \tau (1 - \phi) [\exp(\eta_t^R - \eta_t^S) - 1]}.$$

Figure 7 shows the optimal responses to a positive innovation in  $\eta_t^R$  for three values of its correlation with the innovation to  $\eta_t^S$ : 0.25, 0.50, and 0.75. The correlation makes both optimal policies act in a counter-cyclical way. Optimal prudential policy raises the capital requirement to tame risk taking, and optimal monetary policy raises the deposit rate to tame the effects of the investment boom.

Figure 7: Response to an Increase in the Productivity of the Risky Technology ( $\eta_t^R$ )



## 9 Concluding Remarks

This paper models the optimal interaction of monetary and prudential policies in a setting that views bank-capital requirements as a tool for addressing the risk-taking incentives created by limited liability and deposit insurance. In this section, we highlight the main policy implications of our model and put some of our modeling choices, and the assumptions that are behind our results, in the context of other work and recent commentary motivated by the 2007-2009 financial crisis.

In our model, higher capital requirements increase the costs of banking (because equity finance is costly), and this exacerbates other distortions that make the capital stock too small. Absent

sufficiently stringent capital requirements, however, banks may be tempted to fund an inefficient technology for producing capital goods. Because of banks' limited liability, this technology is profitable to banks when it succeeds, but costly to the deposit insurance fund when it fails. This technology is inefficient in the sense of producing less than the safe technology on average (across the success and failure states). So, banks may fund risky projects only to take advantage of their limited liability – and when they do, they want to invest in a risky project as much as they can hide from regulators because they know that failure of the project will wipe out their equity anyway.

One way to view the 2007-2009 crisis through the lens of our model is to argue that banks found a new way to increase the amount of risk they could hide from regulators. In our model, this would be captured by an increase in the parameter  $\gamma$  – while, in reality, it may have involved obfuscating risks associated with off-balance-sheet positions. An increase in  $\gamma$  raises the critical value  $\kappa_t^*$  of the capital requirement needed to dissuade banks from funding the risky technology. If the capital requirement is left unchanged, the economy may switch from the safe equilibrium to the equilibrium with maximal undetected risk. And the switch in our model is analogous to the capital-quality shock in the models of Gertler and Kiyotaki (2011) and Gertler and Karadi (2011). Mechanically, it amounts to an increase in the depreciation rate of capital, but Gertler and Karadi (2011) offer interpretations in terms of the quality or usefulness of existing capital.<sup>45</sup>

Our benchmark model with perfectly competitive banks and constant marginal costs leads to a simple optimal assignment of tasks to prudential and monetary policies. The locally optimal mandate of prudential policy is to ensure that banks never fund inefficient risky projects, but to achieve this objective with minimal damage in terms of increased bank lending rates and decreased capital stock. In other words, prudential policy should be assigned the primary objective of financial stability, and the secondary objective of minimal banking costs (conditionally on achieving the primary objective). These objectives are achieved when capital requirements are state-dependent and respond to shocks that affect the relative attractiveness of risky and safe projects. Monetary policy, meanwhile, should be assigned the objective of macroeconomic stability, taking into account the effects of prudential policy on the economy. The optimal interaction across monetary and prudential policies then boils down to cutting (raising) interest rates to moderate the contractions (expansions) caused by changes in the capital requirement. We show that this locally optimal policy is globally optimal under some parameter restrictions that can be satisfied under plausible calibrations.

In our benchmark model, monetary policy (setting money-market rates) does not affect risk-taking incentives. As we noted in the Introduction, this is not meant to negate the importance of

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<sup>45</sup>Gertler and Karadi (2011) postulate a large and unexpected one-time drop in capital quality as the shock that led to the financial crisis. In our model, losses in capital production are smaller and may work over several periods (while the bank-capital requirement remains too low).

recent contributions that emphasize the risk-taking channel of monetary policy. Our goal, rather, is to formalize an alternative view that argues for relegating the goal of financial stability to prudential policy. Nonetheless, the quantitative significance of departures from our benchmark policy prescription may well depend on issues we have not modeled. In particular, our model abstracts from how booms (and periods with low interest rates) may lead to expansions of bank balance sheets. In models following Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), for example, banks have an equity stake in firms – so, bank equity rises automatically when the stock market booms. In our model, banks are (narrowly) viewed as lenders, and they act competitively. So, regardless of the state of the economy or the stance of monetary policy, bank lending rates adjust in our model to ensure that our banks make zero profits. In a similar vein, our model abstracts from changes in leverage that were linked to higher risk, according to much of the commentary on the 2007-2009 crisis.

We think our extension with an externality in the cost of banking – albeit ad-hoc and stylized – illustrates how policy interactions are more complex when risk-taking incentives change over the business cycle. In this extension, an increase in the aggregate volume of safe loans increases the costs of originating and monitoring safe loans. This feature, which gives rise to a risk-taking channel of monetary policy, matters for the optimal policy interactions. In particular, it makes both policy instruments counter-cyclical under optimal policy in response to certain shocks (like productivity shocks). However, it does not affect the main implications of the benchmark model for the optimal policy responses to shocks that directly affect risk-taking incentives: in responses to these shocks, prudential policy should still be used to tame risk-taking incentives (including those created by monetary policy when it is accommodative) and monetary policy to mitigate the macroeconomic effects of prudential policy. The presence of a risk-taking channel of monetary policy does not invalidate the separation principle, in the sense that optimal policy can still be implemented by assigning a financial-stability mandate to the prudential authority and a macroeconomic-stability mandate to the monetary authority.

The implementation of optimal state-dependent policy would obviously be complicated in reality because risk-taking incentives are not easy to observe. We have sidestepped the question of how the regulators in our model observe the risk-taking incentives of banks by assuming that risky assets exist in the economy even though banks do not hold these assets under optimal policy.<sup>46</sup> However, the broader point highlighted by our (benchmark and extended) models is that time variation in capital requirements – raising the requirement when risk-taking incentives are likely to be high – can serve to keep the average requirement over the cycle lower. Our welfare calculations suggest

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<sup>46</sup>We have also sidestepped questions about other determinants of the optimal capital requirement in Equation (27), by treating them as known parameters – for example, the maximum amount of risk that banks can hide in their portfolios plays a central role in determining optimal policy, and we have assumed regulators know the value of this parameter.

this can be important. In our models, the optimal capital requirement fluctuates considerably, but the welfare loss associated with these fluctuations is much smaller than the welfare loss associated with a higher steady-state capital requirement.

Our model takes deposit insurance as an institutional feature that does not have to be rationalized within the model.<sup>47</sup> The other institutional feature is our assumption that a tax distortion makes equity finance more expensive than debt finance. We are not aware of any arguments for claiming that this is a feature of optimal policy in some expanded framework. To the contrary, existing discussions of this tax distortion [e.g., Admati et al. (2011), Mooij and Devereux (2011)] note its prevalence in OECD countries and call for removing it. Our motivation for including this policy-induced distortion in our model is this prevalence and the fact that central banks and prudential regulators cannot change the tax code.<sup>48</sup> We think this tax distortion merits more attention in models of how the banking sector matters for monetary-policy analysis.<sup>49</sup>

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<sup>47</sup>Presenting an expanded model in which deposit insurance is optimal (rather than taking it as an exogenous feature) seemed too much of a digression to us, but we could motivate deposit insurance as usual [e.g., following Angeloni and Faia (2011)] in terms of ruling out equilibria with bank runs.

<sup>48</sup>Besides, under an arbitrarily small tax distortion, all our analytical results (from Proposition 1 to Proposition 6) would still hold, as banks would still prefer debt finance to equity finance, though the condition stated in Proposition 6 (the “if” part of this proposition) might not be met.

<sup>49</sup>For one thing, this may account for the fact that banks extend credit using loan contracts in reality, even though loan contracts are not optimal according to most formal models (with the notable exception of models with costly state verification).

## 10 Appendix

### 10.1 Proof of Proposition 1

The bank chooses the set  $I$  and the loan amounts  $l_t(i)$  for  $i \in I$  in order to maximize

$$\mathbb{E}_t \{ \max(0, r_{t+1} - b_t) \},$$

where

$$r_{t+1} \equiv \left(1 + R_t^R\right) \sum_{i \in I} \theta_t(i) l_t^R(i),$$

subject to

$$\sum_{i \in I} l_t^R(i) = l_t^R \quad \text{and} \quad \forall i \in I, l_t^R(i) \geq 0.$$

We focus on the non-trivial case in which  $l_t^R > 0$ , so that  $|I| \geq 1$ , where  $|I|$  denotes the cardinality of  $I$ . We note  $V_t \equiv (1 - \phi) \left(1 + R_t^R\right) l_t^R - b_t$  the value, independent of  $I$  and  $l_t(i)$  for  $i \in I$ , taken by the function  $\mathbb{E}_t \{r_{t+1} - b_t\}$ . The latter function corresponds to the objective function  $\mathbb{E}_t \{ \max(0, r_{t+1} - b_t) \}$  without the  $\max(0, \cdot)$  operator.

In the case where  $l_t^R(i) \geq \frac{b_t}{1+R_t^R}$  for all  $i \in I$ , we have  $r_{t+1} - b_t < 0$  only when  $\theta_t(i) = 0$  for all  $i \in I$ , so that the objective function takes the value  $V_t + \phi^{|I|} b_t$  and is therefore maximized for  $|I| = 1$ .

In the alternative case where  $l_t^R(i) < \frac{b_t}{1+R_t^R}$  for at least one  $i \in I$ , let  $\mathcal{S}$  denote the non-empty set of integers  $i \in I$  such that  $l_t^R(i) < \frac{b_t}{1+R_t^R}$ . The objective function then takes a value of type  $V_t + f[l_t^R(i)|i \in \mathcal{S}]$ , where the function  $f$  is decreasing in each of its arguments.<sup>50</sup> So in this case the optimal value of each  $l_t^R(i)$  for  $i \in \mathcal{S}$  is zero, and we are back to previous case. Proposition 1 follows.

### 10.2 Proof of Proposition 2

To show that there is no equilibrium with  $0 < l_t^R < \gamma l_t^S$ , we suppose that there is such an equilibrium and consider a perturbation satisfying  $dl_t^S(j) = -dl_t^R(j)$  in the loan portfolio of a given bank  $j$ . Note that this perturbation neither tightens nor loosens bank  $j$ 's balance-sheet identity

$$l_t^S(j) + l_t^R(j) = e_t(j) + d_t(j) \tag{29}$$

and its capital requirement

$$e_t(j) \geq \kappa_t \left[ l_t^S(j) + l_t^R(j) \right],$$

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<sup>50</sup>For instance, when  $\mathcal{S} = \{1\}$ ,  $f[l_t^R(i)|i \in \mathcal{S}] = f[l_t^R(1)] = \phi^{|I|} b_t + \phi^{|I|-1} (1 - \phi) [b_t - (1 + R_t^R) l_t^R(1)]$  is decreasing in  $l_t^R(1)$ .

given that  $l_t^S(j) + l_t^R(j)$  is left unchanged. So this perturbation should not increase bank  $j$ 's expected excess return. The derivations of the effect of this perturbation on bank  $j$ 's expected excess return involves two cases, depending on whether bank  $j$  goes bankrupt or not when its risky project fails.

If bank  $j$  goes bankrupt when its risky project fails, then, using (3), the change in bank  $j$ 's expected excess return can be written as

$$(1 - \tau) \left[ \beta (1 - \phi) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \frac{R_t^R - R_t^S}{\lambda_t} + \Psi \right] dl_t^R(j) = (1 - \tau) \left[ (1 - \phi) \frac{R_t^R - R_t^S}{1 + R_t^S} + \Psi \right] dl_t^R(j).$$

As discussed in the main text, we must have  $R_t^R \geq R_t^S$  in equilibrium. Therefore, bank  $j$ 's expected excess return is increasing in  $l_t^R(j)$ . This means that bank  $j$  would like to take more risk, contradicting our conjecture about the existence of an equilibrium with  $l_t^R < \gamma l_t^S$ .

If bank  $j$  does not go bankrupt when its risky project fails, then, using (3) and (15), the change in bank  $j$ 's expected excess return can be written as

$$\begin{aligned} & (1 - \tau) \left[ \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}} \left[ \theta_t(j) (1 + R_t^R) - (1 + R_t^S) \right] \right\} + \Psi \right] dl_t^R(j) \\ = & (1 - \tau) \left[ \frac{(1 - \phi) (1 + R_t^R) - (1 + R_t^S)}{1 + R_t^D} + \Psi \right] dl_t^R(j) \\ = & (1 - \tau) \left\{ \left[ (1 - \phi) \exp(\eta_t^R) - 1 \right] \frac{1 + R_t^S}{1 + R_t^D} + \Psi \right\} dl_t^R(j) \\ \leq & (1 - \tau) \Psi \left( 1 - \frac{1 + R_t^S}{1 + R_t^D} \right) dl_t^R(j) \end{aligned}$$

where the inequality comes from (11). Now, we must have  $R_t^S > R_t^D$  at this equilibrium; otherwise, banks would make negative profits on their safe loans and would like to reduce the volume of safe loans (which would be possible given that  $l_t^R < \gamma l_t^S$ ). Therefore, bank  $j$ 's expected excess return is decreasing in  $l_t^R(j)$ . This means that bank  $j$  would like to take less risk, contradicting our conjecture about the existence of an equilibrium with  $0 < l_t^R$ . Proposition 2 follows.

### 10.3 Proof of Proposition 3

Using (29), one can write the expected excess return of a bank  $j$  as

$$(1 - \tau) \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b(j)}{\lambda_t} \right\} - e_t(j) - (1 - \tau) \Psi l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{R_t^S - R_t^D}{\Pi_{t+1}} l_t^S(j) + \left[ \theta_t(j) \frac{1 + R_t^R}{\Pi_{t+1}} - \frac{1 + R_t^D}{\Pi_{t+1}} \right] l_t^R(j) + \frac{1 + R_t^D}{\Pi_{t+1}} e_t(j) \right\}.$$

In the case where  $\omega_{t+1}^b(j) > 0$  when  $\theta_t(j) = 0$ , using (3), bank  $j$ 's expected excess return can be rewritten as

$$(1 - \tau) \left\{ \frac{R_t^S - R_t^D}{1 + R_t^D} l_t^S(j) + \left[ (1 - \phi) \left( \frac{1 + R_t^R}{1 + R_t^D} \right) - 1 \right] l_t^R(j) + e_t(j) \right\} - e_t(j) - (1 - \tau) \Psi l_t^S(j).$$

Since this expression is strictly decreasing in  $e_t(j)$ , it is maximized when  $e_t(j)$  is minimal, that is to say when  $e_t(j)$  satisfies

$$e_t(j) = \kappa_t \left[ l_t^S(j) + l_t^R(j) \right]. \quad (30)$$

In the alternative case where  $\omega_{t+1}^b(j) = 0$  when  $\theta_t(j) = 0$ , bank  $j$ 's expected excess return can be rewritten as

$$(1 - \tau) (1 - \phi) \left\{ \frac{R_t^S - R_t^D}{1 + R_t^D} l_t^S(j) + \left[ \frac{1 + R_t^R}{1 + R_t^D} - 1 \right] l_t^R(j) + e_t(j) \right\} - e_t(j) - (1 - \tau) \Psi l_t^S(j).$$

Similarly, this expression is strictly decreasing in  $e_t(j)$ , and is therefore maximized for  $e_t(j)$  given by (30). This establishes Proposition 3.

## 10.4 Proof of Proposition 5

To prove Part (a) of Proposition 5, we look for a necessary and sufficient condition on policy instruments for the existence of an equilibrium with  $l_t^R = 0$ . This amounts to looking for a necessary and sufficient condition on policy instruments for the demand and supply curves on the risky-loans market to intersect at one or several points  $(R_t^R, l_t^R)$  with  $R_t^R \geq 0$  and  $l_t^R = 0$ . We proceed in several steps.

**Step 1: condition for zero demand for risky loans.** Given capital producers' programme, the portion of the demand curve that is consistent with  $l_t^R = 0$  is characterized by

$$\frac{1 + R_t^R}{1 + R_t^S} \geq \exp(\eta_t^R). \quad (31)$$

**Step 2: condition for zero supply of risky loans.** The portion of the supply curve that is consistent with  $l_t^R = 0$  can be characterized by a necessary and sufficient condition for an individual bank  $j$  not to deviate from the candidate equilibrium with  $l_t^R = 0$ . We now look for such a condition. Appendix 10.2 implies that, if some deviations are profitable, then the most profitable deviation is  $l_t^R(j) = \gamma l_t^S(j)$ . If bank  $j$  makes this deviation, then, using (29) to eliminate  $d_t(j)$  and (30) to eliminate  $e_t(j)$ , its expected excess return can be rewritten as

$$(1 - \tau) \mathbb{E}_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b(j)}{\lambda_t} \right\} - [\kappa_t (1 + \gamma) + (1 - \tau) \Psi] l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \left[ \frac{R_t^S - R_t^D}{\Pi_{t+1}} + \theta_t(j) \gamma \frac{1 + R_t^R}{\Pi_{t+1}} - \gamma \frac{1 + R_t^D}{\Pi_{t+1}} + \frac{1 + R_t^D}{\Pi_{t+1}} \kappa_t (1 + \gamma) \right] l_t^S(j) \right\}.$$

Using (3), one can rewrite bank  $j$ 's expected excess return as

$$(1 - \tau) \mathbb{E}_t \left\{ \max \left\{ 0, \left[ \frac{R_t^S - R_t^D}{1 + R_t^D} + \theta_t(j) \gamma \frac{1 + R_t^R}{1 + R_t^D} - \gamma + \kappa_t (1 + \gamma) \right] l_t^S(j) \right\} \right\} \\ - [\kappa_t (1 + \gamma) + (1 - \tau) \Psi] l_t^S(j).$$

In the case where the ‘‘max’’ that features in this expression is strictly higher than zero when  $\theta_t(j) = 0$ , that is to say in the case where  $\kappa_t > \tilde{\kappa}$ , we know from Appendix 10.2 that bank  $j$ 's deviation is not profitable. In the alternative case where the ‘max’ is equal to zero when  $\theta_t(j) = 0$ , that is to say in the case where  $\kappa_t \leq \tilde{\kappa}$ , bank  $j$ 's expected excess return is

$$\left\{ (1 - \tau) (1 - \phi) \left[ \frac{R_t^S - R_t^D}{1 + R_t^D} + \gamma \frac{1 + R_t^R}{1 + R_t^D} - \gamma + \kappa_t (1 + \gamma) \right] - \kappa_t (1 + \gamma) - (1 - \tau) \Psi \right\} l_t^S(j).$$

Using (23) to eliminate  $R_t^S$ , we can rewrite it as

$$\left\{ (1 - \tau) (1 - \phi) \gamma \frac{R_t^R - R_t^D}{1 + R_t^D} - [\phi (1 + \gamma) + \gamma \tau (1 - \phi)] \kappa_t - \phi_t (1 - \tau) \Psi \right\} l_t^S(j).$$

Therefore, a necessary and sufficient condition for the deviation not to be profitable is then

$$[\phi (1 + \gamma) + \gamma \tau (1 - \phi_t)] \kappa_t + \phi (1 - \tau) \Psi \geq (1 - \tau) (1 - \phi) \gamma \frac{R_t^R - R_t^D}{1 + R_t^D}. \quad (32)$$

To sum up, the portion of the supply curve that is consistent with  $l_t^R = 0$  is characterized by the condition that either  $\kappa_t > \tilde{\kappa}$ , or  $\kappa_t \leq \tilde{\kappa}$  and (32) holds.

**Step 3: condition for zero risky loans in equilibrium.** It follows from Steps 1 and 2 that the demand and supply curves on the risky-loans market intersect at one or several points  $(R_t^R, l_t^R)$  with  $R_t^R \geq 0$  and  $l_t^R = 0$  if and only if either (i)  $\kappa_t > \tilde{\kappa}$ , or (ii)  $\kappa_t \leq \tilde{\kappa}$ , and (32) holds when (31) holds with equality. Now, if (31) holds with equality, then, using (23), we can rewrite (32) as

$$\kappa_t \geq \kappa_t^* \equiv (1 - \tau) \frac{(1 - \phi) \gamma [\exp(\eta_t^R) - 1] + \Psi [(1 - \phi) \gamma \exp(\eta_t^R) - \phi]}{\phi (1 + \gamma) - \gamma \tau (1 - \phi) [\exp(\eta_t^R) - 1]}, \quad (33)$$

since the denominator on the right-hand side of this inequality is strictly positive:

$$\begin{aligned} \phi (1 + \gamma) - \gamma \tau (1 - \phi_t) [\exp(\eta_t^R) - 1] &= \phi [1 + \gamma (1 - \tau)] + \gamma \tau - \gamma \tau (1 - \phi) \exp(\eta_t^R) \\ &> \phi [1 + \gamma (1 - \tau)] + \gamma \tau \Psi \\ &> 0, \end{aligned}$$

where the last but one inequality comes from (11). As a consequence, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with  $l_t^R = 0$  is that either  $\kappa_t > \tilde{\kappa}$ ,



or  $\kappa_t^* \leq \kappa_t \leq \tilde{\kappa}$ . This condition can be equivalently rewritten as  $\kappa_t \geq \min\{\tilde{\kappa}, \kappa_t^*\}$ . Now, using (11) to replace  $(1 - \phi) \exp(\eta_t^R)$  by  $1 - \Psi$  on the right-hand side of (33), we get

$$\begin{aligned} \kappa_t^* &\leq (1 - \tau) \frac{-\gamma\Psi^2 + \phi(\gamma - \Psi)}{\gamma\tau\Psi + \phi(1 + \gamma - \gamma\tau)} \\ &= \tilde{\kappa} \left\{ 1 - \frac{\gamma\Psi}{\gamma - \Psi} \frac{\gamma\tau + \Psi(1 + \gamma)(1 - \tau)}{\gamma\Psi\tau + \phi(1 + \gamma - \gamma\tau)} \right\} \\ &< \tilde{\kappa}, \end{aligned}$$

where the last inequality comes from our assumption that  $\gamma > \Psi$ . Therefore, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with  $l_t^R = 0$  is simply  $\kappa_t \geq \kappa_t^*$ . Parts (a) and (b) of Proposition 5 follow. Finally, Part (c) of Proposition 5 follows straightforwardly from the fact that the denominator on the right-hand side of (33) is strictly positive, as shown above.

## 10.5 Proof of Proposition 6

Define welfare as the representative household's expected utility at date 0,  $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$ . For any policy  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$ , define the distance from  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  as

$$\varepsilon \equiv \max \left[ \max_{\tau \geq 0} (|R_\tau^D - R_\tau^{D*}|), \max_{\tau \geq 0} (|\kappa_\tau - \kappa_\tau^*|) \right].$$

Let us first compare  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  to policies  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\varepsilon$  is arbitrarily small and  $\exists t \geq 0, \kappa_t < \kappa_t^*$ . Moving from  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  to any such policy triggers a discontinuous increase in the amount of risk, as it makes banks' risky loans  $l_t^R$  move from 0 to  $\gamma l_t^S > 0$  at some date  $t \geq 0$ . Under our inefficiency condition (11), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that  $\varepsilon$  is small enough. As a consequence, welfare is strictly higher under  $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  than under any such policy provided that  $\varepsilon$  is small enough.

Let us now turn to policies  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\varepsilon$  is arbitrarily small and  $\forall \tau \geq 0, \kappa_\tau \geq \kappa_\tau^*$ . Among these policies, the optimal one maximizes the following Lagrangian:

$$W \left[ (X_\tau)_{\tau \geq 0} \right] + \sum_{\tau=0}^{+\infty} \lambda_\tau f \left( X_\tau, R_\tau^D, \kappa_\tau \right) + \sum_{\tau=0}^{+\infty} \mu_\tau (\kappa_\tau - \kappa_\tau^*),$$

where  $W$  is the welfare function;  $X_\tau$  is a vector of endogenous variables set by the private sector and exogenous shocks realized at date  $\tau$  or earlier;  $\lambda_\tau$  is a  $1 \times n$  vector of Lagrange multipliers, where  $n$  denotes the number of structural equations;  $f \left( X_\tau, R_\tau^D, \kappa_\tau \right)$  is a  $n \times 1$  vector such that the

structural equations can be written as  $f(X_\tau, R_\tau^D, \kappa_\tau) = 0$ ; and  $\mu_\tau$  is a scalar Lagrange multiplier. The condition stated in Proposition 6, namely that the right derivative of welfare with respect to  $\kappa_t$  at  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  is strictly negative for all  $t \geq 0$ , can be written as

$$\forall \tau \geq 0, \lambda_\tau \frac{\partial f}{\partial \kappa_\tau} (X_\tau^*, R_\tau^{D*}, \kappa_\tau^*) < 0,$$

where  $(X_\tau^*)_{\tau \geq 0}$  denotes the value of  $(X_\tau)_{\tau \geq 0}$  when  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ . Now, a first-order condition for Lagrangian maximization is

$$\forall \tau \geq 0, \lambda_\tau \frac{\partial f}{\partial \kappa_\tau} (X_\tau^*, R_\tau^{D*}, \kappa_\tau^*) + \mu_\tau = 0.$$

Therefore,  $\mu_\tau > 0$  for all  $\tau \geq 0$ , that is to say that the constraint  $\kappa_\tau \geq \kappa_\tau^*$  is binding for all  $\tau \geq 0$ . Since  $(R_\tau^{D*})_{\tau \geq 0}$  is the monetary policy that is Ramsey-optimal when  $(\kappa_\tau)_{\tau \geq 0} = (\kappa_\tau^*)_{\tau \geq 0}$ , we conclude that the policy  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$  is optimal among all policies  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\varepsilon$  is arbitrarily small and  $\forall \tau \geq 0, \kappa_\tau \geq \kappa_\tau^*$ . Since this policy has been shown above to dominate also policies  $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$  such that  $\varepsilon$  is arbitrarily small and  $\exists t \geq 0, \kappa_t < \kappa_t^*$ , Proposition 6 follows.

## 10.6 Steady State Under Optimal Monetary Policy

In this appendix, we solve for some key variables at the steady state under optimal monetary policy, for a given prudential policy. Whatever the prudential policy in place, optimal monetary policy implies zero steady-state inflation. Therefore, the twelve variables  $c, h, w, y, i, k, z, q, x, l, 1 + R^D$ , and  $1 + R^S$  are determined by the following twelve equations:

Euler equation:

$$1 = \beta(1 + R^D); \tag{34}$$

labor-leisure trade-off:

$$\Xi h^\chi c^\mu = w; \tag{35}$$

rental price of capital:

$$q = 1 - \delta + z; \tag{36}$$

production function:

$$y = k^\nu h^{(1-\nu)}; \tag{37}$$

capital-labor ratio:

$$\frac{k}{h} = \frac{\nu}{1-\nu} \frac{w}{z}; \tag{38}$$

price mark-up:

$$\frac{\sigma}{\sigma-1} \frac{wh}{(1-\nu)y} = 1; \tag{39}$$

capital accumulation:

$$x = (1 - \delta)k + i; \quad (40)$$

loans:

$$l = x; \quad (41)$$

technology:

$$k = \xi_1(\kappa)x; \quad (42)$$

zero-profit condition for safe capital-goods producers:

$$q = 1 + R^S; \quad (43)$$

zero-profit condition for banks:

$$\frac{1 + R^S}{1 + R^D} = \xi_2(\kappa) + \xi_3(\kappa)\kappa; \quad (44)$$

goods-market-clearing condition:

$$y = c + i + \xi_4(\kappa)l; \quad (45)$$

where  $\mu$  is the inverse of the intertemporal elasticity of substitution,  $\chi$  is the inverse of the labor-supply elasticity, and  $\xi_1(\kappa)$ ,  $\xi_2(\kappa)$ ,  $\xi_3(\kappa)$ ,  $\xi_4(\kappa)$  are reduced-form parameters whose value depends on whether the economy is at the safe or the maximum-risk corner. More precisely, when  $\kappa \geq \kappa^*$ ,

$$\begin{aligned} \xi_1(\kappa) = \xi_1^S &\equiv 1, \\ \xi_2(\kappa) = \xi_2^S &\equiv 1 + \Psi, \\ \xi_3(\kappa) = \xi_3^S &\equiv \frac{\tau}{1 - \tau}, \\ \xi_4(\kappa) = \xi_4^S &\equiv \Psi, \end{aligned}$$

and when  $\kappa < \kappa^*$ ,

$$\begin{aligned} \xi_1(\kappa) = \xi_1^R &\equiv \frac{1 + \gamma(1 - \phi) \exp(\eta^R)}{1 + \gamma}, \\ \xi_2(\kappa) = \xi_2^R &\equiv \frac{1 + \gamma}{1 + \gamma \exp(\eta^R)} \left[ 1 + \frac{\Psi}{(1 + \gamma)(1 - \phi)} \right], \\ \xi_3(\kappa) = \xi_3^R &\equiv \frac{1 + \gamma}{1 + \gamma \exp(\eta^R)} \left[ \frac{\tau}{1 - \tau} + \frac{\phi}{(1 - \tau)(1 - \phi)} \right], \\ \xi_4(\kappa) = \xi_4^R &\equiv \frac{\Psi}{1 + \gamma}. \end{aligned}$$

Let us now solve this system for equations  $k$ ,  $c$ , and  $h$ . (40), (41), (42), and (45) give

$$y = c + \left[ \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} - (1 - \delta) \right] k. \quad (46)$$

(35), (37), and (39) give

$$\Xi y^{\frac{\chi+\nu}{1-\nu}} k^{\frac{-\nu(1+\chi)}{1-\nu}} c^\mu = \frac{(\sigma - 1)(1 - \nu)}{\sigma}. \quad (47)$$

(34), (36), (43), and (44) give

$$z = \frac{\xi_2(\kappa) + \xi_3(\kappa)\kappa}{\beta} - (1 - \delta). \quad (48)$$

(38), (39), and (48) give

$$y = [\xi(\kappa)] k, \quad (49)$$

where

$$\xi(\kappa) \equiv \left[ \frac{\xi_2(\kappa) + \xi_3(\kappa)\kappa}{\beta} - (1 - \delta) \right] \frac{\sigma}{\nu(\sigma - 1)}.$$

(46) and (49) give

$$c = \left[ \xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta) \right] k. \quad (50)$$

(47), (49), and (50) give

$$k = \left\{ \frac{(\sigma - 1)(1 - \nu)}{\Xi \sigma [\xi(\kappa)]^{\frac{\chi + \nu}{1 - \nu}} \left[ \xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta) \right]^\mu} \right\}^{\frac{1}{\chi + \mu}}, \quad (51)$$

so that  $k$  is decreasing in  $\kappa$ . (37) and (49) give

$$h = [\xi(\kappa)]^{\frac{1}{1 - \nu}} k. \quad (52)$$

Then,  $c$  and  $h$  are straightforwardly obtained from (51) using (50) and (52).

## 10.7 Steady-State Effect of Prudential Policy on Welfare

In this appendix, we study how welfare varies with capital requirements at the steady state under optimal monetary policy. In particular, we show that optimal prudential policy at the safe corner sets  $\kappa = \kappa^*$ , and we derive the necessary and sufficient condition on the parameters for optimal prudential policy at the maximum-risk corner to set  $\kappa = 0$ . (51) implies

$$\frac{dk}{d\xi(\kappa)} = - \left[ \frac{\chi + \nu}{(1 - \nu)(\chi + \mu)} \right] \frac{k}{\xi(\kappa)} - \left( \frac{\mu}{\chi + \mu} \right) \frac{k}{\xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta)}. \quad (53)$$

(50), (52) and (53) imply

$$\begin{aligned} \frac{du(c, h)}{d\xi(\kappa)} &= c^{-\mu} \frac{dc}{d\xi(\kappa)} - \Xi h^\chi \frac{dh}{d\xi(\kappa)} \\ &= \left[ \xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta) \right]^{-\mu} k^{-\mu} \left\{ k + \left[ \xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta) \right] \frac{dk}{d\xi(\kappa)} \right\} \\ &\quad - \Xi [\xi(\kappa)]^{\frac{\chi}{1 - \nu}} k^\chi \left[ \frac{1}{1 - \nu} [\xi(\kappa)]^{\frac{\nu}{1 - \nu}} k + [\xi(\kappa)]^{\frac{1}{1 - \nu}} \frac{dk}{d\xi(\kappa)} \right] \\ &= \left[ \xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta) \right]^{-\mu} k^{1 - \mu} \left\{ \frac{\chi}{\chi + \mu} - \left[ \frac{\chi + \nu}{(1 - \nu)(\chi + \mu)} \right] \frac{\xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta)}{\xi(\kappa)} \right\} \\ &\quad - \Xi [\xi(\kappa)]^{\frac{\chi + \nu}{1 - \nu}} k^{1 + \chi} \left[ \frac{\mu - \nu}{(1 - \nu)(\chi + \mu)} - \left( \frac{\mu}{\chi + \mu} \right) \frac{\xi(\kappa)}{\xi(\kappa) - \frac{1 + \xi_4(\kappa)}{\xi_1(\kappa)} + (1 - \delta)} \right]. \end{aligned}$$

Using (51) and noting

$$\begin{aligned}
A(\kappa) &\equiv \left[ \frac{(\sigma-1)(1-\nu)}{\Xi\sigma} \right]^{\frac{1-\mu}{\chi+\mu}} [\xi(\kappa)]^{\frac{-(\chi+\nu)(1-\mu)}{(1-\nu)(\chi+\mu)}} \left[ \xi(\kappa) - \frac{1+\xi_4(\kappa)}{\xi_1(\kappa)} + (1-\delta) \right]^{\frac{-\mu(1+\chi)}{\chi+\mu}}, \\
B(\kappa) &\equiv \left[ \frac{\frac{1+\xi_4(\kappa)}{\xi_1(\kappa)} - (1-\delta)}{\frac{\xi_2(\kappa)+\xi_3(\kappa)\kappa}{\beta} - (1-\delta)} \right] \left( \frac{\sigma-1}{\sigma} \right), \\
C(\kappa) &\equiv \chi - \left[ \frac{1-B(\kappa)\nu}{1-\nu} \right] (\chi+\nu) + \left( \frac{\sigma-1}{\sigma} \right) (\nu-\mu) + \mu \left( \frac{\sigma-1}{\sigma} \right) \left[ \frac{1-\nu}{1-B(\kappa)\nu} \right],
\end{aligned}$$

we then get

$$\left[ \frac{\chi+\mu}{A(\kappa)} \right] \frac{du(c, h)}{d\xi(\kappa)} = C(\kappa)$$

and therefore

$$\left[ \frac{\beta(1-\beta)\nu(\sigma-1)(\chi+\mu)}{A(\kappa)\xi_3(\kappa)\sigma} \right] \frac{dU(c, h)}{d\kappa} = C(\kappa),$$

where  $U(c, h) \equiv \sum_{t=0}^{\infty} \beta^t u(c, h)$ , so that  $\frac{dU(c, h)}{d\kappa}$  is of the same sign as  $C(\kappa)$ . Now,  $C(\kappa)$  depends positively on  $B(\kappa)$ . In turn,  $B(\kappa)$  depends negatively on  $\kappa$  for  $0 \leq \kappa < \kappa^*$  and for  $\kappa > \kappa^*$ . Therefore,  $C(0) < 0$  is a necessary and sufficient condition on the parameters for  $U(c, h)$  to be decreasing in  $\kappa$  for  $0 \leq \kappa < \kappa^*$ , and, hence, for optimal prudential policy at the maximum-risk corner to set  $\kappa = 0$ . Finally, for  $\kappa > \kappa^*$ ,  $U(c, h)$  is necessarily decreasing in  $\kappa$ , since  $C(\kappa) < C(\kappa^*) < \frac{-\nu}{\sigma} < 0$ , where the last but one inequality follows from

$$B(\kappa^*) = \left[ \frac{\frac{1+\xi_4^S}{\xi_1^S} - (1-\delta)}{\frac{\xi_2^S+\xi_3^S\kappa^*}{\beta} - (1-\delta)} \right] \left( \frac{\sigma-1}{\sigma} \right) < \left[ \frac{\frac{1+\xi_4^S}{\xi_1^S} - (1-\delta)}{\frac{\xi_2^S}{\beta} - (1-\delta)} \right] \left( \frac{\sigma-1}{\sigma} \right) < 1.$$

Therefore, optimal prudential policy at the safe corner sets  $\kappa = \kappa^*$ .

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