

Breaking the Spell with Credit-Easing*

Self-Confirming Credit Crises in Competitive Search Economies

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May 18, 2015

Abstract

We introduce Self-Confirming Equilibria (SCE) in a competitive credit market to explain market freezes. Lenders post credit lines at a fixed interest rate and borrowers apply to them, as in a directed search model. In a SCE lenders' pessimistic beliefs, about borrowers projects, can result in high interest rates being offered and only risky projects being financed. The competitive environment prevents lenders from experimenting with low interest rates in order to attract safe projects, even if individual experimentation would dissipate their uncertainty about borrowers' reactions. We provide conditions, under which, a planner, maximizing expected social welfare, would intervene implementing an optimal subsidy of lenders' eventual losses. We characterise *credit easing as the optimal policy* that breaks the uncertainty freezing the economy and results in an efficient low-interest rate equilibrium, whenever it exists. Nevertheless, lenders' expected return are kept at zero profits, through competition. Hence, the social value of experimentation can be positive when the private is not. The theory can rationalise the – ex-post, successful – 2009 FRB TALF intervention in the ABS market.

Keywords: unconventional policies, learning, credit crunches, sentiments.

JEL Classification: D53, D83, D84, D92, E44, E61, G01, G20, J64.

*Preliminary draft, do not quote without authors permission. The views expressed in this paper do not necessarily reflect the ones of Banque de France. We thank Marco Bassetto, Manolis Galenianos, and participants in seminars and workshops, where previous versions of this work have been presented, for their comments. We also thank the Fondation Banque de France for their financial support.

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1 Introduction

Most of observed economic crisis are characterized by high uncertainty on the state of the economy. Such uncertainty shows up in credit markets in the form of perceived counterpart risk, resulting in high lending rates, and in some cases, complete market freezes. The common wisdom is that disruptions of liquidity markets are collateral effects of recessions, so that, conventional (and unconventional) policies aiming at lowering the cost of money are appropriate reactions to both. Unfortunately, these policies have seldom translated into a substantial improvement of the credit conditions for private firms, as demonstrates by the prolonged credit crisis in the Euro area.

A notable exception to this framework were some specific *credit-easing* interventions implemented by the Federal Reserve Bank during the 2009 recession in the US. These policies were specifically designed to partly ensure private lenders from the *perceived* counterpart risk in important credit markets under pressure. One of these policies was the TALF whose introduction in the AAA-rated ABS market (completely frozen in late 2008) coincided with a permanent recovery of transactions without that any subsidy was actually dispensed; this means that, in retrospect, the perceived counterpart risk in that market was excessive.

Despite such ‘success’ other central banks – for example, the ECB – have not implemented similar policies, as if the FED had just been lucky or the circumstances – say, in the euro crisis – had been different. Was the Fed right, or better informed, or just lucky? More broadly, how should a central bank, which does not know more than the private sector, react when lower policy rates are not effective in improving credit market conditions?

To explore these questions, we develop a model with a competitive credit market consisting of a continuum of borrowers (e.g. entrepreneurs with projects) and lenders (e.g. banks). The latter, are intermediaries that borrow money from the interbank market, and post credit offers at a fixed interest rate; the former apply for these contracts and choose the projects to be financed. Their expected profits depend on the probability that their loan application is accepted and, if so, on the expected net return of their project. When a risky project fails borrowers only repay the principal of the loan (the *pledgeable* part) and, therefore, the lender also bears part of the risk, which is compensated by the loan’s interest risk-premium. However, borrowers also have the choice of implementing a riskless project paying a fixed cost¹. Safe projects are implemented if the interest rate is low enough to compensate the fixed cost.

Borrowers observe the menu of debt contracts posted by lenders, and choose where to send an application under complete information. On the other hand, lenders do not observe the choice of borrowers and, therefore, have to anticipate the borrower’s reaction to their credit offers in order to maximize the value of their contract.

¹Different interpretations are possible. For instance, that the borrower is – say, a car buyer – subject to liquidity shocks and can hedge against them by paying a fix insurance cost; or, simply, that the borrower at a (separable) effort cost can make a project safe.

Multiplicity of equilibria – in particular, of Pareto ranked equilibria, such as a low-risk and a high-risk equilibrium – plays a central role. However, in our model we do not require that such multiplicity relies on agents’ actions being coordinated in equilibrium, as in Self-Fulfilling equilibria of high and low levels of credit (e.g. [2]), nor that there is a hold-up contracting problem (as in [1]). Instead, we consider Self-Confirming equilibria (SCE), where, departing from the rational expectations hypothesis, ‘subjective beliefs’ only need to be ‘self-confirmed’ in equilibrium and, therefore, rational agents can entertain out-of-equilibrium missperceptions. Nevertheless, a key feature of SCE is that that individual actions can produce the observables that correct these missperceptions². This feature of SCE provide a rigorous theoretical framework to formalize the concept of missperceived counterpart risk. For instance, in our economies lenders can maintain excessive pessimism on the cost of riskless projects and therefore only offer high interest rates to cover losses, seeing these announcements, borrowers may choose only risky projects and, as a result, pessimistic beliefs are self-confirmed. In contrast, with rational expectations, the identity between agents subjective beliefs with the objective distribution of payoffs, means that they could typically figure out an optimal contract to uncover the missing information, with SCE this is not necessarily true as agents do not know out-of-equilibrium reactions of the counterpart and so cannot engineer an optimal contract as ‘rational expectations mechanism design theory’ would vindicate.

Our first contribution is to characterize Self-Confirming Equilibria in a competitive economies. Self-confirming equilibrium was first introduced by [3] in game theory and in macroeconomics has been explored by [7], and lately by [6], to explain the rise and fall of American inflation. In their case, the FED was taking actions based on a wrong theory of the economy, which could not be confuted by the outcomes that the policy itself was determining, at least up to the point where enough experimentation finally revealed the actual working of the economy. In contrast to previous literature, we characterize a Self-Confirming Equilibrium in a search and matching environment with atomistic agents, that, whereas have no power to affect general equilibrium outcomes, can have misspecified beliefs about the counterpart’ incentives within the match and, therefore, their individual actions could dissipate their missperceptions. Moreover, we strengthen the classical notion of SCE by requiring that agents’ subjective beliefs must coincide with rational expectations beliefs not just ‘in equilibrium’ but ‘in a neighborhood of equilibrium’; we call it *Strong Self-confirming equilibrium* (SCCE).

The second contribution of the paper is to show how one can design an optimal Ramsey policy that maximizes social welfare, taking into account the directed search equilibrium constraints, and without the Ramsey planner having more information or less uncertainty than private agents – in our case, lenders – have. In a SCCE, which is not a rational expectations equilibrium (REE), the lender missperceives a local-maximum in her optimization problem as being a global maximum; although she assigns positive

²It should be noticed that in the macro literature sometimes the term Self-confirming equilibrium is (mis)-used without this last key feature.

probability to being wrong, she would choose not to deviate if this probability is low enough, preventing herself from learning about it. Furthermore, as it is well known, competitive directed search is efficient, in the sense that the Hosios' efficient matching condition is satisfied [5]. As we show, Strong Self-confirming directed search equilibria share this property, although, as we emphasize, it is a local property. Nevertheless, it implies that a Ramsey planner with the same (or more pessimistic) beliefs than lenders will also choose not to experiment in a *laissez faire* economy; more precisely, it will choose not to affect the decentralized allocation, unless it has an instrument that changes the contracts being offered and, as a result, the distribution of matches³.

We show that an optimal contingent subsidy is the instrument that allows the planner to implement a globally (constrained) efficient outcome. We characterise *credit easing as the optimal policy* that takes the form of a subsidy to eventual lenders' losses, lump-sum financed by borrowers. Since it is designed using implementability conditions, under fairly general conditions, it is a powerful instrument: if the central bank targets an interest rate, private lenders offer contracts at the targeted interest rate. The subsidy has the property of restoring the Hosios condition at the targeted interest rate. This also implies that if the targeted interest rate is the efficient⁴, low-interest rate REE - which the authority hopes to uncover with the implementation of the policy - then: i) no subsidy will be actually implemented in the favorable case that a missperceived REE is discovered, ii) there will be no need to subsidize after the one period 'experiment'.

The third contribution of the paper is a corollary of the previous results: *the private and social value of experimentation are not the same*, when the central bank can perform a social experiment with *credit easing*. From the perspective of a lender there are two potential benefits from individually experimenting with different interest rates: the profits from attracting borrowers with a new market offer and the learning of updating her beliefs with information the market is not producing. However, with competitive directed search, the result of her experiment is public information after one period: she would not learn more than her competitors, who by not experimenting are not taking any risk. Hence, the eventual positive profits, or losses, produced by an individual experiment of a lender will only last one period – while, had she been a monopolist, extra-profits would last much longer, while relative losses would only last one period (since she can always return to offer the old contracts). As a result, given her subjective beliefs, the *ex-ante* value of private experimentation (individual deviation from the equilibrium) is strictly negative. But the authority, applying an optimal subsidy in a specific credit market, can move the whole matching distribution in that market. Such a one-period move leaves

³For example, in our model, a central bank conventional policy instruments, such as interest rate policies is not effective because typically is a 'local instrument' and therefore in a locally efficient equilibrium should not be used (unless the economy parameters are just at the margin), furthermore, not being an instrument target to specific markets may not be optimal to consider large deviations. Similarly, a less conventional policy, such as QE liquidity provision to banks, is not effective since more liquidity does not change the pessimistic beliefs of the banks.

⁴Efficiency requires that the equilibrium is an interior maximum of lenders' objective.

indifferent the individual lender, who keeps making zero profits, although, as we show, an optimal subsidy maximizes total expected surplus. Moreover, the social *ex-ante* value of experimentation is positive, as long as the planner's assigns positive probability to the existence of a Pareto superior equilibrium. Obviously, with a longer horizon the expected gains can be substantially larger.

In sum, we show, first, that in a SCCE lenders have no incentive to experiment with other interest rates, when they assign a small probability to the event that such experiment would result in profits and changing their beliefs; second, in contrast, a central bank, maximizing social welfare has a strong incentive to experiment with an optimal *credit easing* policy, as long as gives positive probability to the success of such experimentation; third, if the social experiment is successful (i.e. reveals that lenders' beliefs were wrong) unveiling a missperceived efficient REE then, the optimal subsidy is, by construction, a one period (in this sense, unconventional) intervention.

2 Competitive search for credit

2.1 Borrowers and lenders

The economy is characterized by a set of possible projects Ω . At each time t , an atomistic representative borrower $i \in (0, 1)$ has access to a subset of projects $\omega_t \subset \Omega$. Agent i can implement only one unit of one specific project $\rho \in \omega$. Each project last one period, then the investment opportunity vanishes. The process $\{\omega_t\}$ is Markovian with a time-invariant distribution with a density function $\phi(\omega)$ ⁵.

Each project needs to be externally financed. An agent can obtain liquidity to invest in a project from a lender. A lending contract specifies an interest rate R that the borrower pays to the lender at the end of the period. Given credit conditions characterized by an interest rate R , a borrower chooses the type of the project. An investment policy is then

$$\rho^*(R, \omega) = \arg \max_{\{\rho \in \omega\}} \{\pi^b(\rho; R, \omega)\}, \quad (1)$$

where $\pi^b(\rho; R, \omega)$ is the expected return associated with the project ρ of a borrower, given a finance interest rate R and a set of project types ω available to a borrower. We require π^b being differentiable and decreasing in R .

An atomistic representative lender j borrows money in the interbank market, or equivalently at a rate determined by the central bank (CB), R_{CB} and lends to borrowers. The expected return of a loan is $\pi^l(R; \rho, R_{CB})$, where we require π^l being differentiable in R and R_{CB} , increasing in R and decreasing in R_{CB} . However, the lender does not observe the investment opportunities of a lender ω . In particular, if the lender offers a contract at an interest R and the borrower chooses $\rho^*(R, \omega)$ the expected return will be

$$\pi^l(R; \rho^*(R, \omega), R_{CB}), \quad (2)$$

⁵For the ease of notation, we eliminate the time subscript, except when it is necessary.

where ω is not directly observable, but should be inferred by the lender. Such uncertainty generates counterpart risk in the lending contract as the borrower's choice ρ affects the returns of the lender as well determining, for example, the risk of default on the debt. Let us denote $\beta_t(\omega)$ the *subjective* density function of a lender at time t , describing her beliefs about the probability that a borrower has access to a set of choices $\omega \subset \Omega$. In particular, for a given R and R_{CB} ,

$$E^\beta [\pi^l(R; \rho^*(R, \omega), R_{CB})] \equiv \int \pi^l(R; \rho^*(R, \omega), R_{CB}) \beta(\omega) d\omega,$$

denote the expected lender's profit evaluated with the probability distribution induced by β . Note that we allow for subjective beliefs to possibly differ from the objective distribution; i.e. $\beta(\omega)$ not necessarily being the same as $\phi(\omega)$. In particular, we do not assume that the lender knows the underlying Markovian structure of $\phi(\omega)$. Nevertheless, we assume that $\text{supp}(\phi(\omega)) \subseteq \text{supp}(\beta)$ and, moreover, lenders' beliefs cannot contradict the realization of observable variables in a sense that we will specify below as part of the definition of equilibrium. Finally, in what follows we will restrict to the case of a linear economy, i.e. one in which the total surplus $\pi^l + \pi^b$ does not depend directly on the interest rate R , but only indirectly via ρ . In such a case, we have $\pi^{b'}(\rho^*(R, \omega)) = -\pi^{l'}(R, \rho^*)$. An example of linear economy is presented below.

Example: A Simple Specification

Here, we provide our baseline example, which will help us to illustrate some of our results. Borrowers can choose between two kinds of projects, namely a safe and a risky one, which differ for the likelihood of success and cost adoption. The table below summarizes projects payoff which finally determine the incentives of the borrowers. Both types have the same conditional per-unit return: in case of success is $1 + r$, whereas 1 in case of failure. Safe projects do not fail, but their adoption requires a fix per unit cost of k which sums up to the repayment of the bank's loan. Risky project do not have any fix per-unit additional cost, but they can fail with a probability of $1 - \alpha$. In terms of our general formulation: $\Omega = \{[0, 1], \mathbb{R}_+\}$, $\omega = (\alpha, k)$ and $\rho \in \{\alpha, k\}$. That is, at each point in time, the borrower can choose between a *risky* or a *safe* project, which are parameterised by a time invariant pair α and k , respectively.

Projects $\omega = (\alpha, k)$	Safe	Risky
Success	cost: $1 + R + k$ return: $1 + r$ probability: 1	cost: $(1 + R)$ return: $(1 + r)$ probability: α
Failure	cost: 1 return: 1 probability: 0	cost: 1 return: 1 probability: $1 - \alpha$

Table 1. Projects payoffs.

Hence, a *risky* project, $\rho = \alpha$, implies a net per-unit expected return of

$$\pi^b(\alpha; R, \omega) = (r - R)\alpha \quad (3)$$

$$\pi^l(R; \alpha, R_{CB}) = \alpha R - R_{CB} \quad (4)$$

to borrowers and lenders, respectively. Implicitly we assume that a lender cannot default on the interbank market, so that its repayment does not depend on the project success. Participation of both a borrower and a bank require $R \in (r, R_{CB}/\alpha)$. A *safe* project, $\rho = k$, instead gives a net per-unit expected return of

$$\pi_i^b(k; R, \omega) = r - R - k \quad (5)$$

$$\pi_j^l(R; k, R_{CB}) = R - R_{CB} \quad (6)$$

to borrowers and lenders, respectively. Participation of both a borrower and a bank require $R \in (r - k, R_{CB})$.

In the case of a *risky project* the sum of the interim surplus is $\alpha r - R_{CB}$ whereas in the case of the *safe project* is $r - k - R_{CB}$. Therefore as soon as $r - k/(1 - \alpha)$ becomes positive the surplus generated adopting the safe project is larger. Nevertheless the borrower could choose not to adopt it when the offered interest rate is too high. Specifically, for a given R , the borrower will adopt a safe project if it will make higher profits out of it, hence, if $\alpha(r - R) \leq (r - k - R)$ or

$$R \leq \bar{R} \equiv r - \frac{k}{1 - \alpha},$$

that is whenever the offered interest rate is low enough or the gross return high enough. In fact, for a given level of riskiness, the higher the productivity the higher the net return from risky projects. On the other hand, for $\alpha > (r - k)/r$, that is for low enough level of risk, borrowers will never adopt the safe technology no matter which interest rate R is offered. We will refer to \bar{R} as the borrowers' profitability frontier. Finally notice that the cost of liquidity does not affect the choice of projects.

2.2 Matching in the credit market

Lenders and borrowers match to sign a credit contract in the context of a competitive direct search framework as introduced by [5] along a simplified variant described by [8]. Let denote by a and o respectively the measure of applicants and the measure of offers at a given time. The per-period flow of new lender-borrower matches is determined by a standard Cobb Douglas matching function

$$x(a, o) = Aa^\gamma o^{1-\gamma}$$

with $\gamma \in (0, 1)$ ⁶. The search is *directed*, meaning that at a certain interest rate R there is a subset of applications $a(R)$ and offers $o(R)$ looking for a match at that specific

⁶This assumption, which is standard in the literature, ensures a constant elasticity to the fraction of vacancies and applicants.

R . Let $\theta(R) = a(R)/o(R)$ denote the specific tightness⁷ of the submarket R . The number of matches in the submarket R is given then $x(a(R), o(R)) = o(R)x(\theta(R), 1)$. The probability that an application for a unit of credit at interest rate R is considered is $p(\theta(R)) = A\theta(R)^{\gamma-1}$ and the probability that a unit of credit offered R is used is $q(\theta(R)) = A\theta(R)^\gamma$. Both borrowers and banks are free to operate in all submarkets. Once the match is formed the lender lends one unit to the borrower at a rate R . We will say that a submarket is active if there is at least a contract posted.

Lenders are first movers in the search: they choose whether to enter in the market and eventually at which interest rate R they post a contract. Therefore a lender posts an offer R which maximizes

$$E^\beta [V(R)] \equiv E^\beta [q(\theta(R)) \pi^l(R; \rho^*(R, \omega), R_{CB}) - c], \quad (7)$$

with c being the cost of posting an offer. Notice that, in order to solve (7), a lender needs to anticipate the reaction of the borrower $\rho^*(R, \omega)$ that she will meet when posting R . Hence a lender bears the risk associated with the probability that a contract is not filled and the uncertainty, or possible missperception, concerning the payoff incentives of lenders.

Borrowers move once lenders have posted their offers. In particular, borrowers choose to which posted contract to send its application for funds. A borrower sends an application to one R among the set of posted contracts H to maximize

$$J(R) \equiv p(\theta(R)) \pi^b(\rho^*(R, \omega)). \quad (8)$$

Notice that there are no expectations since borrowers observe all the elements to act optimally.

The competitive nature of the economy imposes restrictions on both sides of the market, which together determine an equilibrium tightness for each submarket. On the side of the lenders, free entry guarantees competition, so that the mass of lenders posting a contract in the submarket R , namely $o(R)$, increases (resp. decreases) whenever $V(R) > 0$ (resp. $V(R) < 0$). On the other hand, borrowers are constrained to send applications by the existing set of offered contracts H . Furthermore, the mass of applicants to a submarket $R' \in H$, namely $a(R')$ increases (resp. decreases), whenever $J(R') > J(R'')$ for each $R'' \in H$ (resp. $J(R') < J(R'')$ for at least a $R'' \in H$). Competitive lenders take market tightness as given because as individuals they cannot affect the distributions of offers and applicants in the market.

⁷The tightness is a ratio representing the number of borrowers looking for a credit line *per-unit of vacant open lines*. . This means that the tightness is independent of the absolute number of vacancies open in a certain market. In other words, suppose an equilibrium is associated with a particular probability to obtain credit \bar{p} , then the matching function gives a $\theta = p^{-1}(\bar{p})$ and so a $\bar{q} = q(p^{-1}(\bar{p}))$ that is a probability of filling a vacant line in that submarket irrespective of the number of vacancies open in that particular submarket. Notice that tightness is also defined in the labor literature as $1/\theta$.

3 Equilibria

3.1 Competitive SSCE and REE

Here we present the definition of Strong Self-Confirming equilibrium (SSCE) and we contrast it to the notion of Self-Confirming equilibrium (SCE) and the classical one of rational expectation equilibria (REE).

Definition 1. *Given a density function $\phi(\omega)$, a strong Self-Confirming equilibrium (SSCE) at time t is a set of posted contracts H_t^* such that, for each $R_t^* \in H_t^*$:*

(sc1) *borrowers maximize expected profits*

$$R_t^* = \arg \sup_{R_t \in H_t^*} J(R_t); \quad (9a)$$

resulting in $J^ = J(R_t^*)$*

(sc2) *lenders maximize expected profits*

$$R_t^* = \arg \sup_{R_t \in \mathfrak{R}} E^{\beta_t} [V(R_t)] \quad (10a)$$

subject to

$$J(R_t) = J^* \quad (10b)$$

resulting in $V(R_t^) = 0$ for each $R_t^* \in H_t^*$;*

(sc3) *there is an open neighborhood of R_τ^* , namely $\mathfrak{S}(R_\tau^*)$, such that for any $R \in \mathfrak{S}(R_\tau^*)$ it is*

$$E^{\beta_t} [V(R)] = E^\phi [V(R)], \quad (11)$$

for each $\tau \leq t$, that is, borrowers correctly anticipates lenders' reaction only locally around the realised equilibrium contracts.

The first requirement (sc1) implies that all equilibrium contracts yield the same ex-ante utility to borrowers, therefore they are indifferent among them.

The second condition (sc2) requires that a bank posts a R^* that globally maximizes its expected value of a credit line, subject to the ‘applications adjust to interest rate variations’ constraint (10b). In other words, they take as given that ‘price’ variations are offset by variations of applications; that is, in maximizing (7) lenders take as fixed the equilibrium value achieved by borrowers⁸. Condition (sc2) and (sc1) define the tightness

⁸This restriction is commonly made – locally, around the equilibrium – in the *directed search* literature as a way to eliminate rational expectations equilibria based on (unreasonable) discontinuities of beliefs at equilibrium. Although the local restriction is enough for our purposes, we treat this restriction as part of the definition of the competitive directed search environment; more precisely, that lenders internalize the offsetting effect, through applications, of changing interest rates. As it will become clear in our proofs, the level J^* is not important, what counts is the constancy of J^* , with respect to interest rate variations.

of the submarkets active in equilibrium. Notice that the relevant expectation is the one conditional on the lender's subjective beliefs summarized by β_t .

The third condition (sc3) restricts lenders' beliefs β_t about borrowers' actions to be correct in a neighborhood of an equilibrium R^* . This is also a stronger beliefs' restriction of the one usually assumed in the notion of Self-Confirming Equilibrium (SCE) which does not contemplate any belief restriction out of equilibrium. In fact, at a SCE, condition (sc3) holds punctually for any R_τ^* rather than for any $R \in \mathfrak{R}(R_\tau^*)$.

What is crucial in the definition of SSCE is that it does not require lenders to have correct beliefs about unobserved out-of-equilibrium behavior that is far away from the equilibrium considered. This leaves open the possibility that at a Self-Confirming equilibrium banks are not actually maximizing in the *whole* domain of their actions, *although they believe they do*. Better contracts out of a neighborhood of the equilibrium could be wrongly believed by banks to be strictly dominated by existing ones. Since such contracts will be never posted, then in equilibrium there do not exist counterfactual observations that could confute wrong beliefs. In particular, lenders may missperceive the actions of the borrowers would take – and the resulting risks – when offered lower interest rates.

A REE is a stronger notion than a SSCE requiring that no agent holds wrong out-of-equilibrium beliefs. In the present model this equals to impose that banks' unbiased beliefs about borrowers' payoffs. In such a case the equilibrium contract is the one which objectively yields the highest reward with respect to every possible feasible contract.

Definition 2. *A rational expectation equilibrium (REE) at time t is a Self-Confirming equilibrium H_t^* for which:*

(ree) for any $R \in \mathfrak{R}$ it is

$$E^{\beta_t} [V(R_\tau^*)] = E^\phi [V(R_\tau^*)]. \quad (12)$$

for each $\tau \leq t$, that is, lenders correctly anticipates borrowers' reaction for any possible contract.

A REE obtains from a tightening of condition (sc3) in the definition of a SSCE. This implies that every $R^* \in H^*$ is such that lenders can exactly forecast the out-of-equilibrium value of posting a credit line as they can correctly anticipate borrowers' responses. Therefore, posting in the submarket R^* is a globally dominant strategy both from an objective and a subjective point of view.

3.2 Characterization of equilibria

In this section we provide a characterization of equilibria, developing a non-marginal technique that can be generally used to identify the equilibrium of search and matching economies with non convex payoff structures and potentially multiple local maxima. As we will show the Hosios condition can be recovered from this more general analysis.

We first rewrite the individual lender's problem⁹ as:

$$\max_R E^\beta [q(\theta(R)) \pi^l(R, \rho^*) - c],$$

subject to¹⁰

$$p(\theta(R)) \pi^b(\rho^*(R, \omega)) = \bar{J},$$

and

$$q(\theta(R)) = p(\theta(R))^{-\frac{\gamma}{1-\gamma}},$$

which derives from the definition of matching probabilities. Plugging constraints into the objective we can derive the condition for an equilibrium contract as:

$$R^* = \arg \max \left(E^{\beta_i} \left[\bar{J}^{-\frac{\gamma}{1-\gamma}} \pi_i^b(R, \omega)^{\frac{\gamma}{1-\gamma}} \pi_j^l(R, \rho^*) - c \right] \right),$$

so that after defining

$$\mu(R) \equiv \pi_i^b(R, \omega)^{\frac{\gamma}{1-\gamma}} \pi_j^l(R, \rho^*), \quad (13)$$

we have the following as a direct consequence.

Lemma 1. *Consider two contracts posted respectively at R_1 and R_2 . From the point of view of a single atomistic bank*

$$E^\beta [V(R_1)] \geq E^\beta [V(R_2)]$$

if and only if

$$E^\beta [\mu(R_1)] \geq E^\beta [\mu(R_2)], \quad (14)$$

for any profile of contracts offered by other banks.

Note that the evaluation of R does not depend on \bar{J} , i.e. it does not depend on the level of utility granted to the other side of the market, which a single lender cannot affect. However, lenders partly internalize the welfare of borrowers as contracts that provides better conditions for borrowers are more likely to be signed. In particular, with $\gamma = 0$ when all the surplus is extracted by lenders (13) becomes $E^\beta [\pi^l(R_1, \omega)] \geq E^\beta [\pi^l(R_2, \omega)]$, that is at the equilibrium only the interim payoff of lenders is maximized as borrowers will always earn zero. With $\gamma = 1$ instead when the whole surplus is extracted by borrowers (13) becomes $E^\beta [\pi^b(R_1, \omega)] \geq E^\beta [\pi^b(R_2, \omega)]$, that is only the interim payoff of borrowers is maximized as lenders will always earn nothing.

Let us introduce here a definition of local maxima of μ evaluated by the system of beliefs β and ϕ , respectively.

⁹From now on, to simplify notation, we do not write R_{CB} as an argument of π^l , since we take it as an underlying fixed parameter of the economies. Only when it is not treated as a fixed parameter we revert to the notation of making R_{CB} an explicit argument of π^l

¹⁰In equilibrium $\bar{J} = J^*$. Here we emphasise that from the lender's perspective $p(\theta(R)) \pi^b(\rho^*(R, \omega))$ is constant.

Definition 3. A contract R' is β -local maximum for the lender if there exists a neighborhood of R' , namely $\mathcal{I}(R')$, such that

$$E^\beta [\mu(R')] = \sup_{R \in \mathcal{I}(R')} E^\beta [\mu(R)], \quad (15)$$

with \mathcal{M}^β denoting the set of β -local maxima. An interior β -local maximum is a contract R' satisfying

$$E^\beta \left[\mu(R') \left(\frac{\gamma}{1-\gamma} \frac{\pi^{b'}(\rho^*(R', \omega))}{\pi^b(\rho^*(R', \omega))} + \frac{\pi_j^{l'}(R'^*(R', \omega))}{\pi^l(R^*; \rho^*(R', \omega))} \right) \right] = 0, \quad (16)$$

with $\hat{\mathcal{M}}^\beta \subset \mathcal{M}^\beta$ denoting the set of interior β -local maxima. The corresponding sets of local ϕ -maxima, namely $\hat{\mathcal{M}}^\phi$ and \mathcal{M}^ϕ , obtain for $\beta = \phi$.

Note that in the case of linear economies, on which we are focusing, $\pi^{b'} = -\pi^{l'}$. That allows a simple characterization of the equilibria as follows.

Equilibrium characterization. For a given subjective probability distribution β_t , a set of contracts H_t^* at time t is a SSCE but not REE if any $R_t^* \in H_t^*$ is such that

- $R_t^* = \sup \mathcal{M}^\beta$;
- $R_t^* \in \mathcal{M}^\phi$ but $R_t^* \neq \sup \mathcal{M}^\phi$;

whereas it is a REE if any $R_t^* \in H_t^*$ is such that

- $R_t^* = \sup \mathcal{M}^\beta = \sup \mathcal{M}^\phi$.

The requirement $R_t^* \in \mathcal{M}^\phi$ is a direct consequence of having $\beta = \phi$ locally around the equilibrium. Of course, (16) is satisfied locally by any interior SSCE (or REE), i.e. an equilibrium where neither profitability nor participation constraints are binding. In such a case we can obtain a marginal condition on the elasticity of the $\mu(R)$ function which identifies the local maximum.

Example: SSCE equilibria

The first step to compute equilibria in our example is to work out the set \mathcal{M}^ϕ to which a SSCE belongs. We can use (16) to compute the interior local maximum relative to safe and risky project choice, respectively. Then, we check whether such contracts are within the bounds imposed by profitability and participation constraints. Note that, conditionally a particular project choice, the problems are nicely concave, so that a unique maximum typically arise.

Proposition 2. *The set of ϕ -local maxima is $\mathcal{M}^\phi = \{R_s^*, R_r^*\}$ where $R_s^* = \min(\bar{R}, \hat{R}^s)$ with*

$$\hat{R}^s = (1 - \gamma)(r - k) + \gamma R_{CB}, \quad (17)$$

provided that $R_s^ > R_{CB}$, and $R_r^* = \min(r, \hat{R}^r)$ with*

$$\hat{R}^r = (1 - \gamma)r + \frac{\gamma}{\alpha} R_{CB} \quad (18)$$

and it exists if $R_r^ > \bar{R}$.*

\hat{R}^s and \hat{R}^r represent rational expectation interior local maxima, namely they are the contracts which locally maximize banks' profits when no constraints are binding; R_s^* and R_r^* instead account for the possibility that constraints bind. Notice that at the risky equilibrium lenders' participation constraint $\alpha R_r^* - R_{CB} \geq 0$ is always satisfied whenever borrowers' participation constraint $r - R_r^* \geq 0$ is too: in fact $r \geq R_r^*$ implies $r \geq R_{CB}/\alpha$ which in turn yields $R_r^* \geq R_{CB}/\alpha$. Moreover note that the safe SCE always exists, whereas the risky SCE exists only when $R_r^* > \bar{R}$, that is, the borrowers have incentive to adopt risky projects for the interest rate R_r^* offered. Finally $R_r^* > R_s^*$, that is, ceteris paribus, risky projects imply higher interest rates. Nevertheless, the expected profit of both a borrower and a lender can be higher when a risky project is implemented depending on parameters (for example, when $\alpha = 1$).

Let us now characterize the set of REE, i.e. $\sup \mathcal{M}^\phi$.

Proposition 3. *Given ϕ , there exists a threshold value $\hat{\alpha} \in (\underline{\alpha}, \bar{\alpha})$ with*

$$\underline{\alpha} = \frac{r - \hat{R}^s - k}{r - \hat{R}^s} \quad \text{and} \quad \bar{\alpha} = \frac{r - R_{CB} - k}{r - R_{CB}}$$

such that:

- (i) *if $\alpha < \hat{\alpha}$ then $\sup \mathcal{M}^\phi = \{R_s^*\}$,*
- (ii) *if $\alpha > \hat{\alpha}$ then $\sup \mathcal{M}^\phi = \{R_r^*\}$,*
- (iii) *only for $\alpha = \hat{\alpha}$ then $\sup \mathcal{M}^\phi = \{R_r^*, R_s^*\}$.*

The proposition establishes the existence of two regions in the α -dimension whose threshold $\hat{\alpha}$ is the only point where two REE exists in this model. For higher level of riskiness ($\alpha < \hat{\alpha}$) then the safe equilibrium is the unique REE. For lower level of riskiness instead ($\alpha > \hat{\alpha}$) then the risky equilibrium is the unique REE. Such a threshold lies in $(\underline{\alpha}, \bar{\alpha})$, that is the interval for which $R^s = \bar{R}$ that is a safe equilibrium arises as a corner contract.

If R_r^* (resp. R_s^*) is such that borrowers adopt the risky (safe) technology then such a value is an equilibrium if lenders would like not to deviate from that equilibrium. This is certainly true locally for a REE and a SSCE that is not a REE.

Proposition 4. *Given ϕ , only R_r^* can be SSCE without being REE. In particular, for $R_r^* \in \mathcal{M}^\phi$ it is $\sup \mathcal{M}^\beta = \{R_r^*\}$ without being $\sup \mathcal{M}^\phi = \{R_r^*\}$ if: $\alpha < \hat{\alpha}$ and $E^\beta[k]$ sufficiently high.*

The two conditions for the existence of a risky SSCE that is not a REE are intuitive. First, the safe equilibrium must be globally a strictly dominant contract when evaluated with the objective distribution. Second, lenders must believe that it is sufficiently unlikely that borrowers will adopt the safe project if they lower the interest rate; i.e. they expect to be in a risky REE. A sufficiently high level of k is, for example, $E^\beta[k] > (1 - \alpha)r$. Such a missbelief is sustainable given that no evidence of the existence of unexploited profitable contracts is produced along the equilibrium. In other words, with such beliefs lenders can never infer the realization of k as borrowers will always implement α . In this sense beliefs are self-confirmed.

On the other hand, there could not exist a safe SSCE that is not REE. Suppose such an equilibrium exists, then it would arise as a corner solution posted at the frontier \bar{R} because it turns out that interior safe SSCE are always REE (i.e. they are global maxima when evaluated by ϕ). Nevertheless, by definition of a SSCE, agents would have correct beliefs for marginal deviations from the equilibrium that in this case would provide information about the actual α . Therefore at a SSCE posted along the frontier \bar{R} agents would know the actual α . Hence banks can correctly forecast $\rho(R, \omega)$ at any R , and so they cannot sustain a safe SSCE that is not a REE. A contradiction arises. A safe SCE that is not REE can instead exist. Therefore the model has the important feature of generating a unique determinate SSCE that is not a REE which will provide for excessive credit tightening and risk taking.

We can think about a credit crisis as a progressive exogenous fall in α . When risk increases up to a sufficiently high level (low α), rational expectation would predict that the lenders switch to the low interest rate regime. In the logic of Self-Confirming equilibria instead whether or not the “jump” occurs at the right point, if ever, crucially depends on lenders’ expectations about the counterfactuals reaction of borrowers, which are not observed as long as lenders keep posting at the high interest rate regime.

Figure 1 illustrates a baseline configuration of the economy for $r = 0.03$, $R_{CB} = 0.008$, $k = 0.005$, $\gamma = 0.4$, $c = 0.001$, $A = 0.1$. Let us firstly focus on panel A. The feasible range of equilibrium interest rates compatible with the adoption of a safe (resp. risky) technology is the region below (resp. above) the dotted curve representing the adoption frontier of borrowers. For any α , R_r and R_s are denoted by respectively the upper and lower solid lines. In particular, the red line denotes the unique REE for a given α . For $\alpha < \hat{\alpha}$ the unique SSCE which is not REE is plotted in blue.

Panel B plots the corresponding levels of social welfare for the REE and the SSCE that is not REE equilibria as a function of the aggregate risk in the economy, measured in terms of cost-per-vacancy c . Notice that since lenders run at zero expected profits, then the social welfare coincides with the expected profits of borrowers. Social welfare is increasing in α (and so decreasing in R_r) when the economy is on a risky equilibrium,

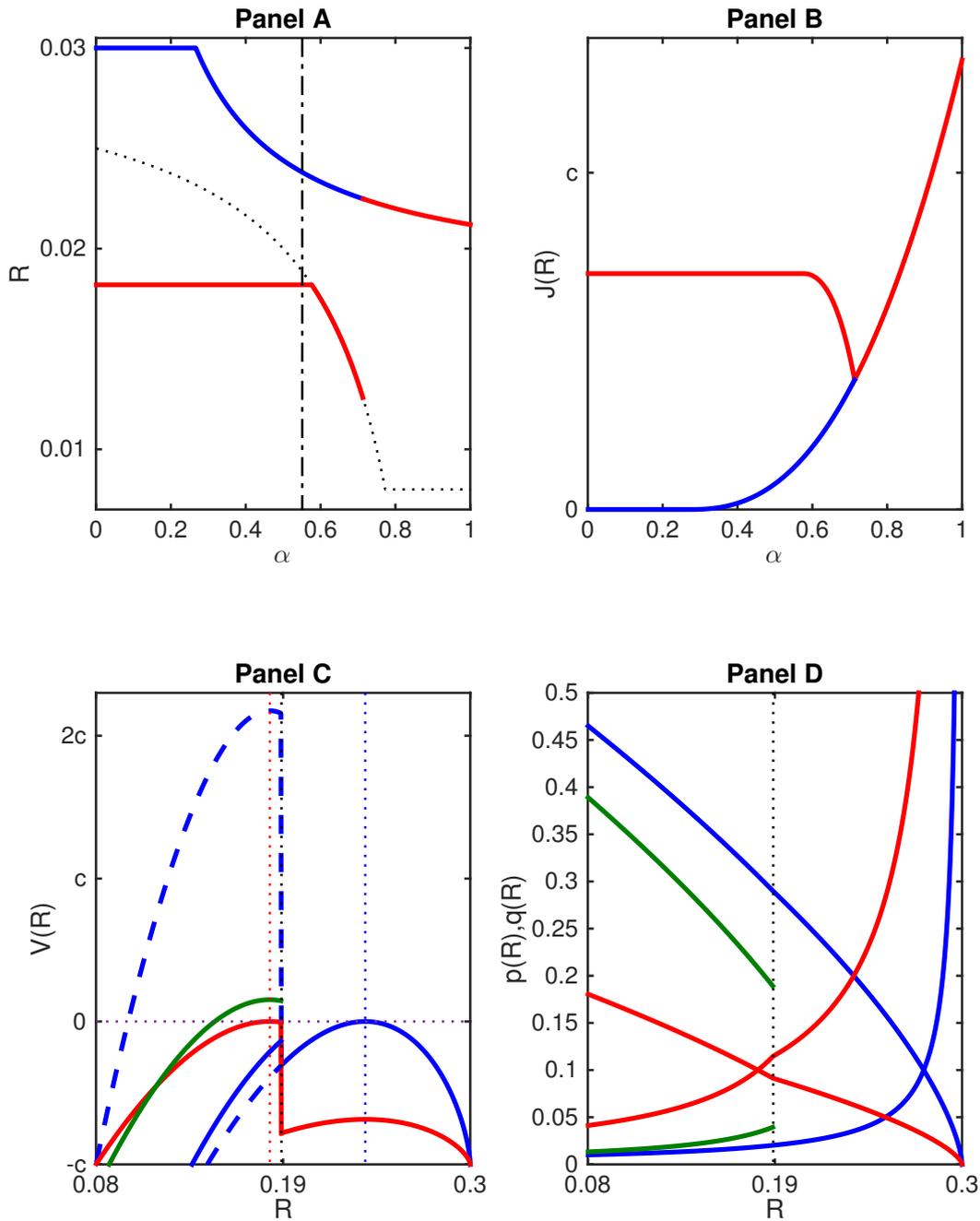


Figure 1: Panel A - the unique REE (red) and the unique SSCE not REE (blue) in the baseline configuration, $r = 0.03$, $R_{CB} = 0.008$, $k = 0.005$, $\gamma = 0.4$, $c = 0.001$, $A = 0.1$; the dotted line represents \bar{R} ; the dashed dotted line is plotted at $\alpha = 0.55$. Panel B - the equilibrium $J(R)$ as a function of α ; Panel C - lender's expected profits as a function of R for $\alpha = 0.55$ (dotted-dashed line in Panel A) in the cases of: the risky SSCE not REE in solid blue (the discontinuity obtains with a 0.07 probability that $k = 0.005$ as opposed to $k = 0.015 > r(1 - \alpha)$), the safe REE in solid red, and the effect on individual profit of the optimal contingent subsidy targeted on R_s^* in solid green; PANEL D: the matching probabilities relative to the cases in panel C.

whereas it is decreasing whenever $\alpha \in (\underline{\alpha}, \bar{\alpha})$ for which the safe equilibrium arises as a corner contract constrained by the borrowers' profitability constraint. For values of $\alpha < \underline{\alpha}$ instead, when the equilibrium contract is \hat{R}_s the aggregate investment is insensitive to α . Red and blue are used to distinguish the REE from the SSCE which is not REE, as in panel A.

Panel C illustrates the individual maximization problem of a single lender for a specific value $\alpha = 0.55$ (dashed-dotted line in Panel A) when all the others post equilibrium contracts at R_r^* . The solid blue line corresponds to the expected profits of lenders when the subjective probability induced by β that $k = 0.005$ is 0.07, whereas the probability that the state is $k = 0.015 > r(1 - \alpha)$ is 0.93. In the case $k = 0.015$ with probability 1 borrowers will never choose the safe technology: the lower dashed blue curve denotes the corresponding expected profits. The higher dashed blue curve represents instead the case $k = 0.005$ for sure. The probability of 0.07 assigned to the safe technology choice by borrowers is too low to induce lenders to individually deviate from the equilibrium. So this situation can sustain a SSCE that is not a REE when, as in our example, an existing globally optimal state for the investor is not believed sufficiently likely. Given this missbeliefs, evidence of the existence of such global maximum is never produced along the equilibrium path. In fact, the risky equilibrium is SSCE but not a REE because lenders could individually lower the interest rate and observe the existence of unexploited profitability opportunities. In such a case, lenders could correct their misspecified beliefs and converge on the unique REE. The REE payoff curve is plotted in red when all lenders post at the REE R_s^* , i.e. $k = 0.007$ is known with certainty. Notice competition dries out lenders' profits so that they always expect to earn zero.

Concluding for now the description of Figure 1 (below we discuss the green line in the bottom panels), Panel D plots the probabilities $q(R)$ (decreasing in R) and $p(R)$ (increasing in R) at the SCE and the REE. Their evolution reflect the effects of directed search. For a given equilibrium, the higher (lower) the interest rate posted by a lender, the lower (higher) the probability of filling that vacancy, and the higher (lower) the probability of borrowers obtaining funds at that rate.

4 Private vs Social Value of Experimentation

In this section we demonstrate how a contingent subsidy from the central bank to the private lenders, financed by borrowers, can be a powerful tool to break a socially inefficient equilibrium and implement, if it exists, a welfare maximizing equilibrium. Under fairly general conditions, we show that if the objective of the CB is to maximize *ex-ante* social welfare, this is equivalent to maximize the *interim* total surplus of a match between a borrower and a lender, when an optimal subsidy is implemented. Moreover, the CB will implement the policy whenever it assigns positive probability to the existence of a REE associated with strictly higher *ex-ante* surplus.

On the other hand, the lenders' private evaluation of the policy does not align with

the social evaluation because, as the subsidy moves the whole distribution of matches, the lenders will always expect to earn zero profit.

4.1 Welfare evaluation of *laissez-faire* economies

In this subsection, we will analyze the problem of a benevolent social planner who maximizes $J(R)$ dictating the interest rate R used by participants into the market, but cannot prevent free entry, and so cannot affect market tightness of submarkets. Importantly we will provide the planner with the same subjective beliefs of the lenders.

The planner's problem is:

$$\max_R E^\beta [p(\theta(R)) \pi_i^b(\rho^*(R, \omega))],$$

subject to

$$c = q(\theta(R)) \pi_j^l(R, \rho^*)$$

and

$$q(\theta(R)) = p(\theta(R))^{-\frac{\gamma}{1-\gamma}},$$

i.e, the social planner maximizes social welfare taking the zero profit condition and the market tightness as a constraint. We will refer to a *laissez-faire* economy as one in which a planner has no other instrument than R to affect the terms of trade.

As before, plugging constraints into the objective we can derive the constrained first-best contract as¹¹

$$R^* = \arg \max \left(E^\beta \left[c^{-\frac{1-\gamma}{\gamma}} \pi_j^l(R, \rho^*)^{\frac{1-\gamma}{\gamma}} \pi_i^b(\rho^*(R, \omega)) \right] \right),$$

so that after defining

$$\hat{\mu}(R) \equiv \pi_j^l(R, \omega)^{\frac{1-\gamma}{\gamma}} \pi_i^b(R, \rho^*), \quad (19)$$

we have a criterion to rank the welfare generated by different contracts.

Lemma 5. *Consider two alternative laissez-faire economies trading at interest rate R_1 and R_2 , respectively. From a the point of view of a planner:*

$$E^\beta [J(R_1)] \geq E^\beta [J(R_2)]$$

if and only if

$$E^\beta [\hat{\mu}(R_1)] \geq E^\beta [\hat{\mu}(R_2)], \quad (20)$$

for any profile of contracts offered by other lenders.

Comparing (20) and (14) we can easily see that the two criteria are necessarily maximized for the same contract, i.e. $R^* = R^*$. We therefore obtain the following proposition, which is a version of the well known result on the efficiency of the directed search competitive equilibrium.

¹¹From here onward, we sill use a \star to denote an outcome determined by the planner, as opposed to $*$, which denotes the outcome determined by private agents.

Proposition 6. *In a laissez-faire economy where lenders and the planner have the same subjective beliefs, the competitive allocation is a solution to the planner's problem.*

The proposition states that in an economy in which the social planner has no other instrument than R to alter the terms of trade, the socially preferred allocation coincides with the one determined by the decentralized market.

4.2 Optimal Subsidy

In this subsection we will introduce the possibility that the social planner can implement a linear transfer between borrowers and lenders. The transfer has two important features.

- i) The subsidy is a lump sum conditional on lenders' losses (from which it is possible to infer the underlying state). This means that it is fixed with respect to individual actions, whereas it is conditional to the realized state.
- ii) The subsidy targets a given contract, i.e. it is calibrated in such a way to induce a particular equilibrium.

To recover the optimal subsidy let us first work out the problem of the authority that, for a given R and each state ω , chooses an optimal subsidy $s^*(R, \omega)$ in order to maximize social welfare. The planner's problem now is:

$$\max_{R, s} E^\beta [p(\theta(R)) (\pi_i^b (\rho^*(R, \omega) - s))],$$

subject to

$$c = q(\theta(R)) (\pi_j^l (R, \rho^*) + s)$$

and

$$q(\theta(R)) = p(\theta(R))^{-\frac{\gamma}{1-\gamma}},$$

i.e, where s denotes a subsidy to lenders financed by taxing borrowers.

Plugging constraints into the objective we can derive the optimal subsidy for a given R and each state ω as:

$$s^*(R, \omega) = \arg_s \max \left(c^{-\frac{1-\gamma}{\gamma}} (\pi_j^l (R, \rho^*) + s)^{\frac{1-\gamma}{\gamma}} (\pi_i^b (\rho^*(R, \omega) - s)) \right),$$

so that after defining

$$\hat{\mu}(R, s^*(R, \omega)) \equiv (\pi_j^l (R, \omega) + s^*(R, \omega))^{\frac{1-\gamma}{\gamma}} (\pi_i^b (R, \rho^*) - s^*(R, \omega)), \quad (21)$$

we have a criterion to rank the welfare generated by different contracts provided the authority implements the optimal subsidy. In particular, the optimal subsidy s^* contingent to a contract R and an event ω is such that

$$\hat{\mu}(R, s^*(R', \omega)) \left(\frac{1-\gamma}{\gamma} \frac{1}{\pi^l (R, \omega) + s^*(R, \omega)} - \frac{1}{\pi^b (R; \rho^*) - s^*(R, \omega)} \right) = 0. \quad (22)$$

Proposition 7. *The optimal subsidy contingent to a couple (R, ω) is:*

$$s^*(R, \omega) = (1 - \gamma)\pi^b(R, \rho^*(R, \omega)) - \gamma\pi^l(R, \omega). \quad (23)$$

Notice that the optimal subsidy implies a split of the total surplus determined by the relative elasticity of the matching function to the mass of applicants relative to offers. In practice, we have

$$\pi^b(R, \rho^*) - s^*(R, \omega) = \gamma\mathcal{S}(R, \omega), \quad (24)$$

$$\pi^l(R, \rho^*) + s^*(R, \omega) = (1 - \gamma)\mathcal{S}(R, \omega). \quad (25)$$

where $\mathcal{S}(R, \omega) \equiv \pi^b(R, \rho^*) + \pi^l(R, \omega)$ is the total surplus generated by the project choice of the borrower as an optimal reaction to an offer R . Finally, plugging (23) back into (21) gives

$$\hat{\mu}(R, s^*(R, \omega)) = \gamma(1 - \gamma)^{\frac{1-\gamma}{\gamma}} \mathcal{S}(R, \omega)^{\frac{1}{\gamma}}, \quad (26)$$

which, therefore, reduces the social evaluation to a simple total expected surplus criterion put forward by the following.

Lemma 8. *Consider two alternative subsidized economies trading at interest rate R_1 with subsidy $s^*(R_1, \omega)$ and R_2 with $s^*(R_2, \omega)$, respectively. From a the point of view of a planner:*

$$\mathbb{E}^\beta [J(R_1, s^*(R_1, \omega))] \geq \mathbb{E}^\beta [J(R_2, s^*(R_2, \omega))]$$

if and only if

$$\mathbb{E}^\beta \left[\mathcal{S}(R_1, \omega)^{\frac{1}{\gamma}} \right] \geq \mathbb{E}^\beta \left[\mathcal{S}(R_2, \omega)^{\frac{1}{\gamma}} \right]. \quad (27)$$

In general the criteria (20) and (27) do not necessarily coincide. The reason is that, without the subsidy, the interest rate R determines at the same time both: i) the incentives of the borrower in the project choice and ii) the incentive of the lender to post an offer, which depends on the split of the expected surplus. The presence of the subsidy makes possible to disentangle this two dimensions. In particular, the subsidy is set such to achieve an efficient share of the surplus, whereas the interest rate can be targeted to induce lenders to select the type of project which maximizes the surplus.

4.3 Implementing *Credit Easing* as a Social Experiment

Since the criteria (20) and (27) may not coincide, there will be situations in which the planner would like to implement the subsidy and change the decentralized allocation in order to achieve a higher *expected* welfare. Moreover, in the case of a SSCE that is not REE, the policy constitutes a social experiment as it is based on subjective beliefs β , which can be misspecified. In such a case the policy has the effect of producing evidence that can correct beliefs and clear uncertainty. Therefore, even in a case where there is any structural externality that prevents the decentralized market to potentially achieve

the social optimum, a temporary policy intervention could be necessary to break the spell of missbeliefs about counterpart risk.

Let us now explain how the authority can implement the preferred allocation. We will show that, in the case of linear economies, the authority can induce whatever equilibrium contract she prefers by means of the subsidy. Suppose the authority targets a certain R^* and implements the subsidy $s^*(R^*, \omega)$. It is simple to show that the lenders' evaluation becomes

$$\mu(R, s^*(R, \omega)) \equiv \hat{\mu}(R, s^*(R, \omega))^{\frac{\gamma}{1-\gamma}}, \quad (28)$$

where $\mu(R, 0)$ is nothing else than (13). In analogy with the equilibrium characterization in the *laissez-faire* economy, let us denote by $\mathcal{M}_{s^*(R, \omega)}^\beta$ (resp. $\hat{\mathcal{M}}_{s^*(R, \omega)}^\beta$) the local (resp. interior) maxima of the function $\mu(R, s^*(R, \omega))$. Therefore the first order condition of $\mu(R, s^*(R, \omega))$ with respect to R (the analogous to (16)), satisfies

$$\mathbb{E}^\beta \left[\mu(R, s^*(R, \omega)) \left(\frac{\gamma}{1-\gamma} \frac{1}{\pi^b(R; \rho^*) - s^*(R, \omega)} - \frac{1}{\pi^l(R, \omega) + s^*(R, \omega)} \right) \right] = 0, \quad (29)$$

exactly at R^* because of (23). The conditions (24), (25) and (29) makes clear that that the optimal subsidy restores *locally* optimality at *any* targeted contract in the sense of the Hosios condition (see [4]). That is, the fraction of the surplus (properly evaluated) going to lenders - the term on the right-hand side - reflects the elasticity of the matching function with respect to the fraction of illiquid borrowers in the market. This is exactly the condition for which lenders internalize the social cost of creating a new contract. We can finally state the following.

Proposition 9. *Suppose the authority targets a contract R^* fixing a contingent subsidy $s^*(R^*, \omega)$, then the lenders' best reply to this policy is to offer:*

$$R^* = \sup \mathcal{M}_{s^*(R^*, \omega)}^\beta = \sup \hat{\mathcal{M}}_{s^*(R^*, \omega)}^\beta$$

We have showed that, in the case of linear economies, for the decentralized market to sustain a certain R^* it is sufficient that the authority commits to $s^*(R^*, \omega)$. Let us now clarify under which conditions the authority will decide to implement a subsidy.

Proposition 10. *Consider a decentralized SSCE equilibrium $R^* \in \mathcal{M}^\phi$ delivering an expected total surplus \mathcal{S}^* . The authority will implement a contingent subsidy $s^*(R^*, \omega)$ targeting a contract R^* whenever $\mathbb{E}^\beta \left[\mathcal{S}(R^*, \omega)^{\frac{1}{\gamma}} \right] \geq \mathbb{E}^\phi \left[(\mathcal{S}^*)^{\frac{1}{\gamma}} \right]$.*

Therefore, the authority will implement the subsidy no matter how small is the subjective probability that total surplus could improve. This result tells us that from the point of view of a planner, the implementation of the subsidy is beneficial irrespective of what agents can eventually learn after exploring new submarkets (notably, that the status quo was not a REE). In other words, although the CB has in principle the power to unveil the true state experimenting on few matches, she finds worth implementing the

policy on the whole distribution to maximize the expected benefit across the population. On the other hand, the fact that all the distribution of posted contracts moves after the subsidy, leaves the lenders at zero expected profits in any case. In this sense the private and social value of experimentation diverge.

Example: Implementation of the Subsidy

In our baseline economy, a large enough k , for example one such that $k > (1 - \alpha)r$, will induce borrowers to always choose the risky project. In this case the surplus is $\mathcal{S}(R, \alpha) = \alpha r - R_{CB}$. The matching instead yields a surplus of $\mathcal{S}(R, k) = r - k - R_{CB}$ in the case the borrowers are offered a positive R such that $R < r - k/(1 - \alpha)$. Notice that whenever such a positive value exists we also have $\mathcal{S}(R, k) > \mathcal{S}(R, \alpha)$. Therefore,

$$E^\beta[k] < (1 - \alpha)r \quad (30)$$

identifies the condition for which a planner, with the same beliefs of lenders, would like to target an interest rate $0 < R < r - E^\beta[k]/(1 - \alpha)$ if the decentralize equilibrium does not belong to this range.

In the case of figure 1, we considered a simplified framework where lenders believe with probability 0.07 that $k = 0.005$, whereas $k = 0.15$ otherwise. Therefore, 0.07 is also the probability that $\mathcal{S}(R, k) = r - k - R_{CB}$ at any $R < r - k/(1 - \alpha)$. In particular, for any $R < r - k/(1 - \alpha)$, it is

$$E^\beta[\mathcal{S}(R, \omega)^{\frac{1}{\gamma}}] > \mathcal{S}(R_r^*, \alpha)^{\frac{1}{\gamma}}. \quad (31)$$

This implies that the authority would like to implement a contingent subsidy at any $R < r - k/(1 - \alpha)$. Let us focus on the case when the authority targets a contract $R^* = R_s^*$ (note that $R < r - k/(1 - \alpha)$), which is the best contract conditional to the realization of the good state $k = 0.005$. In particular, the optimal contingent subsidy is given by

$$s^*(R_s^*, \alpha) = (1 - \gamma)\alpha(r - R_s^*) - \gamma(\alpha R_s^* - R_{CB}), \quad (32)$$

$$s^*(R_s^*, k) = 0, \quad (33)$$

because R_s^* is already locally optimal and so it does not need any correction. The subsidy makes (29) being satisfied, so that lenders - *all of them* - strictly prefer to post offers at R_s^* . This is illustrated by the curve green in panel C of Figure 1, which represents the expected payoff for a $R < \bar{R}$ once the subsidy $s^*(R_s^*, \omega)$ is in play. The pick of the green line is exactly at R_s^* . Therefore the equilibrium at R_r^* is now no longer sustainable, whereas R_s^* is.

In panel D of Figure 1 the green lines plots the matching probability implied by the subsidy. It is useful to contrast those with the blue ones, which denote matching probabilities in absence the subsidy. The subsidy substantially lowers the probability of lenders to find a borrower, which reflects a larger number of lenders in the market attracted by the subsidy. The probability of a borrower finding a lender instead increases by little.

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