Mandatory Disclosure and Financial Contagion*

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Abstract

The paper analyzes the welfare implications of mandatory disclosure of losses at financial institutions when it is common knowledge that some banks have incurred losses but not which ones. We develop a model that features "contagion," meaning that banks not hit by shocks may still suffer losses because of their exposure to banks that are. In addition, banks in our model have profitable investment projects that require outside funding, but which banks will only undertake if they have enough equity. Investors thus value information about which banks were hit by shocks. We find that when the extent of contagion is large, it is possible for no information to be disclosed in equilibrium but for mandatory disclosure to increase welfare by allowing investment that would not have occurred otherwise. Absent contagion, however, mandatory disclosure will not raise welfare, even if markets are otherwise frozen. Our findings provide insight on when contagion is likely to be a concern, e.g. when banks are highly leveraged against other banks, and thus on when mandatory disclosure is likely to be desirable.

JEL Classification Numbers:

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1 Introduction

In trying to explain how the decline in U.S. house prices evolved into a financial crisis in which trade between financial intermediaries nearly ground to a halt, various analysts have singled out the prevailing uncertainty at the time regarding which entities incurred the bulk of the losses associated with the housing market. For instance, Gorton (2008) provides an early analysis of the crisis in which he argues

> "The ongoing Panic of 2007 is due to a loss of information about the location and size of risks of loss due to default on a number of interlinked securities, special purpose vehicles, and derivatives, all related to subprime mortgages... The introduction of the ABX index revealed that the values of subprime bonds (of the 2006 and 2007 vintage) were falling rapidly in value. But, it was not possible to know where the risk resided and without this information market participants rationally worried about the solvency of their trading counterparties. This led to a general freeze of intra-bank markets, write-downs, and a spiral downwards of the prices of structured products as banks were forced to dump assets."

Market participants emphasized the same phenomenon as the crisis was unfolding. Back in February 24, 2007, the *Wall Street Journal* attributed the following to former Salomon Brothers vice chairman Lewis Ranieri, the so-called "godfather" of mortgage finance:

> "The problem ... is that in the past few years the business has changed so much that if the U.S. housing market takes another lurch downward, no one will know where all the bodies are buried. 'I don't know how to understand the ripple effects through the system today,' he said during a recent seminar."

In line with this view, some have argued that an important step in the eventual stabilization of financial markets was the Fed's implementation of bank stress tests. These tests required banks to report to Fed examiners how their respective portfolios would fare under various stress scenarios and thus the losses banks were vulnerable to. In contrast to the traditional confidentiality accorded to bank examinations, the results of these stress tests were publicly released. Bernanke (2013) summarizes the view that the public disclosure of the stress-test results played an important role in stabilizing financial markets:

> "In retrospect, the [Supervisory Capital Assessment Program] stands out for me as one of the critical turning points in the financial crisis. It provided anxious investors with something they craved: credible information about prospective losses at banks. Supervisors' public disclosure of the stress test results helped restore confidence in the banking system and enabled its successful recapitalization."

In this paper, we examine whether uncertainty as to which banks incurred losses – that is, uncertainty as to where the bad apples are located – can lead to market freezes that make it desirable for policymakers to intervene and force banks to disclose their financial position. The feature that turns out to be critical for such intervention to be beneficial in our model is contagion, by which we mean a situation in which shocks that hit some banks lead to losses at other banks that are not themselves hit by these shocks. An example of contagion in the context of the financial crisis is if the losses of banks directly exposed to the subprime market led to losses at banks that held few subprime mortgages in their portfolios.

In what follows, we focus on a model of balance sheet contagion in which banks that are hit by shocks end up defaulting on their obligations to other banks, so that banks not hit by shocks can still have their equity wiped out. We modify this model in two ways. First, we allow banks to raise additional funds from outside investors in order to finance profitable investment projects. However, we introduce an agency problem so that investors only want to invest in banks with sufficient equity. When investors are uncertain about which banks incurred losses, they may refuse to invest in banks altogether. Contagion exacerbates this problem, since investors worry not only that the banks they invest in were hit by shocks that wiped out their equity, but that these banks may be indirectly exposed to such shocks because they have financial obligations from banks that were directly hit. The greater the potential for contagion, the more likely are market freezes to occur.

Second, we allow banks to disclose whether they were hit by shocks. To determine whether mandatory disclosure is desirable, we need to know why banks don't simply hire an external auditor to conduct their own stress test, or else release the information they provide to examiners on their own. We show that when the extent of contagion is small, mandatory disclosure cannot be welfare improving when banks choose not to disclose in equilibrium, even when non-disclosure results in a market freeze where no bank can raise outside funds. But when contagion is large and the cost of disclosure is low, mandatory disclosure can be welfare improving even though banks choose not to disclose their financial situation. Intuitively, contagion implies that information on the financial health of one bank is relevant for assessing the financial health of other banks. Since banks fail to internalize these informational spillovers, too little information will be revealed, creating a role for mandatory disclosure as a welfare improving intervention. Absent these spillovers, banks internalize the benefits of disclosure, and so if they choose not to disclose it must be because the cost of stress-tests exceed the benefits. In that case, forcing them to disclose will not be desirable.

Since our model is somewhat involved, an overview may be helpful. At the heart of our model is a set of banks arranged in a network that reflects the financial obligations across banks. Some of these banks are hit with shocks that prevent them from paying their

obligations to other banks in full. Consequently, even banks not hit by shocks are vulnerable to losses. All banks, including those hit by a shock, have access to profitable projects that require them to raise outside funds. However, because of an agency problem that is present at each bank, outside investors will only want to invest in banks that have enough equity. Banks that want to raise funds from outsiders can disclose at some cost whether they were hit by a shock. This disclosure must be made before a bank knows which other banks were hit with shocks, and thus before it knows its own equity value. Outside investors see all the information that is disclosed and then decide which banks to invest in and under what terms. If enough banks choose not to disclose their state, investors will be uncertain as to which banks were hit by shocks. Finally, banks learn their equity and decide what to do with any funds they raised. In particular, banks that learn their equity has been wiped out will take actions that yield them private benefits at the expense of their investors.

This framework allows us not only to draw the connection between contagion and the desirability of mandatory disclosure, but also to show which features of the underlying financial network are more likely to give rise to contagion and market freezes, e.g. the degree of leverage banks have against other banks in the network, the magnitude of losses, and the number of banks hit by shocks, both relative to the number of banks and in absolute level. In addition, our approach leads us to derive expressions for contagion probabilities for a particular network when there are multiple bad banks, a result that may be of interest for researchers working on contagion independently of our results regarding disclosure.

The paper is structured as follows. Section 2 reviews the related literature. Section 3 develops the model of contagion we use in our analysis. In Section 4 we modify our model so that banks can raise additional funds, and we introduce an agency problem that makes investors leery of investing in banks with little equity. In Section 5, we introduce a disclosure decision. We then examine whether non-disclosure can be an equilibrium outcome, and if so whether mandatory disclosure can be welfare improving relative to that equilibrium. Section 6 considers more general network structures. Section 7 concludes.

2 Literature Review

Our paper is related to several different literatures, specifically work on i) financial contagion and networks, ii) disclosure, iii) market freezes, and iv) stress tests.

Turning first to the literature on contagion, various channels for contagion have been described in the literature. For a survey, see Allen and Babus (2009). We focus on models of contagion based on balance sheet effects in which a bank hit by a shock is unable to pay its obligations, making it difficult for other banks to meet their obligations. Examples of

papers that explore this channel include Kiyotaki and Moore (1997), Allen and Gale (2000), Eisenberg and Noe (2001), Gai and Kapadia (2010), Caballero and Simsek (2012), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013), and Elliott, Golub, and Jackson (2013). These papers are largely concerned with how the pattern of obligations across banks affects the extent of contagion, and whether certain network structures can reduce the extent of contagion. Our focus is quite different: Rather than exploring which policies might mitigate the extent of contagion, we examine whether policies can be used to mitigate the fallout due to contagion once it occurs, e.g. restarting trade in markets that would otherwise remain frozen.

Since our model posits that banks connected via a network can communicate information about themselves, we should point out that there is some work on communication and networks, e.g. DeMarzo, Vayanos, and Zwiebel (2003), Calvó-Armengol and de Martí (2007), and Galeotti, Ghiglino, and Squintani (2013). However, these papers study environments in which agents communicate to others *on the network*. By contrast, we study an environment where agents communicate information *about* the network, specifically the location of bad nodes in the network, *to outsiders*.

The other major literature our work relates to concerns research on disclosure. Two good surveys of this literature include Verrecchia (2001) and Beyer et al. (2010). A key result in this literature, first established by Milgrom (1981) and Grossman (1981), is an "unravelling principle" which holds that all private information will be disclosed because agents with favorable information will want to avoid being pooled together with inferior types and receive worse terms of trade. Beyer et al. (2010) summarize the various conditions subsequent research has established that are necessary for this unravelling result to hold: (1) disclosure must be costless; (2) outsiders know the firm has private information; (3) all outsiders interpret disclosure in the same way, i.e. outsiders have no private information (4) information can be credibly disclosed, i.e. the information disclosed is verifiable; and (5) agents cannot commit to a disclosure policy ex-ante before observing the relevant information. Violating any one of these conditions can result in equilibria where not all relevant information is conveyed. We show that non-disclosure can be an equilibrium outcome in our model even when all of these conditions are satisfied. We thus highlight a distinct reason for the failure of the unravelling principle that is due to informational spillovers: In order to know whether a bank in our model is safe to invest in, outside investors need to know not just the bank's own balance sheet, but also the balance sheets of other banks.

Ours is certainly not the first paper to explore disclosure in the presence of informational spillovers. A particularly important predecessor is Admati and Pfleiderer (2000). Like in our model, their setup allows for informational spillovers and gives rise to non-disclosure equilibria. However, these equilibria rely crucially on disclosure being costly; when the cost

of disclosure is zero in their model, information will be disclosed. The reason our framework allows for non-disclosure even when disclosure is costless is because it allows for informational complementarities that are not present in their model. In particular, disclosure by a bank in our model is not enough to establish whether that bank has positive equity, since this requires information about other banks in the network. This feature, which has no analog in their model, is why non-disclosure equilibria can arise in our framework despite satisfying all of the conditions listed above. However, Admati and Pfleiderer (2000) are similar to us in showing that informational spillovers can make mandatory disclosure welfare-improving.[1] Another difference between our model and theirs is that in their model agents commit to disclosing information before they learn it, while in our model banks know their losses before they choose whether to disclose it. In addition, our setup allow us to study the role of contagion for disclosure, something that cannot be deduced from their setup.

Our paper is also related to the literature on market freezes. As in our model, this literature has emphasized the importance of informational frictions. Some of these papers emphasize the role of private information, where agents are reluctant to trade with others for fear of being exploited by others who are more informed than them. Examples include Rocheteau (2011), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2012), Camargo and Lester (2011), and Kurlat (2013). Other papers have focused on uncertainty concerning each agent's own need for liquidity and the liquidity needs of others which discourages trade. Examples include Caballero and Krishnamurthy (2008) and Gale and Yorulmazer (2013). One difference between our framework and these papers concerns the source of informational frictions. Since in our framework the uncertainty concerns information that can in principle be verified such as the bank's balance sheet, it naturally focuses attention on the possibility that the information agents are uncertain about will be revealed. By contrast, previous papers have focused on private information on individual assets that may be more difficult to verify, or information that no agents are privy to and thus cannot be disclosed.

Finally, there is an emerging literature on stress tests. On the empirical front, Peristian, Morgan, and Savino (2010), Bischof and Daske (2012), Ellahie (2012), and Greenlaw et al. (2012) have looked at how the release of stress-test results in the US and Europe affected bank stock prices. These results are complementary to our analysis, which is more concerned with normative questions regarding the desirability of releasing stress-test results. There are also several recent theoretical papers on stress tests, e.g. Goldstein and Sapra (2013), Goldstein and Leitner (2013), Shapiro and Skeie (2012), Spargoli (2012), and Bouvard, Chaigneau, and de Motta (2013). In these papers, banks are not allowed to disclose information on their own.

[1] Foster (1980) and Easterbrook and Fischel (1984) also argue that spillovers may justify mandatory disclosure, although these papers do not develop formal models to study this.

Thus, these papers sidestep the main question we are after, namely whether it is possible for banks to choose not to disclose even when forcing all banks to disclose is desirable.

3 A Model of Contagion

We begin with a bare-bones version of our model where banks make no decisions. This allows us to highlight how contagion works in our model and to motivate our measure of contagion.

Our approach to modelling contagion follows Allen and Gale (2000), Eisenberg and Noe (2001), Gai and Kapadia (2010), Caballero and Simsek (2012), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) in focusing on the role of balance sheet effects. Formally, there are n banks indexed by $j \in \{0, ..., n-1\}$. Each bank i is endowed with a set of financial obligations $\Lambda_{ij} \geq 0$ to each bank $j \neq i$. Following Eisenberg and Noe (2001), we take these obligations as given without modelling where they come from. For much of our analysis, we follow Caballero and Simsek (2012) in restricting attention to the special case in which

$$\Lambda_{ij} = \begin{cases} \lambda & \text{if } j = (i+1) \pmod{n} \\ 0 & \text{else} \end{cases} \tag{1}$$

This case is known as a ring network or circular network, since these obligations can be depicted graphically as if the n banks are located along a circle as shown in Figure 1, with each bank owing λ units of resources to the bank that sits clockwise from it. In Section 6, we show that our analysis can be extended to a larger class of networks. However, since the circular network is expositionally convenient, we prefer to focus on this network initially.

In addition to the obligations Λ_{ij}, each bank is endowed with some assets that can be liquidated if needed. We do not explicitly model the value of these assets, and simply set their value fixed at some value $\pi > 0$.

A fixed positive number of banks b are hit with negative net worth shocks, where $1 \leq b \leq n-1$. We refer to these as "bad" banks. We thus generalize Caballero and Simsek (2012), who only consider the case of $b = 1$. Each bad bank incurs a loss ϕ, where ϕ represents a claim on the bank by an outside sector, i.e. by an entity that is not any of the remaining banks in the network. The obligation ϕ is senior to the obligations to other banks in the network. That is, all of a bank's available resources must first be used to pay its senior claimant, and only then can bank j make payments to bank $j+1$ from any remaining funds. For example, ϕ could represent a margin call against the bank following a drop in the value of some asset the bank used as collateral. We shall refer to all remaining banks as "good."

Let $S_j = 1$ if j is a bad bank and 0 otherwise. The vector $S = (S_0, ..., S_{n-1})$ denotes the state of the banking network. By construction, $\sum_{j=0}^{n-1} S_j = b$. Shocks are equally like to hit

any bank, i.e. each of the $\binom{n}{b}$ possible locations of the bad banks within the network are equally likely. In particular, $\Pr(S_j = 1) = \frac{b}{n}$ for any bank j.

We now analyze the financial position of banks given our seniority rules. Banks can be either insolvent – meaning they are unable to fully repay their obligation λ to another bank – or solvent and able to fully repay, although they may have to liquidate some of their endowment to do so. The main feature we wish to highlight is that even good banks may be forced to liquidate their assets or wind up insolvent because of their exposure to bad banks.

Let x_j denote the payment bank j makes to bank $j+1$, and y_j denote the payment bank j makes to the outside sector. Bank j has $x_{j-1} + \pi$ resources it can draw on to meet its obligations. Given our restrictions on the seniority, it must first pay the outside sector. Let $\Phi_j \equiv \phi S_j$ denote the obligation to the outside sector. Then the payment y_j must satisfy

$$y_j = \min\{x_{j-1} + \pi, \Phi_j\} \tag{2}$$

Bank j can then use any remaining resources to pay bank $j+1$, to which it owes λ. Hence, the payment bank j makes to bank $j+1$ is given by

$$x_j = \min\{x_{j-1} + \pi - y_j, \lambda\} \tag{3}$$

Substituting in for y_j yields a system of equations involving only the payments between banks, $\{x_j\}_{j=0}^{n-1}$, that characterizes these payments:

$$x_j = \max\{0, \min(x_{j-1} + \pi - \Phi_j, \lambda)\}, \qquad j = 0, ..., n-1 \tag{4}$$

(4) involves n equations and n unknowns. Given a solution $\{x_j\}_{j=0}^{n-1}$, we can define the implied equity of bank j as the value of any residual resources after a bank settles all payments, i.e.

$$e_j = \max\{0, \pi - \Phi_j + x_{j-1} - x_j\} \tag{5}$$

Although e_j is redundant given the payments x_j, equity will turn out to be important later on when we expand the model. While x_j and e_j both depend on the state of the network S, i.e. $x_j = x_j(S)$ and $e_j = e_j(S)$, we shall omit the explicit reference to S when this dependence does not play an essential role. Our first result is to establish that (4) has a generically unique solution $\{x_j^*\}_{j=0}^{n-1}$.[2]

Proposition 1: For a given S, the system (4) has a unique solution $\{x_j^*\}_{j=0}^{n-1}$ if $\phi \neq \frac{n}{b}\pi$.

[2]Our result is a special case of Theorem 2 in Eisenberg and Noe (2001) and Proposition 1 in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013). The latter establishes uniqueness for a generic network Λ_{ij} but does not provide exact conditions for non-uniqueness as we do for the particular network we analyze.

In the knife-edge case where total losses across bad banks, $b\phi$, are equal to the aggregate value of the asset endowments of banks, $n\pi$, there can be multiple solutions if λ is sufficiently large. However, these solutions are equivalent to one another in the sense that across all such solutions, the outside sector is paid in full, so $y_j = \Phi_j$ for all j, and the equity values $\{e_j\}_{j=0}^{n-1}$ of all banks are the same, so $e_j = 0$ for all j. The only difference across solutions are the notional amounts banks default on to other banks.[3]

In what follows, we will initially restrict attention to the case of $\phi < \frac{n}{b}\pi$, so total losses incurred by bad banks $b\phi$ cannot be so large that they exceed the total resources of the banking system, $n\pi$. Although Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) show that explicitly allowing for large losses can yield important insights on the nature of contagion, allowing for large shocks yields few insights for our purposes. In particular, when $\phi > \frac{n}{b}\pi$, there are two possible outcomes depending on the value of λ. When λ is small, the distribution of equity values $\{e_j\}_{j=0}^{n-1}$ is independent of ϕ, and so the implications of this case can be understood even if we restrict $\phi < \frac{n}{b}\pi$. When λ is large, none of the n banks have equity when $\phi > \frac{n}{b}\pi$. Since we are interested in decisions when banks are unsure about their equity, the case where banks know their equity to be zero is of little interest.

At the same time, we don't want the loss per bank ϕ to be too small, since as the next proposition shows, $\phi \leq \pi$ implies bad banks are solvent and so there is no contagion.

Proposition 2: If $\phi \leq \pi$, then $x_j^* = \lambda$ for all j and $e_j = \pi$ for any j for which $S_j = 0$.

The above insights suggest the following restriction on ϕ:

Assumption A1: Losses at bad banks ϕ satisfy $\pi < \phi < \frac{n}{b}\pi$.

When $\phi > \pi$, bad banks will be insolvent: Even if these banks receive the full amount λ from the bank that is indebted to them, they will have less than λ resources to pay their obligations. The equity of each bad bank must therefore be 0 under Assumption A1.

To understand the nature of contagion in this economy, it will help to begin with the case of one bad bank, i.e. $b = 1$, as in Caballero and Simsek (2012). Without loss of generality, let bank $j = 0$ be the bad bank. Given that bank 0 receives x_{n-1} from bank $n-1$, the total amount of resources bank 0 can give to bank 1 is $\min\{x_{n-1} + \pi - \phi, 0\}$. We show in Proposition 3 below that under Assumption A1, there is at least one bank that is solvent and can pay its obligation λ in full. From this, it follows that bank $n-1$ must be solvent, since if any bank $j \in \{1, ..., n-2\}$ were solvent, it would pay bank $j+1$ in full, who in turn will pay bank $j+2$ in full, and so on, until we reach bank $n-1$. Hence, $x_{n-1} = \lambda$.

Given that $x_{n-1} = \lambda$, deriving the equity of the remaining banks is straightforward. Since

[3] Eisenberg and Noe (2001) also show in their Theorem 1 that $\{e_j\}_{j=0}^{n-1}$ is unique even if $\{x_j\}_{j=0}^{n-1}$ is not.

$\phi > \pi$, the bad bank will fall short on its obligation to bank 1 by the amount

$$\Delta_0 = \min\{\phi - \pi, \lambda\}.$$

Since bank 1 is endowed with $\pi > 0$ resources, it can use them to make up some of the shortfall it inherits when it pays bank 2. If the shortfall $\Delta_0 > \pi$, bank 1 will also be insolvent, although its shortfall will be π less than shortfall it receives. The first bank that inherits a shortfall that is less than or equal to π will be solvent, with an equity position that is at least 0 but strictly less than π. Hence, we can classify banks into three groups: (1) Insolvent banks with zero equity, which includes both the bad bank and possibly several good banks; (2) Solvent banks whose equity is $0 \leq e_j < \pi$. When $b = 1$, there will be exactly one such bank; and (3) Solvent banks that are sufficiently far from the bad bank and have equity equal to π.

Since equity will figure prominently in our analysis below, it will be convenient to work with the case where e_j can take on only two values, 0 or π. For $b = 1$, this requires that $\Delta_0 = \min\{\phi - \pi, \lambda\}$ be an integer multiple of π. For general values of b, we will need to impose that both ϕ and λ are integer multiples of π. Formally, we have

Assumption A2: ϕ and λ are both integer multiples of π.

For $b = 1$, Assumption A2 implies that the one solvent bank with equity less than π has exactly zero equity. The number of good banks with zero equity when $b = 1$ is thus

$$k = \frac{\Delta_0}{\pi} = \min\left\{\frac{\phi}{\pi} - 1, \frac{\lambda}{\pi}\right\} \tag{6}$$

Caballero and Simsek (2012) refer to k as the size of the "domino effect" of a bad bank on good banks. With more than one bad bank, k still captures the *potential* for contagion of any given bad bank, in that at most bk good banks can have their equity wiped out. But for reasons that will become apparent when we turn to the case where $b > 1$, the actual number of good banks whose equity falls because of their exposure to bad banks can fall below this amount. As such, we will need to introduce a different metric to measure contagion. This metric will depend on k as well as on other parameters.

Two conditions are required for k in (6) to be large. First, a large k requires losses ϕ to be large. This is because when ϕ is small, a bad bank will still be able to pay back a large share of its obligation λ, and so fewer banks will ultimately be affected by the loss. Second, a large k requires the obligation λ be large. Intuitively, when λ is small, banks are not very indebted to one another, and in the limit as $\lambda \to 0$, there will be no contagion to good banks regardless of how large losses ϕ at bad banks are. As λ rises, what matters is not so much that the bad bank's obligation to one of the other good banks grows, but that

more of the resources of the banking system end up in the hands of the bad bank, where they are diverted to senior claimants. This starves the banking system of equity, leaving fewer resources for banks located downstream from the bad bank. The higher is λ, the fewer resources that remain for banks, and hence the larger the number of good banks that fall victim to contagion. Indeed, we show in Proposition 4 below that for sufficiently large λ, the outside sector will incur no losses, and all losses will be borne by banks within the network.[4]

Armed with this intuition, we can now move to the general case of an arbitrary number of banks, i.e. $1 \leq b \leq n-1$. We begin with a preliminary result that under Assumption A1, at least one bank will be solvent and can pay its obligation in full.

Proposition 3: If $\phi < \frac{n}{b}\pi$, there exists at least one solvent bank j for which $x_j = \lambda$, and among solvent banks there exists at least one bank j with positive equity, i.e. $e_j > 0$.

As in the case with $b = 1$, there will be three types of banks when $b > 1$: (1) Insolvent banks with zero equity; (2) Solvent banks whose equity is $0 \leq e_j < \pi$; and (3) Solvent banks that are sufficiently far away from a bad bank whose equity $e_j = \pi$. Since we know there is at least one solvent bank j, we can start with this bank and move to bank $j+1$. If bank $j+1$ is good, it too will be solvent and its equity will be $e_{j+1} = \pi$. We can continue this way until we eventually reach a bad bank. Without loss of generality, we refer to this bad bank as bank 0. By the same argument as in the case where $b = 1$, Assumption A2 implies that banks $1, ..., k$ will have zero equity, where k is given by (6): Even if all of these banks are good, each will inherit a shortfall of at least π and will have to sell off its π assets. If any of these banks are bad themselves, the shortfall subsequent banks will inherit will be even larger, and so equity at the first k banks will be zero.

If bad banks are sufficiently spread out across the network, i.e. if there are at least k banks between any two bad banks, the number of good banks with zero equity would equal bk, this in addition to the b bad banks whose equity is also wiped out. But more generally, a bank that is exposed to a bad bank may be bad itself. In this case, the number of good banks with zero equity may fall below bk, depending on the size of the obligations across banks λ. We now show that when λ is large, the location of the bad banks within the network will not matter, and exactly bk good banks will have zero equity regardless of whether bad banks are spaced out or not. But when λ is small, the number of good banks with zero equity can be smaller than bk and will depend on how close bad banks are located to one another.

We begin by showing that for sufficiently large λ, all banks will be able to make some

[4]Per Elliott, Golub, and Jackson (2013), increasing λ in our setup implies more integration but not more diversification. However, unlike in their model where greater integration means firms swap their own equity for that of other firms, here greater integration implies greater exposure to shocks at other banks while leaving banks equally vulnerable to their own shocks. Hence, the effect of higher λ is (weakly) monotone.

payment to the bank they are obligated to regardless of where the bad banks are located, i.e. regardless of the state of the banking network $S = (S_0, ..., S_{n-1})$.

Proposition 4: Under Assumption A1, $x_j(S) > 0$ for all j and all S iff $\lambda > b(\phi - \pi)$. When $\lambda \leq b(\phi - \pi)$, there exist realizations of S for which $x_j(S) = 0$ for at least one j.

If each bank j can pay some positive amount to bank $j+1$, then each bank j must pay the outside sector in full given whose claims are senior to all other claims. Hence, Proposition 4 implies that for large λ, senior claimants will be fully paid in all states. This in turn implies that the the total amount of resources left within the banking network is the same regardless of where bad banks are located. Since Assumption A2 implies banks can have equity equal to either 0 or π and total equity is the same for all S, it follows that the number of banks with zero equity is the same for all S whenever $\lambda > b(\phi - \pi)$. Formally:

Proposition 5: Under Assumptions A1 and A2, if $\lambda > b(\phi - \pi)$, the number of good banks with zero equity is equal to bk regardless of the state of the banking network S.

Next, consider the case where λ is small. With fewer resources flowing to bad banks, some bad banks may default on senior claimants. But the more banks default on their obligations to the outside sector, the larger the number of banks who can maintain their equity.

For sufficiently low values of λ, specifically when $\lambda < \phi - \pi$, we can explicitly characterize the distribution of the number of banks with zero equity. At such low values of λ, bad banks would default on senior claimants, and thus default in full on their obligation λ to the next bank. This implies that regardless of where the bad banks are located, the contagion from each bad bank is limited to wiping out the equity of the $k = \frac{\lambda}{\pi}$ banks that come after it. Denote the total number of banks with zero equity, including the b bad banks, by ζ. The number of banks with zero equity ζ is now a random variable, with a support that ranges from $b + k$, when all bad banks are located next to each other, to $bk + b$ when there are at least k good banks between any two bad banks. By contrast, for $\lambda > b(\phi - \pi)$, the number of banks with zero equity ζ has a degenerate distribution with all of its mass at $bk + b$.

To obtain an exact distribution for ζ for $\lambda < \phi - \pi$, we exploit the fact that for $\lambda < \phi - \pi$, our model corresponds to a discrete version of a well-studied geometric problem in applied probability known as the circle-covering problem first introduced by Stevens (1939). In this problem, a given number of points are drawn at random locations along a circle of length 1, and then arcs of a given length less than 1 are drawn starting from each of these points and proceeding clockwise. The only randomness is the location of the arcs. The circle-covering problem involves determining the probability that the circle is covered by the arcs and the distribution of the length of the region that is not covered. In our setting, the number of bad banks is analogous to the number of points drawn at random, while the potential for

contagion k, expressed relative to the total number of banks in the network, corresponds to the length of each arc. The region of the circle covered by arcs is analogous to the fraction of banks with zero equity. The discrete version of this circle-covering problem has been analyzed in Holst (1985), Ivchenko (1994), and Barlevy and Nagaraja (2013). As Holst (1985) notes, the discrete version can be analyzed using Bose-Einstein statistics. This insight can be used to obtain an exact expression for the distribution of ζ. However, for our purposes only the expected value of $E[\zeta]$ matters, which can be obtained using results in Ivchenko (1994) and Barlevy and Nagaraja (2013). This expectation is summarized in the next lemma.

Lemma 1: Under Assumptions A1 and A2, the expected number of good and bad banks with zero equity, ζ, is given by

$$E[\zeta] = n - \frac{(n-b)!\,(n-k-1)!}{(n-1)!\,(n-b-k-1)!}$$

where k is defined by (6) and is equal to $\frac{\lambda}{\pi}$ given $\lambda < \phi - \pi$.

Finally, for intermediate values of λ between $\phi - \pi$ and $b(\phi - \pi)$, the number of banks with zero equity ζ will again be random, with support ranging between $b + \frac{\lambda}{\pi} > b + k$ and $b\frac{\phi}{\pi} = bk + b$, where k is defined in (6). For these intermediate values of λ, the distribution of banks with zero equity is analogous to a circle covering problem in which the length of the arcs is not fixed but rather depends on the location of the points drawn at random. As far as we know, this variation of the circle-covering problem case has yet to be studied. However, in Proposition 6 below we establish some comparative static results for $E[\zeta]$ for this case.

To recap, when $b > 1$, how many good banks will end up with zero equity can be random. To summarize the extent of contagion in this case, consider what happens if we chose a good bank at random. The extent to which good banks are exposed to losses at bad banks will be reflected in the distribution of the equity of this good bank, i.e. how likely it will be to have to liquidate its endowment and end up with an equity below π. The smaller the probability that the equity value is equal to π, the more good banks that tend to have equity below π, and thus the greater the extent of contagion. Formally, define p_g as the probability that a good bank retains all of its equity, i.e.,

$$p_g = \Pr(e_j = \pi | S_j = 0) \tag{7}$$

We will use p_g as our measure of contagion: A value of p_g close to 1 implies a good bank is highly likely to avoid liquidating its resources, so losses at bad banks have a small effect on good banks, while a value of p_g close to 0 means a good bank will be very likely to be wiped out because of direct or indirect exposure to bad banks. As we discuss below, for more

general networks e_j will take on more than just two values, and we will need to track the distribution of e_j. For now, using the definition of k in (6), we can compute p_g as follows:

$$\begin{aligned} p_g &= \sum_{z=b+k}^{bk+b} \Pr\left(e_j = \pi | S_j = 0, \zeta = z\right) \Pr\left(\zeta = z\right) \\ &= \sum_{z=b+k}^{bk+b} \frac{n-z}{n-b} \Pr\left(\zeta = z\right) = \frac{n - E\left[\zeta\right]}{n-b} \ . \end{aligned}$$

Intuitively, the expected number of banks with positive equity is $n - E\left[\zeta\right]$. Since only good banks can have positive equity, and there are always $n - b$ good banks, the fraction of good banks with equity equal to π is just the ratio of the two. The next proposition summarizes how p_g varies in our model depending on the underlying parameters:

Proposition 6. Under Assumptions A1 and A2,

$$p_g = \begin{cases} \prod_{i=1}^{\lambda/\pi} \left(\frac{n-b-i}{n-i}\right) & \text{if } 0 < \lambda < \phi - \pi \\ \Psi\left(b, n, \frac{\phi}{\pi}, \frac{\lambda}{\pi}\right) & \text{if } \phi - \pi \leq \lambda \leq b\left(\phi - \pi\right) \\ 1 - \frac{b}{n-b}\left(\frac{\phi}{\pi} - 1\right) & \text{if } b\left(\phi - \pi\right) < \lambda \end{cases} \tag{8}$$

where the function Ψ is weakly decreasing in ϕ/π and in λ/π.

Proposition 6 reveals that p_g depends on the magnitude of the losses at bad banks ϕ, the depth of financial ties λ, the number of bad banks b, and the total number of banks n. One feature we point out now and revisit below is that the effect of bank losses ϕ on p_g depends on λ. For small λ, specifically for $\lambda < \phi - \pi$, changes in ϕ have no effect on p_g. This is because increasing ϕ only affects senior claimants but not other banks in the network. For large λ, increasing ϕ lowers p_g. That is, when banks are more strongly integrated, a shock that results in bigger losses at bad banks will wipe out the equity of a larger number of good banks. Essentially, high values of λ allow losses at bad banks to affect more good banks. For much of our analysis we can treat p_g as fixed, although we will occasionally return to the comparative statics of what drives p_g.

For $b = 1$, p_g reduces to $\frac{k}{n-1}$ and reflects both the probability a good bank has zero equity and the fraction of good banks with zero equity. For $b > 1$, the fraction of good banks with zero equity may be a random variable, so p_g reflects the probability a good bank has zero equity and the *average* fraction of good banks with zero equity.

Remark 1: For some applications, it would be more convenient to have the fraction of banks with zero equity also deterministic. One way to achieve this for general b is to increase the number of banks n and exploit the law of large numbers. In particular, suppose we hold

the potential for contagion k in (6) fixed and keep the fraction of bad banks $\frac{b}{n}$ constant at some value θ, but let $n \to \infty$. Let ζ_n denote the (random) number of banks with zero equity when there are n banks in the network. When $\lambda < \phi - \pi$, we can appeal to Theorem 4.2 in Holst (1985) to establish that $\frac{\zeta_n}{n}$ converges to a constant as $n \to \infty$. Likewise, the fraction of good banks with zero equity, $\frac{n-\zeta_n}{n-b}$, also converges to a constant. This constant will equal p_g, which recall is just the *expected* fraction of good banks with zero equity. Taking the limit of (8) as $n \to \infty$ for the case where $\lambda < \phi - \pi$ reveals that p_g converges to a simple expression:

$$\lim_{n\to\infty} p_g = (1-\theta)^k \tag{9}$$

Intuitively, a good bank will only have positive equity if each of the k banks located clockwise from him are good. As $n \to \infty$, the probability that any one bank is bad converges to θ independently of what happens to any finite collection of banks around it. Hence, the probability that all of the relevant k neighbor banks are good is $(1-\theta)^k$. While the location of banks with zero equity remains random when the size of the network becomes large, the fraction of good banks with positive equity $\frac{n-\zeta_n}{n-b}$ will exhibit no randomness in the limit. For any given θ, the limiting value of p_g can range between 0 and 1 as k varies from 0 to arbitrarily large integer values. Note that since $k = \min\left\{\frac{\lambda}{\pi}, \frac{\phi}{\pi} - 1\right\}$, values of k that exceed $\frac{1}{\theta} - 1$ will violate the second inequality in Assumption A1, which requires that $\frac{\phi}{\pi}$ be less than $\frac{n}{b} = \frac{1}{\theta}$. However, this restriction can essentially be dispensed with for large values of n, since the probability that equity is wiped out at all banks becomes exceedingly small even without this assumption. The limiting case as $n \to \infty$ is thus useful not only for eliminating uncertainty regarding the extent of contagion, but also for demonstrating that the contagion measure p_g in a circular network can assume the full range of possible values, from nearly no contagion ($p_g \to 1$) to nearly full contagion ($p_g \to 0$). ■

Finally, in some of our subsequent analysis we will need the unconditional probability that a given bank chosen at random has positive equity. Denote this probability by p_0. Since there are exactly b bad banks and $n-b$ good banks, and since all bad banks have zero equity under Assumption A1, p_0 can be expressed directly in terms of p_g:

$$p_0 = \frac{n-b}{n} p_g + \frac{b}{n} \times 0 = \left(1 - \frac{b}{n}\right) p_g \tag{10}$$

4 Outside Investors and Bank Equity

We now build on the model of contagion from the previous section by allowing banks to raise external funds in order to finance productive opportunities. Although all banks can use the

funds they raise profitably regardless of their equity position, we introduce a moral hazard problem that implies only banks with enough equity will use the funds as intended. Specifically, we allow banks to divert the funds they raise to achieve private gains, a temptation that is mitigated by the equity a bank would give up in that case. More generally, there are various actions banks can undertake when their equity is low that would be against the interests of outside investors, e.g. investing in risky projects or gambling for resurrection.

In this section, we focus on the full-information benchmark in which banks and outside investors know which banks are bad and thus the equity of each bank. In this case, allowing banks to raise funds has no impact on contagion. In particular, since outside investors are only willing to finance banks with enough equity, banks that would have had zero equity in the original model will not be able to raise new funds. Letting banks raise funds merely accentuates the inequality between banks with zero and positive equity. While this leads to no new insights regarding contagion, it does introduce a reason for why bank equity can matter for the allocation of resources: Bank equity facilitates gains from trade that would not occur in its absence. When we allow banks to withhold information about whether they were hit by shocks or not, as we do in the next section, policy can potentially affect what agents believe about the equity at any given bank and thus whether trade takes place.

Formally, suppose that outside investors – which can be the same original outsiders that have senior claims against banks or a new group of outsiders – can choose whether to invest with any of the n banks in the network. Banks have profitable projects they can undertake, but funding these projects requires outside financing. For simplicity, we assume that each bank has a finite number of profitable projects it can undertake. We set the capacity of the bank to 1 unit of resources. On their own, outside investors can earn a gross return of $\underline{r}$ per unit of resources. Banks can earn a gross return of R on the projects they undertake, where $R > \underline{r}$. Thus, there is scope for gains from trade.

We restrict banks and outside investors to transact through debt contract that are junior to all of the bank's other obligations. Allowing for equity contracts would not resolve the moral hazard problem we introduce below, and so we invoke this assumption for convenience only. Let r_j^* denote the equilibrium gross interest rate bank j offers outside investors for any funds they invest in the bank. We assume that the outside sector is large enough that r_j^* is set competitively, i.e. the expected gross returns from investing in a bank equal $\underline{r}$. Hence, $r_j^* \geq \underline{r}$, and the most a bank can earn from raising funds is $R - \underline{r}$.

After banks raise funds from outsiders, they can choose to either invest the funds they raised and earn a return R, or divert the funds to a project that accrues a purely private benefit v per unit of resources. These private benefits cannot be seized by outsiders. Outside investors cannot monitor banks and prevent them from diverting funds. However, if the bank

fails to pay the required obligation r_j^*, they can go after any assets the bank owns.

We want v to be large enough to ensure that banks with zero equity would choose to divert – so the moral hazard problem is binding – but not so large that even a bank that keeps its π worth of assets will be tempted to divert funds. To satisfy the first condition, we need $v > R - \underline{r}$, i.e. the private benefit v exceeds the most a bank can earn from undertaking the project. To ensure that a good bank will not be tempted, we need to make sure that the payoff after undertaking the project, $\pi + R - r_j^*$, exceeds the payoff from diverting funds, $v + \max\left\{\pi - r_j^*, 0\right\}$, i.e. the bank would earn v in private benefits but would have to liquidate at least some of its assets to meet the promised obligation of r_j^*. Comparing the two expressions implies we need $v < R - \max\left\{r_j^* - \pi, 0\right\}$. Since a bank that can be entrusted not to divert funds need not offer more than $\underline{r}$ to outsiders, the condition that ensures banks with assets worth π can credibly promise to invest the funds they raise is if $v < R - \max\{\underline{r} - \pi, 0\}$. The conditions on v we need can be summarized as follows:

Assumption A3: The private benefits v to a bank from diverting 1 unit of resources it raises from outsiders are neither too high nor too low, specifically

$$R - \underline{r} < v < R - \max\{\underline{r} - \pi, 0\} \tag{11}$$

Note that the second inequality in (11) implies $v < R$, so diversion is socially wasteful.

In the full information benchmark, banks know the state S, i.e. they know the location of the bad banks. In Section 3, we showed that when banks had no option to raise and invest funds, there were ζ banks with zero equity and $n - \zeta$ with equity π. We now show that when banks can raise funds, the ζ banks that originally had no equity will not be able to raise funds and will thus remain with zero equity, while the remaining $n - \zeta$ banks would be able to raise funds and raise their equity to $\pi + R - \underline{r}$. Allowing banks to raise funds under full information would not change the pattern of contagion in our original model.

To derive this result, define a new variable $I_j \in [0, 1]$ as the amount outsiders invest in bank j. Since Assumption A3 involves strict inequalities, banks will either divert the funds they raise or invest. Let $D_j = 1$ if bank j decides to divert the funds and 0 otherwise. Recall that y_j denotes the obligation of bank j to its most senior creditors and x_j its payment to bank $j + 1$. Let w_j denote its payment to outsiders who invest in bank j. Then we have

$$\begin{aligned}
y_j &= \min\{x_{j-1} + \pi + R(1 - D_j) I_j, \Phi_j\} \\
x_j &= \min\{x_{j-1} + \pi + R(1 - D_j) I_j - y_j, \lambda\} \\
w_j &= \min\left\{x_{j-1} + \pi + R(1 - D_j) I_j - y_j - x_j, r_j^* I_j\right\}
\end{aligned}$$

Finally, the equity at each bank j is given by

$$e_j = \max\{0, x_{j-1} + \pi + R(1 - D_j) I_j - y_j - x_j - w_j\}$$

Let $\{\widehat{y}_j, \widehat{x}_j\}_{j=1}^n$ denote the payments to senior creditors and to banks, respectively, if outside investors could not fund any bank, i.e. if $I_j = 0$ for all j. Likewise, define $\{\widehat{e}_j\}_{j=1}^n$ as the equity positions given $\{\widehat{y}_j, \widehat{x}_j\}_{j=1}^n$, i.e.

$$\widehat{e}_j = \max\{0, \pi - \Phi_j + \widehat{x}_{j-1} - \widehat{x}_j\}$$

Note that $\widehat{e}_j$ corresponds to the equity positions we solved for in the previous section. Our claim is that under full information, $e_j = 0$ whenever $\widehat{e}_j = 0$, and $e_j > 0$ whenever $\widehat{e}_j > 0$.

Proposition 7: Given Assumption A1-A3, with full information, $e_j = 0$ for any bank j for which $\widehat{e}_j = 0$, and $e_j > 0$ if $\widehat{e}_j > 0$. Moreover, $I_j = 0$ if and only if $\widehat{e}_j = 0$.

Proposition 7 shows that even though allowing bankrupt banks to raise funds potentially provides these banks with a way to make up their shortfalls, under full information such banks would not be able to raise funds. Thus, with full information, contagion from bad banks to good banks persists as before. The role of allowing firms to raise funds will turn more interesting when we allow for incomplete information, i.e. when outsiders are unsure which banks are bad. This is the case we turn to in the next section, where we allow firms to choose whether to disclose their financial position. However, even under full information, allowing banks to raise funds introduces one novelty. In particular, we can now assign a social cost to contagion, even though policymakers can do nothing to prevent it in our model: When bank balance sheets are linked, shocks drain more equity away from the banking system and redirect it to senior creditors, reducing the scope for trade. Absent this reduction in trade, contagion merely redistributes resources between bankers and senior claimants.

5 Disclosure

We now introduce the last component into our model – allowing banks to decide whether to disclose their financial position before raising funds. If enough banks decide not to disclose, outsiders must decide whether to invest in banks not knowing exactly where all of the bad banks are located. This allows us to explore the main questions we are after: Under what conditions will market participants be unsure about which banks incurred losses, and in those cases would it be advisable to compel banks to reveal their financial position?

This section is organized as follows. After we describe how we model disclosure, we

provide conditions under which there exists a non-disclosure equilibrium where no bank discloses its S_j. We then examine whether mandatory disclosure can improve welfare relative to this equilibrium. While we give a rigorous answer to this question, our essential insight is captured in Theorem 1, which shows that mandatory disclosure cannot improve welfare when contagion is small but can improve welfare when contagion is large and disclosure costs are not too large. Finally, we examine whether other equilibria besides non-disclosure are possible. While we provide conditions under which multiple equilibria exist, we argue that our main result points to a general tendency for insufficient disclosure in the presence of contagion rather than to the need to help coordinate agents to a superior equilibrium.

5.1 Modelling Disclosure

To model disclosure, suppose that after nature chooses the location of the b bad banks, each bank j observes S_j, but not S_i for $i \neq j$. At this point, all banks simultaneously choose whether to incur a utility cost $c \geq 0$ and disclose their own S_j. The cost c is meant to capture the effort of conducting and documenting the result of stress-test exercises. In principle, c could reflect the cost of revealing information about trading strategies that rival banks can exploit. But it is not obvious whether we should treat these as costs a social planner would face, so we prefer to interpret c as the costs of running stress-tests.

Outside investors observe these announcements and then decide what terms to offer banks (if any). After outsiders choose whether to invest in banks, the state of the network S is revealed, and banks learn their own equity. Only then do banks decide whether to invest the funds they raised or divert them. Finally, profits are realized and obligations are settled. Note that a bad bank with $S_j = 1$ will never find it beneficial to disclose if $c > 0$. As such, we can describe each bank's decision by $a_j \in \{0, 1\}$, where $a_j = 1$ means bank j discloses it is good and $a_j = 0$ means it does not disclose any information. Outside investors thus observe the vector $a = (a_1, ..., a_n)$ and choose whether to provide funds to any of the banks. For simplicity, we force outsiders to only offer debt contracts, so the terms offered to banks can be summarized as an amount of resources each bank j receives, $I_j^*(a)$, and an interest rate $r_j^*(a)$ bank j must repay its investors. As will become clear, allowing for equity contracts would not be of much help given the problem is that banks already have too little equity.

5.2 Existence of a Non-Disclosure Equilibrium

Our first question is under what conditions non-disclosure can be an equilibrium, i.e. each bank is willing to set $a_j = 0$ if it expects $a_i = 0$ for $i \neq j$. This case is of interest since it implies outsiders must be uncertain as to the location of bad banks, in line with our

discussion in the Introduction. As our equilibrium concept, we use the notion of sequential equilibria introduced by Kreps and Wilson (1982), which requires that off-equilibrium beliefs correspond to the limit of beliefs from a sequence in which players choose all strategies with positive probability but the weight on suboptimal actions tends to zero. This rules out arguably implausible off-equilibrium path beliefs. For example, without this restriction, off the equilibrium path outsiders could believe all banks that don't report are bad, even though only b banks are bad. Likewise, without this restriction outsiders can form any beliefs about the neighbors of bank j if bank j deviates from equilibrium and chooses not to disclose, even though bank j knows nothing about other banks when it decides on disclosure.

We now show that the existence of a non-disclosure sequential equilibrium depends on two parameters – the cost of disclosure c and the degree of contagion p_g. For non-disclosure to be an equilibrium, each good bank must weakly prefer not to disclose, i.e. set $a_j = 0$, when it anticipates other banks will not disclose. To solve for the optimal disclosure decision, we need to establish which banks if any outsiders fund when no bank discloses and when a single good bank discloses, since this determines the bank's payoffs. If no bank discloses, the probability that a random bank has positive equity is $p_0 = \left(1 - \frac{b}{n}\right) p_g$ as defined in (10). Under Assumption A3, banks that learn they have zero equity would divert funds and leave nothing for investors. Assumption A2 implies remaining banks have equity π. Whether these banks invest or divert depends on how much r_j^* they promise outside investors in equilibrium. The next lemma summarizes when banks would divert funds:

Lemma 2: Assume Assumption A3 holds. For any bank j where pre-investment equity is π, $D_j = 0$ is optimal if and only if $r_j^*(a) \leq \overline{r} \equiv \pi + R - v$.

In other words, if outside investors charge a rate above some threshold $\overline{r}$, banks will divert funds regardless of their equity. In principle, outsiders might still fund banks at a rate above $\overline{r}$, since they can count on grabbing the equity of banks with positive equity. However, it turns out that the equilibrium interest rate charged to any bank never exceeds $\overline{r}$:

Lemma 3: Assume Assumptions A2 and A3 hold. In any equilibrium, $r_j^*(a) \leq \overline{r}$ for any bank j that receives funding, i.e. for which $I_j^*(a) = 1$.

Under Assumption A3, the maximal rate $\overline{r}$ is bigger than the outside option of outside investors $\underline{r}$.[5] We now argue that if p_0 is small, specifically if $p_0 < \underline{r}/\overline{r} < 1$, then outsiders will not finance any bank in a non-disclosure equilibrium, i.e. $I_j^* = 0$ for all j. Absent any information on S, the rate outside investors must charge to earn as much as from their outside option is $\frac{\underline{r}}{p_0}$. From Lemma 3, banks cannot charge above $\overline{r}$ in equilibrium. Hence, the only possible non-disclosure equilibrium when $p_0 < \underline{r}/\overline{r}$ is if $I_j^* = 0$ for all j, or else outsiders

[5] Consider the two cases $\underline{r} > \pi$ and $\underline{r} \leq \pi$. If $\underline{r} > \pi$, the second inequality in (11) implies $\underline{r} < R + \pi - v \equiv \overline{r}$. If $\underline{r} \leq \pi$, the second inequality in (11) implies $v < R$, and hence $\overline{r} = \pi + R - v > \pi \geq \underline{r}$.

must charge banks a rate above $\overline{r}$, which contradicts Lemma 3. Conversely, when $p_0 > \underline{r}/\overline{r}$, a non-disclosure equilibrium requires $I_j^* = 1$ for all j. Otherwise, there exists a rate $r_j < \frac{\underline{r}}{p_0} < \overline{r}$ that ensures an expected return above $\underline{r}$ to investors so both investors and banks would prefer this to no trade. Note that since p_0 is proportional to p_g from (10), the cutoff for p_0 can be expressed in terms of p_g, i.e. $I_j^* = 0$ if $p_g < \frac{n}{n-b}\underline{r}/\overline{r}$ and $I_j^* = 1$ if $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$.

So far, we have shown that in a non-disclosure equilibrium, outsiders either invest in all banks or none, depending on the value of p_g. We now use this insight to verify whether a good bank would prefer not to disclose S_j knowing that no other bank will disclose. Consider first the case where $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$, which implies $I_j^* = 1$ for all j if no disclosure is an equilibrium. Since a bank can attract funds even without disclosing itself, the only benefit to a good bank from disclosing is that it can pay outside investors less than it would have to otherwise. In particular, disclosure will increase the probability outsiders attach to the bank having positive equity from p_0 to p_g. This would allow a bank to borrow at a lower rate than the $\frac{\underline{r}}{p_0}$ it must pay in equilibrium, the rate that ensures outside investors just earn their outside option in expectation. Formally, the payoff to a good bank from not disclosing is given by

$$p_g\left(\pi + R - \frac{\underline{r}}{p_0}\right) + (1 - p_g)\, v \tag{12}$$

Since a good bank knows it is good, the payoff in (12) is computed using the conditional probability p_g, even though outsiders assign probability p_0 that the bank will have positive equity. If the bank opts to disclose it is good, it will be able to still attract funding if it offered any rate between $\frac{\underline{r}}{p_g}$ and $\frac{\underline{r}}{p_0}$. Hence, when no other good bank chooses to disclose, good banks will be willing not to disclose their own financial position if and only if the disclosure cost exceeds the maximal gain from lowering the rate they are charged, i.e.

$$c \geq p_g\left(\frac{\underline{r}}{p_0} - \frac{\underline{r}}{p_g}\right) = \frac{b\underline{r}}{n-b} \tag{13}$$

Hence, when $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$, a non-disclosure equilibrium exists if and only if $c > \frac{b\underline{r}}{n-b}$, i.e. when disclosure costs are large. In this case, the unique non-disclosure equilibrium is one where all banks receive funding. While this is the unique non-disclosure equilibrium, there may be other equilibria with partial or full disclosure given these values for p_g and c, an issue we return to below. For now, our only interest is in conditions under which there exists an equilibrium in which no information on S is revealed.

Next, consider the case where $p_g < \frac{n}{n-b}\underline{r}/\overline{r}$. Recall that in this case, a non-disclosure equilibrium involves no investment in any of the banks, i.e. $I_j^* = 0$ for all j. We need to verify that no good bank would wish to disclose its position given no other bank discloses.

Since $I_j^* = 0$ in equilibrium, the only way a bank could benefit from disclosure is if revealing it is good will induce outsiders to fund it. Hence, non-disclosure can be an equilibrium if either unilateral disclosure does not induce outsiders to invest in a bank, or if unilateral disclosure induces investment but the cost of disclosure exceeds the gains from attracting investment.

Given our restriction to sequential equilibria, a good bank that discloses unilaterally should expect outside investors to assign probability p_g that it has equity π. Hence, outsiders will demand at least $\frac{\underline{r}}{p_g}$ from it. From Lemma 2, we know that if $\frac{\underline{r}}{p_g} > \overline{r}$, a bank will not be able to both pay enough to outsiders and credibly commit not to divert funds. Hence, if $p_g < \underline{r}/\overline{r}$, a good bank will not be able to attract investment if it discloses unilaterally. In this case, non-disclosure is an equilibrium for any $c \geq 0$. The fact that non-disclosure is an equilibrium even when $c = 0$ is of particular interest, since it shows that our model gives rise to non-disclosure equilibria in cases not already encompassed in the survey of Beyer et al. (2010) we discussed above. That is, our model satisfies each of the conditions they identify for non-disclosure to unravel. Our non-disclosure is instead due to an informational spillover in which information from multiple agents is required to deduce whether a bank has sufficient equity to be worth investing in. This feature has no analog in previous work on disclosure, including work on informational spillovers such as Admati and Pfleiderer (2000). In their model, a firm can disclose all relevant information about itself even when other firms fail to disclose, and their model yields non-disclosure equilibria only when disclosure is costly.[6]

The only remaining case is where $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\underline{r}/\overline{r}$. In this case, $p_0 < \underline{r}/\overline{r} < p_g$. This means that outsiders will be too worried about default to invest when no bank discloses, but will invest in a bank if it alone reveals it is good. In particular, since $p_g > \underline{r}/\overline{r}$, a bank that discloses can offer a rate below $\overline{r}$ that remains competitive with the return $\underline{r}$ outsiders can earn. By disclosing and attracting investment, the bank will achieve an expected payoff of

$$p_g \left(R - \underline{r}/p_g\right) + \left(1 - p_g\right) v - c$$

Hence, non-disclosure is an equilibrium only when c makes disclosure unprofitable, i.e.

$$p_g \left(R - v\right) + v - \underline{r} < c$$

In short, non-disclosure is an equilibrium if either the probability of contagion p_g is small,

[6] Okuno-Fujiwara, Postlewaite, and Suzumura (1990) obtain a result that is closer in spirit to our finding. They provide several examples where non-disclosure can be an equilibrium. In one of these (Example 4), a firm can disclose information it has about another firm, which is similar to our framework. In their setup, a firm does not benefit from disclosing unfavorable information about its competitor because without disclosure the firm's competitor is already at a corner and would act the same way if the firm disclosed unfavorable information about it. Hence, disclosure doesn't matter. By contrast, in our case disclosure matters – outside investors update their beliefs on banks following disclosure – but the impact isn't enough on its own.

enough to render unilateral disclosure ineffective, or if the cost of disclosure c is large. Formally, we can collect our findings into the following:

Proposition 8. Assume that Assumptions A2 and A3 hold. Then

1. A non-disclosure equilibrium *with no investment* can only exist if $p_g \leq \min\left(1, \frac{n}{n-b}\underline{r}/\overline{r}\right)$. Such an equilibrium exists if either

$$\begin{array}{ll} \text{(i)} & p_g \leq \underline{r}/\overline{r}; \text{ or} \\ \text{(ii)} & \underline{r}/\overline{r} < p_g \leq \frac{n}{n-b}\underline{r}/\overline{r} \text{ and } c \geq p_g(R-v) + v - \underline{r} \end{array}$$

2. A non-disclosure equilibrium *with investment* can exist only if $p_g \geq \frac{n}{n-b}(\underline{r}/\overline{r})$. Such an equilibrium exists if

$$\begin{array}{ll} \text{(i)} & \frac{b}{n} \leq 1 - \underline{r}/\overline{r} \text{ to ensure } \frac{n}{n-b}(\underline{r}/\overline{r}) < 1; \text{ and} \\ \text{(ii)} & c \geq \frac{b}{n-b}\underline{r} \end{array}$$

Figure 2 illustrates these results graphically. The shaded region in the figure corresponds to the region in non-disclosure equilibria exists. Since the thresholds for c are not generally comparable for $p_g < \frac{n}{n-b}\underline{r}/\overline{r}$ and $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$, these two cases are shown separately.

Since the degree of contagion as reflected in p_g depends on primitives that govern the financial network of banks, we can relate our existence results to features such as the magnitude of losses ϕ and the size of the obligations λ across banks. As an illustration, observe that when ϕ is small, p_g will be close to 1. If there is a non-disclosure equilibrium, then as long as b/n is small, it will be one in which all banks attract funds. Now, suppose news arrives that losses at banks increased, so ϕ is higher. How this effects the non-disclosure equilibrium depends on λ. Recall that we showed in Section 3 that for $\lambda < \phi$, a change in ϕ has no effect on p_g. Thus, for small λ the news of large losses at some banks will have little observable effect: Banks will continue to attract funds. But if λ is large, p_g will fall with ϕ. If p_g falls sufficiently, then the only possible non-disclosure equilibrium is one in which no bank attracts funds. Hence, the model suggests that large degrees of leverage against other banks as measured by λ allow shocks to give rise to market freezes that would not occur when λ is smaller. In the next subsection, we show that higher leverage may be related not only to the occurrence of market freezes but to whether mandating disclosure is desirable.

5.3 Mandatory Disclosure and Welfare

We now turn to the question of whether if a non-disclosure equilibrium exists, mandating disclosure can be welfare-improving. Recall that there are two types of non-disclosure equilibria,

depending on whether banks raise funds or not. We consider each of these in turn.

We begin with equilibria with no investment, i.e. when $p_g < \frac{n}{n-b}\underline{r}/\overline{r}$. In this case, mandatory disclosure would "unfreeze" markets in the sense that instead of no bank receiving funding, some banks would now receive funding and invest these at a higher return R than what investors can earn on their own. Thus, mandatory disclosure creates surplus. However, this comes at the cost of forcing all banks to incur disclosure costs. To determine whether the additional surplus created exceeds the cost, note that the expected number of banks that will have positive equity and will be able to attract funds is $(n-b)\,p_g$. Each of these banks creates a surplus of $R-\underline{r}$. The cost of forcing all banks to produce information about their losses is cn. Hence, the surplus created exceeds the cost of disclosure iff

$$(n-b)\,p_g\,(R-\underline{r})-cn>0. \tag{14}$$

We will refer to mandatory disclosure as a *welfare improvement over no-disclosure* if (14) holds. Strictly speaking, a Pareto improvement may require redistribution from the banks that benefit to the banks that incur costs but do not attract funds, and one needs to verify such a redistribution scheme does not create incentives for banks to divert funds. Even if this is not possible, condition (14) still implies that mandatory disclosure is desirable *ex-ante*, i.e. banks will prefer it before knowing which banks are bad.

We can now examine whether the conditions that ensure the existence of a non-disclosure equilibrium are compatible with welfare improving mandatory disclosure. From Propostion 8, we know that when $p_g < \underline{r}/\overline{r}$, a non-disclosure equilibrium exists regardless of c. By contrast, (14) implies that forcing all firms to disclose will be valuable if the cost of disclosure c is not too large. Hence, the region in which no disclosure is an equilibrium but mandatory disclosure is welfare improving is non-empty. Formally,

Proposition 9. Assume Assumptions A2 and A3 hold. If $0 < p_g \leq \underline{r}/\overline{r}$ and $c \leq (R-\underline{r})\,\frac{n-b}{n}p_g$, mandatory disclosure is a welfare improvement over no-disclosure.

Intuitively, at low values of p_g, a good bank that unilaterally discloses its S_j will not be able to attract investment. It may therefore be individually optimal for each bank not to disclose even though all banks could be made better off if they coordinated to disclose.

The remaining case of non-disclosure with no investment is when $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\underline{r}/\overline{r}$. For these values of p_g, non-disclosure equilibria exist only for large c, while mandatory disclosure is a welfare improvement for small c. In contrast to the case where $p_g < \underline{r}/\overline{r}$, a good bank now knows it can attract funds by disclosing. Thus, if the gains from trade are sufficiently high to make mandatory disclosure desirable, unilateral disclosure should appeal to good banks. It is therefore not obvious that the existence of non-disclosure equilibria is compatible with

mandatory disclosure being welfare improving. However, since the private incentives to disclose need not coincide with a planner's incentives, the possibility of welfare improvement remains under some circumstances.

The precise conditions for when a non-disclosure equilibrium exists that can be improved upon for $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\underline{r}/\overline{r}$ but which can nonetheless be are summarized in Proposition 10 below. Two conditions are necessary for this to occur. First, we need $v < \underline{r}$, i.e. diversion of funds is socially inefficient since private benefits are less than what outsiders could earn on their own. Without this condition, whenever it is socially optimal to force mandatory disclosure, the private gains from unilateral disclosure will be even higher: A bank benefits not just if it has positive equity but also from diverting funds if it does not. But if v is below $\underline{r}$, banks will fail to take into account the value of disclosure due to avoiding wasteful diversion. Second, the fraction of bad banks $\frac{b}{n}$ cannot be too large. Intuitively, for a bank considering disclosing unilaterally, the cost of communicating to investors that it is good is c. But for a policymaker who does know in advance which banks are good, the cost of disclosure per good bank is $\frac{n}{n-b}c$ since all banks disclose rather than just good banks. This implicitly higher cost of disclosure can make mandatory disclosure undesirable, and so for mandatory disclosure to be welfare improving we need the fraction of bad banks to be small. Formally:

Proposition 10. Assume Assumptions A2 and A3 hold. If $\underline{r}/\overline{r} < p_g < \frac{n}{n-b}\,\underline{r}/\overline{r}$, then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot be welfare improving over non-disclosure.

2. If $v < \underline{r}$, then

   (a) If $\frac{b}{n} > \left(\frac{\overline{r}}{\underline{r}} - 1\right)\frac{\underline{r}-v}{R-\underline{r}}$, there exists no non-disclosure equilibrium that can be welfare improved via mandatory disclosure.

   (b) If $\frac{b}{n} \leq \left(\frac{\overline{r}}{\underline{r}} - 1\right)\frac{\underline{r}-v}{R-\underline{r}}$, a non-disclosure equilibrium exists upon which mandatory disclosure is welfare improving whenever

      i. $\underline{r}/\overline{r} < p_g < \min\left\{\frac{n}{n-b}\underline{r}/\overline{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})}\right\}$, and

      ii. $(R-v)p_g + (v-\underline{r}) \leq c \leq \frac{n-b}{n}p_g\,(R-\underline{r})$.

      Since $\min\left\{\frac{n}{n-b}\underline{r}/\overline{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})}\right\} < 1$, condition (i) requires that $p_g < 1$.

Note that Proposition 10 implies that a non-disclosure equilibrium can be improved upon only if p_g is strictly below 1. That is, mandatory disclosure will only be desirable if there is sufficiently high contagion from bad banks to good banks.

Finally, we turn to the case where $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$. Recall from Proposition 8 that in this case, a non-disclosure equilibrium implies all banks can raise funds. This does not mean

that banks no longer have a reason to disclose: A bank that reveals it is good will be able to promise a lower interest to outside investors. This represents a purely private gain: A bank is able to keep more of the surplus it creates, but disclosure creates no new surplus. As Jovanovic (1982) points out, when disclosure is costly and driven by purely private gains, mandating disclosure is typically undesirable: It represents a costly activity with no social gains. Fishman and Hagerty (1989) similarly show that when disclosure is driven by rent-seeking, forcing more disclosure than occurs in equilibrium may not be desirable. By contrast, since our model exhibits informational spillovers, mandatory disclosure may be desirable even though each bank's decision to disclose is entirely driven by rent-seeking. To see this, observe that the expected resources available to banks in equilibrium is given by

$$(n-b)\,p_g\,(\pi+R)+(n-(n-b)\,p_g)\,v \tag{15}$$

That is, on average $(n-b)\,p_g$ banks have positive equity and invest the funds they raise, while the remainder divert their funds for private gains. By contrast, under mandatory disclosure, all banks with zero equity will be refused funding and outsiders deploy these funds on their own. Expected available resources are then equal to

$$(n-b)\,p_g\,(\pi+R)+(n-(n-b)\,p_g)\,\underline{r} \tag{16}$$

Although v represents private benefits that cannot be redistributed, comparing (15) and (16) still turns out to be the key to whether mandatory disclosure can be welfare improving. This is because if fewer resources are available under mandatory disclosure, it will be impossible to keep everyone as well off, so mandatory disclosure cannot improve welfare. But if more resources are available under mandatory disclosure, this will be without any resources used to obtain private benefits. Hence, as long as the additional resources exceed disclosure costs cn, there will be enough to leave outsiders equally well off but give more to banks. The welfare gain in this case is not due to unfreezing markets, but to preventing wasteful diversion that allows bank to keep more of the surplus they create. Although banks benefit from mandatory disclosure, unilateral disclosure will not be enough to prevent diversion. Comparing the difference between (15) and (16) to disclosure costs reveals that mandatory disclosure will be welfare improving when c satisfies

$$\frac{b\underline{r}}{n-b}<c<\left(1-\frac{n-b}{n}p_g\right)(\underline{r}-v) \tag{17}$$

Once again, for this range to be non-empty, two conditions must be satisfied. First, $v<\underline{r}$, i.e. diversion must be socially wasteful. Second, the fraction of bad banks $\frac{b}{n}$ cannot be too large.

Again, a larger fraction of bad banks raises the effective cost of mandatory disclosure relative to the considerations that determine whether an individual bank would like to disclose. Formally, the case where $p_g > \frac{n}{n-b}\underline{r}/\overline{r}$ can be summarized with the following proposition:

Proposition 11. Assume Assumptions A2 and A3 hold. Suppose $p_g \geq \frac{n}{n-b}\,\underline{r}/\overline{r}$. Then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot be welfare improving over non-disclosure.

2. If $v < \underline{r}$, then

   (a) If $\frac{b}{n} > \frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\overline{r})+\underline{r}}\,(1-\underline{r}/\overline{r})$, there exists no non-disclosure equilibrium that can be welfare improved via mandatory disclosure.

   (b) If $\frac{b}{n} \leq \frac{\underline{r}-v}{(\underline{r}-v)(1-\underline{r}/\overline{r})+\underline{r}}\,(1-\underline{r}/\overline{r})$, a non-disclosure equilibrium exists upon which mandatory disclosure is welfare improving whenever

      i. $\frac{n}{n-b}\underline{r}/\overline{r} \leq p_g \leq \frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right)$, and

      ii. $\frac{b}{n-b}\,\underline{r} \leq c \leq (1-\frac{n-b}{n}\,p_g)(\underline{r}-v)$.

   Since $\frac{n}{n-b}\left(1-\frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right) < 1$, condition (i) only holds for $p_g < 1$.

Note the parallel with Proposition 10. Once again, mandatory disclosure can only be welfare improving if p_g is strictly below 1, i.e. when there is enough contagion.

Summarizing Propositions 9-11 yields the following result regarding the desirability of mandatory disclosure as a function of the extreme values p_g can assume:

Theorem 1. Assume Assumptions A2 and A3 hold. For p_g close to 1, mandatory disclosure cannot improve upon a non-disclosure equilibrium. Conversely, for p_g close to but not equal to 0, if the c is low, the non-disclosure equilibrium is improvable.

Remark 2: Note that neither Theorem 1 nor Propositions 8-11 require Assumption A1. Thus, our key results do not depend on being in what Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) describe as the "small shock" regime. This also suggests Assumption A1 is inessential for small values of n and not just large values as we argued in Remark 1. ■

Note that while mandatory disclosure can be welfare improving when p_g is small but positive, this will not be true in the limit when $p_g = 0$. In that case, there are no banks worth investing in, and so disclosure serves no role. For values of p_g where mandatory disclosure can be welfare improving, the size of the gain is not monotonic in p_g. On the one hand, for $p_g < \frac{n}{n-b}\underline{r}/\overline{r}$, the gains from mandatory disclosure are increasing in p_g, since a higher p_g implies a larger fraction of banks could invest if they received funding. This

illustrates a tension inherent in our model: Contagion makes it more likely that mandatory disclosure improves welfare, but it also makes the gains from such intervention smaller. When $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$, the benefits of mandatory disclosure are instead decreasing in p_g. This is because for these values, investors finance all banks and the benefit of mandatory disclosure instead comes from avoiding socially wasteful diversion. A higher p_g implies more such diversion. In this case, more contagion makes it both more likely that mandatory disclosure can be welfare improving and increases the gains from such intervention. But now there is a different tension: Although more contagion makes mandatory disclosure more desirable, it also makes non-disclosure equilibria in which investors invest in the absence of any information less likely.

Finally, we can relate our results to features of the underlying network of banks. Recall that a low value of ϕ will imply p_g will be close to 1. Thus, when losses at bad banks are small, mandatory disclosure would not be desirable. If losses at bad banks rise, p_g will be unchanged if λ is small but will fall if λ is large. Thus, higher leverage against other banks will not only lead to market freezes, but may justify public disclosure of information that was previously kept confidential. Note that for such a shock to make mandatory disclosure desirable does not require markets to freeze up, since mandatory disclosure can be welfare improving for moderate degrees of contagion when banks are able to attract funds. In fact, mandatory disclosure may turn desirable as contagion exacerbates and outsiders continue to invest in banks, but will turn undesirable if contagion causes markets to freeze. While Theorem 1 states that mandatory disclosure is only desirable with enough contagion, the desirability of intervention need not be monotonic in the degree of contagion.

5.4 Multiple Equilibria

So far, we have argued it is possible for no bank to disclose its status in equilibrium even though forcing all banks to disclose improves welfare. This does not preclude the possibility that there may be other equilibria in which some or even all good banks disclose their status. Since our primary motivation is to determine whether it would ever be appropriate to mandate disclosure when uncertainty about which banks incurred losses is paralyzing markets, we have ignored the possibility of multiple equilibria and simply asked whether a non-disclosure equilibrium exists. However, whether multiple equilibria exist can still be useful to understand some features of the model. For example, does our result that mandatory disclosure can improve welfare arise from a need for someone to coordinate to a different equilibrium? or does our model imply there is too little disclosure?

We now argue that while our model does give rise to multiple equilibria under some circumstances, our model is best understood to imply that contagion leads to inefficiently low disclosure. However, we begin with a result that appears to suggest the opposite, namely

that as long as the number of bad banks b is large enough, whenever mandatory disclosure is a welfare improvement over non-disclosure in the absence of investment, there must exist another equilibrium in which all good banks reveal themselves to be good. In other words, for large b, when mandatory disclosure is beneficial by unfreezing markets, good banks should be able to coordinate to all disclose information without requiring any intervention.

Formally, recall that if markets are frozen in the absence of disclosure, forcing disclosure improves welfare if $(n-b)p_g(R-\underline{r}) > cn$, i.e. if the expected surplus created under full revelation exceeds the cost of forcing disclosure. We will now show that this condition ensures that all good banks disclosing must also be an equilibrium for sufficiently large b.

Proposition 12: Suppose $(n-b)p_g(R-\underline{r}) > cn$. Then given Assumptions A2 and A3, $a_j = 1$ for all good banks j is an equilibrium if $b > \frac{\overline{r}}{\underline{r}} - 1$.

To appreciate the role of the number of bad banks b, note that the usual approach to showing the existence of full-disclosure equilibria is to appeal to "skeptical" beliefs in which outsiders believe that any type that does not disclose is the worst possible type. Since we restrict attention to sequential equilibria, there is a limit to how negative beliefs can be. Suppose that starting from an equilibrium in which all good banks disclose, one good bank decided to deviate. In that case, $b+1$ banks would fail to announce, and outsiders will assign equal probability that each of these is bad, namely $\frac{b}{b+1}$. When b is large, this probability will converge to 1, so beliefs are skeptical. But for small values of b, a bank that deviates will still be perceived as having a high probability of being good, and so the penalty for deviating may not be severe enough to deter deviations.

While Proposition 12 might seem to suggest that when mandatory disclosure is desirable it essentially helps agents coordinate on a superior equilibrium, this conclusion is not correct in general. To see this, we make two observations.

First, Proposition 12 only holds for large values of b. For small values, it need not be the case that whenever mandatory disclosure is desirable, there is some other Pareto-superior equilibrium that good banks can coordinate on. In Appendix B, we give an example where $b = 1$ for a slightly modified version of the model in which no disclosure is the unique equilibrium and yet mandatory disclosure is welfare-improving. In particular, not disclosing is a dominant strategy for each good bank. Thus, mandatory disclosure can make agents better off even without another equilibrium they could coordinate to on their own.[7]

[7]Since the existence of multiple equilibria is often related to the presence of strategic complementarity in actions, we should note that disclosure decisions in our model are not always strategic complements. That is, there are examples in which if we restrict other banks to a common probability of disclosing, the incentive to disclose can fall with the common probability that others disclose. This is because there are two offsetting forces in our model: As more other banks disclose, each remaining banks will be perceived as more likely to be bad, encouraging disclosure. At the same time, if enough of the banks you are exposed to reveal they are good, outsiders may invest in you even if you do not disclose, reducing your own incentive to disclose.

Second, even when b is large, the divergence between private and social incentives implies a role for intervention beyond just addressing a coordination failure. For example, Proposition 10 shows that when p_g assumes intermediate values, a bank could ensure itself funding by unilaterally disclosing even when other banks do not. In this case, coordination is not a problem, and yet mandatory disclosure can still improve welfare because banks value the information they disclose less than the planner. Another way to confirm this claim is to introduce global games elements that prevent agents from coordinating, and then asking whether the unique equilibrium in that environment is efficient. For example, suppose banks and outsiders receive private signals about ϕ, the magnitude of the loss. Good banks that learn ϕ is small would deduce p_g is close to 1 and would prefer to disclose, while good banks that learn ϕ is large would prefer not to disclose if $c > 0$. While we have not analyzed this case, we conjecture that since banks do not internalize all of the value of the information they disclose, banks would choose not to disclose at lower signals than would be optimal.

6 Alternative Network Structures

So far, our model of financial contagion assumed a particular network structure in which the amount bank i owes some other bank j is given by $\Lambda_{ij} = \lambda$ for $j = i + 1 \pmod{n}$. We now argue that our result extends to a larger class of networks as defined by Λ_{ij}.

A general network corresponds to a specification of liabilities across banks that can be summarized by an $n \times n$ matrix Λ with zeros along the diagonal. We restrict attention to networks in which the pre-investment equity at any given bank in the absence of any shocks is the value π of the assets it owns. This requires that each bank have a zero net position with the remaining banks in the network, i.e.

$$\sum_{j \neq i} \Lambda_{ij} = \sum_{j \neq i} \Lambda_{ji} \tag{18}$$

Using network theory terminology, (18) implies Λ is a regular weighted directed network.

As in the case of the circular network, we assume the network is hit by a shock process governed by two parameters: b, the number of bad banks, and ϕ, the losses at each bad bank, where each of the $\binom{n}{b}$ possible locations of the bad banks within the network is equally likely.

Since each bank can in principle be obligated to any of the other $n - 1$ banks, the set of payments is now given by $\{x_{ij}\}_{i \neq j}$ as opposed to just n payments as before. Given that banks can have obligations to multiple banks, we need a priority rule for how available resources be divided if banks are unable to pay all of their obligations. We follow Eisenberg and Noe (2001) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) in assuming that an insolvent

bank pays the same pro-rata share of whatever resources it has to each of the banks it is indebted to. In particular, define Λ_i as bank i's obligations, i.e.

$$\Lambda_i = \sum\nolimits_{j=0}^{n-1} \Lambda_{ij} \tag{19}$$

If bank i is insolvent, it will pay each bank j it is obligated to a fraction $\frac{\Lambda_{ij}}{\Lambda_i}$ of the resources it has. This implies that the set of payments x_{ij} solve the system of equations

$$x_{ij} = \frac{\Lambda_{ij}}{\Lambda_i} \max \left\{ \min \left\{ \Lambda_i \, , \, \pi - S_i \phi + \sum\nolimits_{j=0}^{n-1} x_{ji} \right\} \, , \, 0 \right\} \text{ for all } i \neq j \tag{20}$$

where recall $S_i = 1$ if bank i is bad. We can then define the pre-investment equity of bank i, meaning the equity of bank i if it did not raise any outside funds, as

$$e_i = \pi + \sum\nolimits_{j=0}^{n-1} x_{ji} - S_j \phi - \sum\nolimits_{j=0}^{n-1} x_{ij} \tag{21}$$

A convenient feature of the circular network we have analyzed thus far is that it implies a particular symmetry: Every good bank is equally likely to have its equity wiped out regardless of its identity. This allowed us to summarize contagion with a single statistic, p_g, the probability that a good bank will be unaffected by contagion, as opposed to requiring a vector of probabilities, one for each bank. We now argue that for networks that exhibit a strengthened version of this symmetry property, our main results regarding extreme degrees of contagion and the desirability of mandatory disclosure go through. This stronger symmetry property involves the distribution of pre-investment equity e_j:

Definition: A financial network Λ is *symmetrically vulnerable to contagion* given the shock process $\{b, \phi\}$ if the distribution of pre-investment equity for a good bank does not depend on its identity, i.e. $\Pr(e_j = x | S_j = 0)$ is independent of j for all $x \in [0, \pi]$.

One way to ensure that a network is symmetrically vulnerable to contagion is if the debt obligations that define the network are themselves symmetric. To motivate the exact symmetry that debt obligations must satisfy, suppose we have n distinct physical locations. Once we assign banks to different physical locations, the obligations between banks will give rise to a directed network across locations. Maintaining network theory terminology, we define a *symmetric network* as a network where obligations across banks are such that observing the links across physical locations does not reveal enough information to narrow down the location of any individual bank. That is, obligations across banks are such that any bank j can be located at any one of the possible locations and other banks can be arranged so that the implied network across physical locations remains the same. Formally,

Definition: A network Λ is *symmetric* if for any pair k and ℓ in $\{0, ..., n-1\}$ there exists a bijective function $\sigma_{k,\ell} : \{0, ..., n-1\} \to \{0, ..., n-1\}$ such that (i) $\sigma_{k,\ell}(k) = \ell$ and (ii) for any pair i and j in $\{0, ..., n-1\}$, $\Lambda_{\sigma_{k,\ell}(i),\sigma_{k,\ell}(j)} = \Lambda_{ij}$.

One example of a symmetric network is a circulant network, i.e. a network in which it is possible to order banks in such a way that the matrix of obligations Λ is a circulant, meaning that Λ_{ij} can be expressed solely as a function of the distance $i - j \pmod n$ between banks. The fact that obligations depend only on distance guarantees that observing links across locations reveals nothing about where individual banks are located, since rotating banks across physical locations would leave the pattern of obligations across locations unchanged. Circulant networks include the circular network, together with several other networks that have figured prominently in the literature on financial networks, e.g. complete financial networks where banks maintain equal liabilities with all other banks so $\lambda_{ij} = \lambda$ for all $i \neq j$, partially complete networks where banks have liabilities to some but not all other banks such as the interconnected ring network in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013), and multiple disconnected symmetric networks, e.g. isolated pairs of banks. While circulant networks are symmetric, not all symmetric networks are circulant; we give an example of a symmetric network that is not a circulant in Appendix C. Our results thus hold for a broader class of networks than circulant networks.

Our next result establishes that requiring the network to be symmetric is enough to ensure it will be symmetrically vulnerable to contagion.

Lemma 4: Any regular symmetric network Λ is symmetrically vulnerable to contagion.

Remark: It is not necessary for a network to be symmetric to be symmetrically vulnerable to contagion. In Appendix C, we give an example of an asymmetric network that is symmetrically vulnerable to contagion for a particular b and ϕ. That is, we give an example of a network where observing obligations across physical locations fully reveals where each bank is located, and yet the network can be symmetrically vulnerable to contagion. Thus, our results hold for a broader class of networks than just symmetric networks.

For the circular network we have focused on so far, Assumption A2 ensures that the pre-investment equity e_j for a good bank could only assume two values, 0 and π. Hence, this distribution can be summarized by a single parameter, $p_g = \Pr(e_j = \pi | S_j = 0)$. In the general case, the support of the distribution may contain more than two points. Nevertheless, we can still establish an analog to Theorem 1 which shows that the degree of contagion, as reflected in the likelihood of good banks having to liquidate assets and lowering their pre-investment equity to below π, is related to the desirability of mandatory disclosure:

Theorem 2. Suppose Λ is regular and symmetrically vulnerable to contagion, $\pi < \phi$, and

Assumption A3 holds. If $Pr(e_j = \pi | S_j = 0)$ is sufficiently close to 1, mandatory disclosure cannot improve upon non-disclosure. Conversely, there exists an equity level $e^* > 0$ with $0 < e^* < \pi$ such that if $Pr(e_j \geq e^* | S_j = 0)$ is sufficiently close to but not equal to 0, mandatory disclosure will be welfare improving over non-disclosure for low enough c.

Theorem 2 strictly generalizes Theorem 1. However, it would be incorrect to conclude from this that the structure of the network is irrelevant for whether mandatory disclosure is desirable. This is because the network structure determines the extent of contagion. As an example, consider the complete network in which $\lambda_j = \lambda$ for all $j \neq 0$. In this case, the exact location of bad banks is irrelevant, since the equity of any good bank will be the same regardless of which banks are bad. Mandatory disclosure can serve no positive role in this case. Consistent with this, note that since the location of banks is irrelevant, the distribution of equity at good banks is degenerate. Hence, for a given e^*, the probability $Pr(e_j \geq e^* | S_j = 0)$ jumps from 1 to 0 as we change e^*, a cutoff which depends on the network. But Theorem 2 tells us that mandatory disclosure is welfare improving only if $Pr(e_j \geq e^* | S_j = 0)$ is close to but strictly above 0. Even though the conditions that ensure mandatory disclosure can be desirable do not depend on the particular network structure, whether these conditions can be satisfied does.

7 Conclusions and Future Work

This paper shows that when contagion is substantial and disclosure costs are not too high, mandatory disclosure may result in a Pareto improvement relative to an equilibrium without disclosure. The suggests that stress tests, at least insofar as they include mandatory disclosure of information clauses, are socially beneficial provided that there is enough dependence on their counterpart risk or enough "contagion." These insights are arguably relevant for the recommendation that derivatives trade migrate from over-the-counter trading to centralized exchanges.[8] One of the reasons for this recommendation is the fragility due to chains of indirect exposure of counterparty risk our model aims to capture. While we do not model the equivalent to migrating to an exchange, we view the policy of mandatory disclosure of information as a potential substitute to the migration of trade to exchanges, i.e. we view it as a policy that can address some of the shortcomings of over-the-counter trade which motivate the policy of migration to centralized exchanges.

Since our model is relatively simple, which makes our arguments, we hope, transparent, it leaves out many features which we briefly mention here.

[8]For a discussion see, for example, Duffie and Zhu (2011) and Duffie, Li, and Lubke (2010) and the references therein.

The simplicity of our game between banks and outside investors relies, in part, on our restrictions to networks that satisfy a particular symmetry property. This excludes several interesting cases. First, our set up excludes more realistic networks in which some banks are more centrally located than others. One might be able to gain some insights on how this asymmetry matters by looking at sparsely parameterized core-periphery networks as in Babus and Kondor (2013). For example, when is mandatory disclosure only desirable for core banks, in line with the fact that stress tests in practice were limited to large core banks, and when is it necessary to force the periphery to disclose as well? Second, we might allow the severity of the shocks to vary across banks, or the probability of a shock to hit a bank to vary across banks. This type of analysis may suggest better ways of performing stress tests.

More generally, one can use our framework to think about what optimal disclosure policy might be. Mandatory disclosure treats all banks fairly, but it is also inefficient; requiring only $n-1$ banks to disclose in our model is equally informative but less costly. Still more targeted policies do even better, i.e. policies which pay banks as a function of the outcome of their disclosure, and then make the outcome public. For example, if less than half of all banks are bad, rewarding banks that disclose they are bad will be preferable to forcing all banks to disclose, as would rewarding banks that disclose they are good if less than half of banks are good. The optimal policy will thus depend on the exact details of the environment.

Another feature of our model that is worth investigating is the importance of our assumption that disclosure is simultaneous. Allowing banks to move sequentially can potentially facilitate coordination. Since we show that our result cannot be entirely attributed to coordination failures, we suspect that some of our results would carry over to dynamic environments. However, sequential disclosure is likely to introduce new issues, such as informational cascades and herding where information gets "trapped" if banks that are exposed to bad banks choose not to reveal their own state, discouraging the banks exposed to them from disclosing their status.

Another assumption we impose that may be worth relaxing is that banks can provide incontrovertible proof of their state. A more realistic model would allow banks to give an informative yet imperfect signal. This opens new possibilities which may be relevant for the difference between the social and private value of information disclosure. Still another assumption in our model worth relaxing is that banks disclose actual losses. In practice, stress tests ask banks about potential future losses. Whether this matters for our results remains an open question.

Finally, our analysis focuses on stress tests as a source of information. But in the US, these tests were accompanied by capital injections for weak banks. The framework we propose here may be a useful start for exploring such questions.

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A Proofs

Proof of Proposition 1: We can rewrite the system of equations in (4) as

$$x_j = T_j\left(x_{j-1}\right) \equiv \max\left\{0, \min\left(x_{j-1} + \pi - \Phi_j, \lambda\right)\right\}$$

By repeated substitution, we can reduce this system of equations to a single equation

$$x_0 = T^*\left(x_0\right)$$

where

$$T^*\left(x_0\right) \equiv T_n \circ T_{n-1} \circ \cdots \circ T_1\left(x_0\right)$$

The mapping T^* is continuous, monotone, bounded. Moreover, for any x and y in $[0, \lambda]$, we have $\left|T^*\left(x\right) - T^*\left(y\right)\right| \leq \left|x - y\right|$. Let

$$\begin{aligned} \underline{x} &= \lim_{m\to\infty} \left(T^*\right)^m\left(0\right) \\ \overline{x} &= \lim_{m\to\infty} \left(T^*\right)^m\left(\lambda\right) \end{aligned}$$

These limits exist given T^* is monotone and bounded. By continuity, $\underline{x}$ and $\overline{x}$ must both be fixed points of T^*, i.e.

$$\underline{x} = T^*\left(\underline{x}\right) \text{ and } \overline{x} = T^*\left(\overline{x}\right)$$

Moreover, by monotonicity, $\left(T^*\right)^m\left(0\right) \leq \left(T^*\right)^m\left(\lambda\right)$ for any m. Taking the limit, $\underline{x} \leq \overline{x}$. Hence, the set of fixed points of T^* is nonempty.

Suppose $\underline{x} < \overline{x}$. Then for any $\mu \in (0, 1)$, the value $x_\mu = \mu\underline{x} + (1-\mu)\,\overline{x}$ must also be a fixed point of T^* , i.e.

$$x_\mu = T^*\left(x_\mu\right)$$

For suppose

$$x_\mu > T^*\left(x_\mu\right)$$

In this case, we have

$$\begin{aligned} x_\mu - \underline{x} &> T^*\left(x_\mu\right) - \underline{x} \\ &= T^*\left(x_\mu\right) - T^*\left(\underline{x}\right) \geq 0 \end{aligned}$$

But this counterfactually implies

$$\left|x_\mu - \underline{x}\right| > \left|T^*\left(x_\mu\right) - T^*\left(\underline{x}\right)\right|$$

Likewise, if

$$x_\mu < T^*(x_\mu)$$

then we can show that

$$\begin{aligned} \overline{x} - x_\mu &> \overline{x} - T^*(x_\mu) \\ &= T^*(\overline{x}) - T^*(x_\mu) \geq 0 \end{aligned}$$

which again counterfactually implies

$$|x_\mu - \underline{x}| > |T^*(x_\mu) - T^*(\underline{x})|$$

We conclude that $T^*(x) = x$ for all $x \in [\overline{x}, \overline{x}]$. Next, we argue that for $x \in [\underline{x}, \overline{x}]$, for all $j \in \{1, ..., n\}$,

$$T_j \circ \cdots \circ T_1(x) = T_{j-1}(x) + \pi - \Phi_j$$

For suppose not. That is, there exists some j such that either

$$\begin{array}{ll} \text{(i)} & T_{j-1}(x) + \pi - \Phi_j > \lambda \\ \text{(ii)} & T_{j-1}(x) + \pi - \Phi_j < 0 \end{array}$$

But then by continuity there must exist at least two values $x' \neq x''$ from $[\underline{x}, \overline{x}]$ such that

$$T_j(x') = T_j(x'')$$

and hence $T^*(x') = T^*(x'')$, which requires $x' = x''$, a contradiction. It follows that

$$T^*(x) = x + \sum_{j=1}^{n} (\pi - \Phi_j)$$

for all $x \in [\underline{x}, \overline{x}]$. But since $T^*(x)$ must equal x in this interval, we must have

$$\sum_{j=1}^{n} (\pi - \Phi_j) = 0$$

This implies that $\underline{x} = \overline{x}$, i.e. the fixed point of T^* is unique, whenever

$$\sum_{j=1}^{n} (\pi - \Phi_j) \neq 0$$

This completes the proof for the case where $n\pi \neq b\phi$. ■

Proof of Proposition 2: Since $\phi \leq \pi < \frac{n}{b}\phi$, we know from Proposition 1 that the (4) has a unique solution. It will suffice to verify that $x_j = \lambda$ is a solution. For any $j \in \{1, ..., n\}$, we have

$$x_j = \max\{0, \min(\lambda + \pi - \Phi_j, \lambda)\}$$

Since $\pi - \Phi_j \geq 0$ whenever $\phi < \pi$, then $x_j = \lambda$ solves the system of equations (4). ■

Proof of Proposition 3: Suppose $e_j = 0$ for all j. By construction, $e_j \geq \pi - \Phi_j + x_{j-1} - x_j$. Summing up over all j yields

$$\begin{aligned} \sum_{j=1}^{n} e_j &\geq \sum_{j=1}^{n} (\pi - \Phi_j + x_{j-1} - x_j) \\ &= n\pi - b\phi \\ &> 0 \end{aligned}$$

This contradicts the fact that $e_j = 0$ for all j. Hence, there must exist at least one j for which $x_j = \lambda$.

Next, we argue that the fact that $e_j > 0$ for some j implies $x_j = \lambda$ for some j. For suppose not. Since $x_j = \max\{0, \min\{x_{j-1} + \pi - \Phi_j, \lambda\}\}$, it follows that $x_{j-1} + \pi - \Phi_j < \lambda$ for all j. Hence, $x_j = \max\{0, x_{j-1} + \pi - \Phi_j\}$. From this, it follows that $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = 0$, since either $x_{j-1} + \pi - \Phi_j < 0$ in which case $x_j = 0$ and e_j is the maximum of a negative expression and 0, and thus equal to 0, or else $x_j = x_{j-1} + \pi - \Phi_j$ and so $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = \max\{0, 0\} = 0$. ■

Proof of Proposition 4: Define $\hat{S}$ as a state in which all the bad banks are located next to one another. Without loss of generality, we can order banks so that $\hat{S}_j = 1$ for $j = 0$ and $j \in \{n - b + 1, ..., n - 1\}$. We now establish the claim through a sequence of steps. First, we argue that if the state of the network is given by $\hat{S}$, then for λ sufficiently large, all banks will transfer some positive resources to other banks on the network.

Result 1: Suppose $\lambda > b(\phi - \pi)$. Then if $S = \hat{S}$, the fixed point x_j that solves (4) satisfies $x_j > 0$ for all j.

Proof of Result 1: Suppose $x_0 = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, ..., n - b\}$. Since $n\pi > b\phi$ under A1, we can subtract $b\pi$ from both sides to get

$$(n - b)\pi > b(\phi - \pi)$$

Set $\lambda = b(\phi - \pi) + \varepsilon$ where $\varepsilon > 0$. Choose ε sufficiently small so that

$$(n - b)\pi > b(\phi - \pi) + \varepsilon$$

Then $x_{n-b} = \lambda = b(\phi - \pi) + \varepsilon$. Since banks $n - b + 1$ through $n - 1$ are bad, we have

$$x_0 = \min\{0, x_{n-b} - b(\phi - \pi)\} = \varepsilon$$

Therefore, $x_0 > 0$, a contradiction. Since $T^*(x)$ is weakly increasing in λ, then if $T^*(0) > 0$ for $\lambda = b(\phi - \pi) + \varepsilon$, then $T^*(0) > 0$ for any $\lambda' > b(\phi - \pi) + \varepsilon$. ■

Let $T_j(x; S)$ denote the operator T_j for a particular state of the network S. Likewise, let $T^*(x; S)$ denote the composition of $T_j(x; S)$ for $j = 1, ..., n$ for a particular S. The proof of Result 1 involves showing that for $\lambda > b(\phi - \pi)$, $T^*(0; \hat{S}) > 0$ whenever $\lambda > b(\phi - \pi)$. The next result can establishes that as long as $\lambda > b(\phi - \pi)$, then for any vector S that corresponds to the possible location of the b bad banks, $T^*(0; S) > 0$. From this, it follows that as long as $\lambda > b(\phi - \pi)$, the fixed point x_j that solves (4) is positive for all x_j for all S.

Result 2: $T^*(0;S) \geq T^*(0;\hat{S})$ for all S.

Proof: Observe that starting from $\hat{S}$, we can reach any state $S \neq \hat{S}$ with a finite number of steps where each step involves swapping a pair of adjacent banks, one good bank with a lower index and one bad bank with a higher index, so that after swapping them the bad bank has the lower index and the good bank has the higher index. Formally, there exists a sequence of vectors $(S^0, S^1, ..., S^Q)$ where $Q < \infty$ such that $S^0 = \hat{S}$, $S^Q = S$, and for each q

$$S_i^{q+1} = \begin{cases} S_i^q & \text{if } i \notin \{j_q - 1, j_q\} \\ 1 - S_i^q & \text{if } i \in \{j_q - 1, j_q\} \end{cases}$$

for some j_q where $S_{j_q-1}^q = 1$. Intuitively, we can achieve any desired spacing between the bad banks by first moving bank 0 clockwise, then moving bank $n-1$, and so on, until finally we move bank $n-b+2$.

For each q and an initial x_0, define x_j^q as the payment bank j makes if bank 0 pays x_0 to bank 1 and the state of the network is S^q. We can likewise define x_j^{q+1} when the state of the network is S^{q+1}. Formally,

$$\begin{aligned} x_j^q &= T_j \circ \cdots \circ T_1 (x_0; S^q) \\ x_j^{q+1} &= T_j \circ \cdots \circ T_1 \left(x_0; S^{q+1}\right) \end{aligned}$$

By construction, $S_j^q = S_j^{q+1}$ for $j \leq j_q - 2$, which implies $x_{j_q-2}^q = x_{j_q-2}^{q+1}$.

Let $G(\xi)$ denote the payment a good bank will make if it receives a payment ξ from its neighboring bank, and let $B(\xi)$ denote the payment a bad bank will make. Then

$$\begin{aligned} G(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi\}\} \\ B(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi - \phi\}\} \end{aligned}$$

By definition

$$B(\xi) = G(\xi - \phi) \tag{22}$$

Note that $G(\xi)$ is weakly increasing in ξ with slope bounded above by 1. We can now characterize the payment made by bank j_q when $S = S^q$ and $S = S^{q+1}$ using $G(\cdot)$ and $B(\cdot)$ as follows

$$\begin{aligned} x_{j_q}^q &= B\left(G\left(x_{j_q-2}^q\right)\right) \\ x_{j_q}^{q+1} &= G\left(B\left(x_{j_q-2}^{q+1}\right)\right) \end{aligned}$$

For any real number ξ, (22) implies

$$\begin{aligned} G(B(\xi)) &= G(G(\xi - \phi)) \\ B(G(\xi)) &= G(G(\xi) - \phi) \end{aligned}$$

Since $G(\cdot)$ has a slope bounded above by 1, then since $\phi > 0$,

$$G(\xi - \phi) \geq G(\xi) - \phi$$

Applying $G(\cdot)$ to both sides and using the fact that $G(\cdot)$ is monotone yields

$$G(G(\xi-\phi)) \geq G(G(\xi)-\phi)$$

or alternatively

$$G(B(\xi)) \geq B(G(\xi))$$

Setting $\xi = x^q_{j_q-2} = x^{q+1}_{j_q-2}$, we have

$$\begin{aligned} x^q_{j_q} &= B\left(G\left(x^q_{j_q-2}\right)\right) \\ &\leq G\left(B\left(x^q_{j_q-2}\right)\right) \\ &= G\left(B\left(x^{q+1}_{j_q-2}\right)\right) = x^{q+1}_{j_q} \end{aligned}$$

In other words, the state of the network that minimizes the resources bank 0 has at its disposal is when bank 0 and the $b-1$ banks that come before it are bad.■

Result 2 implies that for any S, a bank that pays nothing will be left with positive resources with which it can pay. This contradiction proves that if $\lambda > b(\phi-\pi)$, the fixed point of (4) must be strictly positive in all its terms.

Finally, we show that if $\lambda \leq b(\phi-\pi)$, there exists a state a fixed point with $x_j = 0$ for at least one j whenever $S = \hat{S}$.

Result 3: If $\lambda \leq b(\phi-\pi)$, then $x_j = 0$ for some j when $S = \hat{S}$.

Proof of Result 3: The proof is by construction. Suppose $S = \hat{S}$, and consider $x_0 = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, ..., n-b\}$. Since $n\pi > b\phi$, subtracting $b\pi$ from both sides yields

$$(n-b)\pi > b(\phi-\pi)$$

Hence, $x_{n-b} = \lambda \leq b(\phi-\pi)$. Since the next b banks are bad, it follows that

$$x_0 = \min\{0, x_{n-b} - b(\phi-\pi)\} = 0$$

This confirms $x_0 = 0$ is a fixed point of (4). ■

Proof of Proposition 5: From Proposition 4, we know that $x_j > 0$ for all $j \in \{0, ..., n-1\}$. Hence,

$$x_j = \min\{\lambda, x_{j-1} + \pi - \Phi_j\}$$

Equity is then given by

$$e_j = \max\{0, x_{j-1} + \pi - \Phi_j - x_j\}$$

We consider each of the two cases for x_j. If $x_j = x_{j-1} + \pi - \Phi_j$, then

$$e_j = x_{j-1} + \pi - \Phi_j - x_j = 0$$

If instead $x_j = \lambda$, then $x_{j-1} + \pi - \Phi_j \geq \lambda$ and so

$$e_j = \max\{0, x_{j-1} + \pi - \Phi_j - \lambda\} = x_{j-1} + \pi - \Phi_j - \lambda$$

Either way, we have

$$e_j = x_{j-1} + \pi - \Phi_j - x_j$$

Summing up the equity values across banks yields

$$\sum_{j=1}^{n} e_j = n\pi - b\phi$$

Hence, the sum of equity values is the same, regardless of S. Assumption A2 implies $e_j \in \{0, \pi\}$. But this implies the cardinality of the set $\{j : e_j = 0\}$ is the same for all S. Let $\zeta \equiv \#\{j : e_j = 0\}$. Then we have

$$\sum_{j=1}^{n} e_j = (n - \zeta)\pi = n\pi - b\phi$$

Since $\lambda > b(\phi - \pi)$, then $\min\{\phi - \pi, \lambda\} = \phi - \pi$. From this, it follows that

$$k \equiv \frac{\min\{\phi - \pi, \lambda\}}{\pi} = \frac{\phi - \pi}{\pi}$$

and so $\phi = (k+1)\pi$. Hence,

$$(n - \zeta)\pi = n\pi - b(k+1)\pi$$

which gives

$$\zeta = b(k+1)$$

as claimed. ■

Proof of the Proposition 6: For $0 < \lambda < \phi - \pi$, Lemma 1 implies

$$\begin{aligned} p_g &= \frac{n - E[\zeta]}{n - b} \\ &= \frac{(n-b)!\,(n-k-1)!}{(n-b)\,(n-1)!\,(n-b-k-1)!} \\ &= \prod_{i=1}^{k} \left(\frac{n-b-i}{n-i} \right) \end{aligned}$$

Since $k = \frac{\min\{\phi-\pi,\lambda\}}{\pi} = \frac{\lambda}{\pi}$, we can rewrite p_g in this case as

$$p_g = \prod_{i=1}^{\lambda/\pi} \left(\frac{n-b-i}{n-i} \right)$$

For $\lambda > b(\phi - \pi)$, Proposition 5 implies $\zeta = bk + b$ with probability 1. Hence,

$$\begin{aligned} p_g &= \frac{n - bk - b}{n - b} \\ &= 1 - \frac{bk}{n - b} \end{aligned}$$

Since $b \geq 1$, from (6), $\lambda > b(\phi - \pi)$ implies $\lambda > \phi - \pi$, and so $k = \frac{\phi}{\pi} - 1$, and so

$$p_g = 1 - \frac{b}{n - b}\left(\frac{\phi}{\pi} - 1\right)$$

Finally, for $\phi - \pi \leq \lambda \leq b(\phi - \pi)$, we have

$$p_g = \frac{n - E[\zeta]}{n - b}$$

Hence, the derivative of p_g with respect to any parameter has the opposite sign as the derivative of $E[\zeta]$ with respect to that parameter.

Let $\zeta(s)$ denote the number of banks with zero equity when the state of the network $S = s$. Let Ω denote the set of all possible values S can take, i.e.

$$\Omega = \left\{ x \in \{0,1\}^n : \sum_{j=1}^{n} x_j = b \right\}$$

The expectation $E[\zeta]$ is given by

$$E[\zeta] = \sum_{s \in \Omega} \Pr(S = s) \times \zeta(s)$$

Note that while $\zeta(s)$ depends on λ, ϕ, and π, the probability $\Pr(S = s)$ only depends on n and b. To establish the claim, it will suffice to show that $\zeta(s)$ is weakly increasing in $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ for all $s \in \Omega$.

Define $d_j(s) = \lambda - x_j(s)$ as the amount bank j falls short in their payment to bank $j+1$. The event in which bank j has zero equity can be described in terms of the variables d_j. In particular, if bank j has zero equity, one of two things must be true: Either $d_j(s) > 0$, implying its equity must be zero given the priority of payments, or else $d_j(s) = 0$ and $d_{j-1}(s)$ is exactly equal to π, implying bank j pays his obligation to bank $j+1$ in full but has to make up a shortfall of π to meet its obligation, exhausting its endowment. Otherwise, if $d_j(s) = 0$, the only remaining possibility is if $d_{j-1}(s) < \pi$, in which case $e_j > 0$. Hence, the number of banks with zero equity $\zeta(s)$ when $S = s$ can be expressed as follows:

$$\zeta(s) = \sum_{j=0}^{n-1} 1\{d_j(s) > 0 \cup (d_j(s) = 0 \cap d_{j-1}(s) = \pi)\}$$

We now show that for a given s, the vector $\{d_j(s)\}_{j=0}^{n-1}$ is weakly increasing in $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$. This in turn implies that $\zeta(s)$ must be increasing in $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$, since increasing $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ can only increase the value of the indicator function.

From Proposition 3, Assumption A1 implies that for every s, there exists at least one bank j for which $x_j(s) = \lambda$, and so $d_j(s) = 0$. Since $x_j(s)$ is continuous in ϕ and λ, so is $d_j(s)$. Given that the number of banks is discrete and finite, it follows that for every λ, there exists an $\varepsilon > 0$ such that for all $\lambda' \in [\lambda, \lambda + \varepsilon)$, there exists some $j \in \{0, ..., n-1\}$ such that $d_j(s) = 0$ for all values of λ'. Simliarly, for every ϕ, there exists an $\varepsilon > 0$ such that for all $\phi' \in [\phi, \phi + \varepsilon)$, there exists some $j \in \{0, ..., n-1\}$ such that $d_j(s) = 0$ for all values of ϕ'. Without loss of generality, we label this bank as $j = 0$, and so $d_0(s) = 0$ over the interval $[\lambda, \lambda + \varepsilon)$ or $[\phi, \phi + \varepsilon)$.

Next, using (4), we have

$$d_{j+1}(s) = \begin{cases} \max\{d_j(s) - \pi, 0\} & \text{if } s_{j+1} = 0 \\ \min\{d_j(s) + \phi - \pi, \lambda\} & \text{if } s_{j+1} = 1 \end{cases} \tag{23}$$

Observe that the solution to the system of equations given by (23) and $d_0(s) = 0$ is homogeneous of degree 1 in (ϕ, λ, π). Hence, $\zeta(s)$ must be homogeneous of degree 0 in (ϕ, λ, π), and so $\zeta(s)$ can be written as a function of the ratios $\frac{\phi}{\pi}$ and $\frac{\lambda}{\pi}$ alone. From (23), it is clear that the solution $\{d_j(s)\}_{j=0}^{n-1}$ is weakly increasing in both π and ϕ, and so $\zeta(s)$ is weakly increasing in these terms as well. This establishes the claim. ■

Proof of Proposition 7: Our proof is by construction. We know from Proposition 3 that there exists at least one bank for which $\widehat{e}_j > 0$. Start with this bank and move to bank $j+1$, continuing on until reaching the first bad bank. Without loss of generality, we can refer to this as bank 1. Moreover, we know that $\widehat{x}_0 = \lambda$, i.e. if outsiders did not invest in any of the banks, then bank 0 would be able to pay its obligation to bank 1 in full.

First, we argue that $x_0 = \lambda$, i.e. when banks can raise outside funds, it will still be the case that bank 0 will be able to pay its debt obligation to bank 1 in full. To see this, define

$$\begin{aligned} T_j(x) &= \max\{0, \min\{x + \pi + R(1 - D_j) I_j - \Phi_j, \lambda\}\} \\ &\geq \max\{0, \min\{x + \pi - \Phi_j, \lambda\}\} \equiv \widehat{T}_j(x) \end{aligned}$$

As before, the payment x_0 must solve the fixed point

$$x_0 = T^*(x_0) = T_n \circ \cdots \circ T_1(x_0) \tag{24}$$

Since $T^*(x_0) \geq \widehat{T}^*(x_0)$, then we know that

$$T^*(\lambda) \geq \widehat{T}^*(\lambda) = \lambda$$

But $T^*(x) \leq \lambda$ for all x. Hence, $T^*(\lambda) = \lambda$, and so $x_0 = \lambda$ is a fixed point of (24).

Now, suppose bank 1 was able to raise funding, i.e. $I_1 = 1$. Let r_1 denote the rate bank 1 is charged. If bank 1 diverted the funds it obtained, its expected payoff would be v. If it

invested the funds, it would get to keep

$$\max\{\lambda + \pi + (R - r) - y_1 - x_1, 0\}$$

where

$$\begin{aligned} y_1 &= \min\{\phi, \lambda + \pi + (R - r_1)\} \\ x_1 &= \min\{\lambda + \pi + R - y_1, \lambda\} \end{aligned}$$

If $y_1 = \lambda + \pi + (R - r_1)$, bank 1 would get to keep 0, which is less than v. If $y_1 = \phi$, bank 1 would get to keep

$$\max\{\lambda + \pi + (R - r_1) - \phi - x_1, 0\}$$

which is 0 if $x_1 = \lambda + \pi + R - y_1$ and $\pi + (R - r_1) - \phi$ if $x_1 = \lambda$. Since $\phi > \pi$ under Assumption A1, this is less than $R - r_1$. Moreover, since $r_1 \geq \underline{r}$ in any equilibrium, $R - r_1 \leq R - \underline{r} < v$, where the last inequality follows from Assumption A3. Thus, bank 1 will not be able to raise outside funds, i.e. $I_1 = 0$. From this we can conclude that $e_1 = 0$, since bank 1's resources $\lambda + \pi - \phi$ are less than its obligation of λ to bank 2.

We now proceed by induction. Suppose $e_1 = \cdots = e_{j-1} = 0$ and $I_1 = \cdots = I_{j-1} = 0$. Assumption A2 implies $\widehat{e}_j$ is equal to either 0 or π. We consider each case in turn.

Suppose first that $\widehat{e}_j = 0$. We argue that $I_j = 0$, i.e. if bank j would have zero equity in the absence of investment, then bank j would be unable to raise funds when investment is allowed. For suppose not. Given $x_0 = \widehat{x}_0 = \lambda$ and $I_1 = \cdots = I_{j-1} = 0$, it follows that

$$x_{j-1} = \widehat{x}_{j-1}$$

Since $\widehat{e}_j = 0$, we know that under Assumptions A1 and A2, $\widehat{x}_{j-1} \leq \lambda - \pi$. Now, suppose bank j were able to raise funds. Then if bank j diverts the funds it obtains, its payoff would be v. In particular,

$$\begin{aligned} y_j &= \min\{\Phi_j, x_{j-1} + \pi\} = \widehat{y}_j \\ x_j &= \max\{0, \min\{x_{j-1} + \pi - y_j, \lambda\}\} = \widehat{x}_j \end{aligned}$$

and since

$$\widehat{e}_j = \max\{0, \widehat{x}_{j-1} + \pi - \widehat{y}_j - \widehat{x}_j\} = 0$$

then even before paying back outside investors w_j, the bank would have no resources left. By contrast, if the bank did not divert, then since $\widehat{x}_{j-1} \leq \lambda - \pi$, its payoff will be at most $R - r_j \leq R - \underline{r} < v$, where $r_j \geq \underline{r}$ is the rate bank j will be charged by outside investors. Hence, $I_j = 0$ as claimed. Since $I_j = 0$ implies $x_j = \widehat{x}_j$, it follows that $e_j = \widehat{e}_j = 0$.

Next, suppose $\widehat{e}_j = \pi$. Note that this implies $S_j = 0$, i.e. j must be a good bank. We want to show that $I_j = 1$ and $x_j = \lambda$. That is, if bank j would have full equity in the absence of investment, then bank j would raise funds when investment is allowed. To see this, observe

that $\widehat{e}_j = \pi$ implies $x_{j-1} = \widehat{x}_{j-1} = \lambda$. Hence, we have

$$\begin{aligned} y_j &= \min\{\Phi_j, \lambda + \pi\} = 0 \\ x_j &= \max\{0, \min\{\lambda + \pi + R(1 - D_j) I_j, \lambda\}\} = \lambda \end{aligned}$$

If the bank obtained funds from outside investors, i.e. $I_j = 1$, and did not divert funds, it would earn $\pi + R - r_j$. If it chose to divert funds, it would receive $v + \min\{\pi - r_j, 0\}$. At $r_j = \underline{r}$, Assumption A3 ensures that the bank would prefer to invest than to divert the funds. Since outsiders can observe the state of each bank, it follows that the unique equilibrium is one where $r_j = \underline{r}$ and $I_j = 1$.

So far, we have established that starting from bank 1, continuing through all the consecutive banks for which $\widehat{e}_j = 0$ implies $I_j = 0$. Once we reach the first bank for which $\widehat{e}_j = \pi$, we know that $x_j = \lambda$, and we can keep going until we reach the next bad bank. Since this bank receives λ, the analysis would be the same as for bank 1. The claim then follows. ■

Proof of Lemma 2: If bank j has positive equity in equilibrium, it must be that $x_{j-1} = \lambda$, i.e. bank j is paid in full. This is because Assumptions A1 and A2 imply that if $x_{j-1} < \lambda$, then $\widehat{e}_j = 0$, i.e. such a bank would have no equity prior to raising any funds from outside investors. But we know from Assumption A3 that such a bank would divert funds, i.e. $D_j = 1$, and so such a bank would have no equity. Given this, a bank that receives outside funding would choose to invest the funds it raises rather than divert them iff

$$v + \max\left\{\pi - r_j^*, 0\right\} < \pi + R - r_j^* \tag{25}$$

Suppose $r_j^* < \pi$. In this case, $\max\left\{\pi - r_j^*, 0\right\} = \pi - r_j^*$. But then Assumption A3 tells us that (25) must hold, since it reduces to $v < R$. Next, suppose $r_j^* \geq \pi$. In this case, $\max\left\{\pi - r_j^*, 0\right\} = 0$. In that case, (25) only holds if $\pi \leq r_j^* \leq \pi + R - v$. Since $v < R$, this bound exceeds π. It follows that $D_j = 0$ if and only $r_j^* \leq \pi + R - v$. ■

Proof of Lemma 3: From Lemma 2, the only scenario we have to explore is whether there exists an equilibrium with $r_j > \overline{r}$ in which a bank with positive equity chooses to divert, i.e. $D_j = 1$. Let p_j denote the probability that bank j has positive equity in equilibrium. Then the expected payoff to bank j is given by $p_j\left(r_j^*(1 - D_j) + \min\left\{\pi, r_j^*\right\} D_j\right)$. When $D_j = 1$, this payoff collapses to $= p_j\pi$. But suppose an outside investor were to charge $r_j = \pi + \varepsilon$ where ε is sufficiently small so ensure that $r_j < \overline{r}$. In that case, the bank would be strictly better off since it is charged a lower rate. Moreover, since $\pi + \varepsilon < \overline{r}$, the bank will invest and pay $r_j = \pi + \varepsilon$ in full, so the investor that charges this amount will be better off. But then the original outcome with $r_j^* > \overline{r}$ could not have been an equilibrium. ■

Proof of Proposition 10: First, suppose $v \geq \underline{r}$. Then for any $p_g \in (0, 1)$, we have

$$\begin{aligned} (R - v) p_g + (v - \underline{r}) &= p_g (R - \underline{r}) + (1 - p_g)(v - \underline{r}) \\ &\geq p_g (R - \underline{r}) \\ &> p_g \tfrac{n-b}{n} (R - \underline{r}) \end{aligned}$$

Mandatory disclosure is preferable to no investment if

$$c < (R - \underline{r})\tfrac{n-b}{n} p_g$$

But from above it follows that

$$c < (R - v)p_g + (v - \underline{r})$$

Since $p_g > \underline{r}/\overline{r}$ implies a good bank that unilaterally discloses will be able to raise funds, while the above inequality implies the benefits from attracting funds exceed the disclosure cost, it follows that non-disclosure cannot be an equilibrium whenever mandatory disclosure is preferable to no investment.

Next, suppose $v < \underline{r}$. For any $p_g > \underline{r}/\overline{r}$, a non-disclosure equilibrium with no investment will exist if

$$p_g \leq \frac{n}{n-b}\underline{r}/\overline{r} \text{ and } c \geq (R - v)p_g + (v - \underline{r})$$

and mandatory disclosure will be preferable to no investment if

$$c \leq p_g \frac{n-b}{n}(R - \underline{r})$$

The only way both inequalities involving c can be satisfied is if

$$(R - v)p_g + (v - \underline{r}) \leq p_g \frac{n-b}{n}(R - \underline{r})$$

Rearranging, improveability on a non-disclosure equilibirum with no investment is possible only if

$$p_g \leq \frac{\underline{r} - v}{(R - v) - \frac{n-b}{n}(R - \underline{r})}$$

For this bound to exceed $\underline{r}/\overline{r}$ requires

$$\frac{(\underline{r} - v)}{(R - v) - \left(1 - \frac{b}{n}\right)(R - \underline{r})} > \underline{r}/\overline{r}$$

which, rearranging, implies

$$\frac{b}{n} < \left(\frac{\overline{r}}{\underline{r}} - 1\right)\frac{\underline{r} - v}{R - \underline{r}}$$

Finally, from A3,

$$\begin{aligned} \frac{\underline{r} - v}{(R - v) - \left(1 - \frac{b}{n}\right)(R - \underline{r})} &= \frac{\underline{r} - v}{(\underline{r} - v) + \frac{b}{n}(R - \underline{r})} \\ &< 1 \end{aligned}$$

which completes the proof. ■

Proof of Proposition 11: First, suppose $v \geq \underline{r}$. The expected amount banks pay to

investors is $\underline{r}$ both when there is no disclosure and when there is mandatory disclosure. For a good bank, then, the expected payoff under the non-disclosure equilibrium with investment is $p_g R + (1 - p_g) v - \underline{r}$. Under mandatory disclosure, the expected payoff for a good bank is $p_g (R - \underline{r})$, which is strictly lower. This confirms some party will be made worse off with mandatory disclosure, so mandatory disclosure cannot be Pareto improving.

Next, suppose $v < \underline{r}$. A non-disclosure equilibrium with investment can only exist if $c > \frac{b\underline{r}}{n-b}$. At the same time, mandatory disclosure will be Pareto improving relative to an equilibrium where outsiders invest in all banks only if $c < \left(1 - \frac{n-b}{n} p_g\right)(\underline{r} - v)$. For mandatory disclosure to be Pareto improving and for there to exist a non-disclosure equilibrium with investment, we need

$$\frac{b\underline{r}}{n-b} < \left(1 - \frac{n-b}{n} p_g\right)(\underline{r} - v)$$

or, rearranging, if

$$p_g \leq \frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r} - v}\right)$$

If this inequality is violated at $p_g = \frac{n}{n-b}\frac{\underline{r}}{\overline{r}}$, then it will be violated for all $p_g \geq \frac{n}{n-b}(\underline{r}/\overline{r})$. Hence, a necessary condition for the existence of a Pareto-improveable non-disclosure equilibrium is for

$$\frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r} - v}\right) \geq \frac{n}{n-b}\underline{r}/\overline{r}$$

Rearranging, we have the condition

$$\frac{b}{n} \leq \frac{\underline{r} - v}{(\underline{r} - v)(1 - \underline{r}/\overline{r}) + \underline{r}}\left(1 - \frac{\underline{r}}{\overline{r}}\right)$$

Hence, without this condition, there exists no Pareto-improveable non-disclosure equilibrium with investment. With this condition, the interval $\left[\frac{n}{n-b}\underline{r}/\overline{r}, \frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r}-v}\right)\right]$ will be non-empty. For any p_g in this interval, and so the as long as $c \in \left[\frac{b}{n-b}\underline{r}, (1 - \frac{n-b}{n}p_g)(\underline{r} - v)\underline{r}/\overline{r}\right]$, which is necessarily non-empty given the restriction on $\frac{b}{n}$, a non-dislcosure equilibrium with investment is Pareto-improveable. Finally, observe that since $v < \underline{r}$, then

$$\frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r} - v}\right) < \frac{n}{n-b}\left(1 - \frac{b}{n-b}\right)$$

But then we have

$$\begin{aligned}\frac{n}{n-b}\left(1 - \frac{b}{n-b}\frac{\underline{r}}{\underline{r} - v}\right) &< \frac{n}{n-b}\left(\frac{n-2b}{n-b}\right) \\ &= \frac{n^2 - 2nb}{n^2 - 2nb + b^2} < 1\end{aligned}$$

Proof of Lemma 4. We want to show that for any pair j and k, the distribution of equity e_j for bank j conditional on bank j being good ($S_j = 0$) is the same as the distribution of equity e_k for bank k conditional on k being good ($S_k = 0$).

Once again, let Ω denote the set of all realizations for S, i.e.

$$\Omega = \left\{ x \in \{0,1\}^n : \sum_{j=1}^{n} x_j = b \right\}$$

Note that for any two realizations s and s' in Ω, $\Pr(S = s) = \Pr(S = s')$. Suppose we show that there exists a bijective mapping $\varphi : \Omega \to \Omega$ such that (i) $s_j = \varphi_k(s)$, and (ii) $e_j(s) = e_k(\varphi(s))$ for all $s \in \Omega$, i.e. the state and equity of bank j when $S = s$ is the same as the state and equity of bank k when $S = \varphi(s)$. Since all states have the same probability, it follows that $Pr(e_j = x | S_j = 0) = Pr(e_k = x | S_k = 0)$.

Heuristically, we can establish the existence of φ as follows. Suppose we place each bank j at the physical location j. We then construct a directed network across physical locations. Given a vector $s \in \Omega$ that implies which are the b bad banks, we can compute the equity of bank j when $S = s$.

Symmetry implies that for any bank k, we can rearrange banks across locations so that bank k lies in location j and the directed network across physical locations remains unchanged. Suppose we leave the shocks at the same physical locations implied by s. Since we have rearranged banks across locations, this implies the identity of the b bad banks is now generally different. In particular, the state of each bank will be given by $\varphi(s) = (s_{\sigma_{kj}}(0), ..., s_{\sigma_{kj}}(n-1))$ for σ_{kj} as defined in the text.

By construction, $s_k = 1$ when $S = \varphi(s)$ iff $s_j = 1$ when $S = s$. Moreover, by construction, payments across physical locations are the same given payments depend only on flows across locations. This ensures that for any bank i, the equity e_i when $S = s$ is the same as the equity of bank $\sigma_{kj}(i)$ when $S = \varphi(s)$. In particular, the equity of bank k when $S = \varphi(s)$ is the same as the equity of bank j when $S = s$. This completes the proof. ■

Proof of Theorem 2: Suppose a bank is able to raise funds from outsiders at a rate r. Once the bank learns its pre-investment equity is e_j, it knows it will earn $e_j + R - r$ if it invests, and $v + \max\{e_j - r, 0\}$ if it diverts. We begin by observing that a bank charged r will prefer to invest if $e_j > e^*(r)$ and to divert funds if $e_j < e^*(r)$, where

$$e^*(r) = v - R + r \tag{26}$$

Note that since $v < R$ from Assumption A3, $e^*(r) < r$. Now, suppose $e_j < e^*(r)$. Since $\max\{e_j - r, 0\} = 0$, the fact that $e_j < e^*(r)$ implies

$$e_j + R - r < e_j^* + R - r = v = v + \max\{e_j - r, 0\}$$

and so the bank would prefer to divert. Next, suppose $e^*(r) < e_j \leq r$. In that case, $\max\{e_j - r, 0\} = 0$. In this case, $e_j > e^*(r)$ implies

$$e_j + R - r > e_j^* + R - r = v = v + \max\{e_j - r, 0\}$$

and so the bank will prefer to invest. Finally, suppose $e_j > r > e^*(r)$. In that case,

$\max\{e_j - r, 0\} = e_j - r$. Since $v < R$ under Assumption A3, we have

$$\begin{aligned} R + e_j - r &> v + e_j - r \\ &= v + \max\{e_j - r, 0\} \end{aligned}$$

and so the bank will prefer to invest in this case as well.

Note that under Assumption A3, $0 < e^*(r) \leq \pi$ for $r \in [\underline{r}, \overline{r}]$. In particular, the first inequality in (11) implies that for any $r \geq \underline{r}$,

$$e^*(r) = v + r - R \geq v + \underline{r} - R > 0$$

Since $r \geq \underline{r}$ in equilibrium, the inequality holds in equilibrium. In the other direction, the highest equilibrium rate charged to any bank is $\overline{r} = \pi + R - v$. For $r \leq \overline{r}$ we have

$$e^*(r) = v + r - R \leq v + \overline{r} - R = \pi$$

Given a network that is symmetrically vulnerable to contagion, $Pr(e_j = x | S_j = 0)$ is the same for all j for any value of x. Hence, we can define

$$p_g^*(r) = \Pr(e_j \geq e^*(r) | S_j = 0)$$

That is, $p_g^*(r)$ is the probability that if bank j is good, it will have equity of at least $e^*(r)$, or alternatively the probability that a good bank that raises funds and is charged r will be willing to invest the funds after it learns its equity.

We now derive the analog to Propositions 8-11 to determine when a non-disclosure equilibrium exists and whether mandatory disclosure can improve upon it. The role of $Pr(e_j = \pi | S_j = 0)$ is now replaced with $p_g^*(\tilde{r})$ where $\tilde{r} = \arg\max_r r p_g^*(r)$, i.e. the interest rate that maximizes the expected return to the lender.

First, if a non-disclosure equilibrium exists, we need to determine whether it will involve investment or not. Since $\phi > \pi$, bad banks would divert funds. Hence, outsiders only earn money from the $n - b$ good banks, and then only from those whose equity is high enough that they will choose not to divert the funds they raise. If the maximal expected amount lenders expect to collect is below $\underline{r}$, a non-disclosure equilibrium must involve no investment. This condition is given by

$$\sup_{r \in [\underline{r}, \overline{r}]} \frac{n-b}{n} r p_g^*(r) < \underline{r} \tag{27}$$

If (27) is reversed, then a non-disclosure equilibrium would involve investment; otherwise, a lender and bank could enter a trading relationship that would make both of them better off.

Next, we want to derive conditions for when non-disclosure is an equilibrium. Suppose no other bank disclosed. If a good bank were to deviate and announce it was good, outsiders would expect that if they charged this bank r, the probability they would be paid r is $p_g^*(r)$. Hence, no disclosure is an equilibrium for any $c \geq 0$ if charging the r that maximizes the outside lender's expected return will not yield an expected return to the lender of at least $\underline{r}$, or

$$\sup_{r \in [\underline{r}, \overline{r}]} r p_g^*(r) < \underline{r} \tag{28}$$

When (28) is violated, a good bank could raise funds by disclosing it is good. In that case, a non-disclosure equilibrium exists if the cost of disclosure exceed the benefits. In particular, for values of $\sup_{r\in[\underline{r},\overline{r}]} rp_g^*(r)$ such that

$$\underline{r} < \sup_{r\in[\underline{r},\overline{r}]} rp_g^*(r) < \frac{n}{n-b}\underline{r} \tag{29}$$

non-disclosure can be an equilibrium only if

$$p_g^*(r^*)R + (1-p_g^*(r^*))v - \underline{r} \leq c \tag{30}$$

where r^* is the equilibrium interest rate. The condition above makes use of the fact that in equilibrium, $r^* = \underline{r}/p_g^*(r^*)$. Finally, for

$$\sup_{r\in[\underline{r},\overline{r}]} rp_g^*(r) > \frac{n}{n-b}\underline{r} \tag{31}$$

the only possible non-disclosure is one where outsiders invest in all banks. Let r^* denote the equilibrium rate charged to banks. If a bank were to deviate and reveal itself, it could lower the rate it was charged from r^* to $\frac{n-b}{n}r^*$. Given the cutoff $e^*(r)$ below which a bank would divert is less than r, we know that a bank that diverted would have no equity left. The expected payoff in equilibrium is given by

$$p_g^*(r^*)(R-r^*) + (1-p_g^*(r^*))v \tag{32}$$

while the expected payoff from deviating is given by

$$p_g^*\left(\frac{n-b}{n}r^*\right)\left(R-\frac{n-b}{n}r^*\right) + \left(1-p_g^*\left(\frac{n-b}{n}r^*\right)\right)v \tag{33}$$

For non-disclosure to be an equilibrium, the difference between the second and the first expression must be less the cost of disclosure c.

To establish the theorem, define $e^* = e^*(\underline{r})$. Consider the limit as $p_g^*(\underline{r}) \to 0$. Since $p_g^*(r)$ is decreasing in r, it follows that

$$\sup_{r\in[\underline{r},\overline{r}]} rp_g^*(r) < rp_g^*(\underline{r}) \tag{34}$$

Hence, in the limit, we have $\sup_{r\in[\underline{r},\overline{r}]} rp_g^*(r) \to 0$ implying the only non-disclosure equilibrium is one where no investment takes place. Moreover, from (28), we know that non-disclosure will be an equilibrium for any $c \geq 0$.

Next, consider the limit as $p_g^*(\overline{r}) \to 1$, i.e. letting $Pr(e_j = \pi | S_j = 0)$ tend to 1. Since $p_g^*(r)$ is decreasing in r, then $p_g^*(r) \to 1$ for all $r \in [\underline{r}, \overline{r}]$, and the argument is identical to the one behind Theorem 1. ■

B An Example of Unique Equilibrium Dominated by Mandatory Disclosure

In this Appendix, we construct an example in which non-disclosure is the unique equilibrium but mandatory disclosure still Pareto dominates the non-disclosure equilibrium.

To construct this example, we need to modify the model slightly, for reasons we discuss below. In particular we assume that if a good bank does not disclose its state S_j, the value of S_j may be revealed to outsiders before they invest but after banks make their disclosure decisions. In particular, if a good bank j chooses not to disclose, its value of S_j might still be revealed to outsiders with probability ρ. This modification weakens the incentive of each good bank to disclose, since the bank might be able to reveal it is good to outsiders for free without incurring the cost c.

The signals on each bank's state are i.i.d. across good banks. Note that the model in the paper is just a special case of this generalization in which $\rho = 0$.

We want to find parameter values for which (1) there exists a non-disclosure equilibrium in which no firm chooses to incur the cost of disclosure; (2) mandatory disclosure nevertheless improves welfare relative to this equilibrium; (3) no equilibrium in which some good banks disclose with positive probability exists.

Consider the following values:

$$\begin{array}{ll} n = 10 & b = 1 \\ k = 8 & \rho = .2 \\ \underline{r} = 1.05 & R = 2 \\ v = 1 & \pi = 1.5 \end{array}$$

We now verify the three conditions in reverse order.

Detailed calculations to follow ....

C Examples of Symmetrically-Vulnerable-to-Contagion Networks

In this section, we provide some examples of networks that are symmetrically vulnerable to contagion to highlight the breadth of networks for which our results apply.

Example 1: A symmetric non-circulant network

Our first example demonstrates that the class of symmetric networks is larger than the class of circulant networks, i.e. networks in which we can order banks in such a way that Λ_{ij} is a function of $(i - j) \pmod n$, i.e. the distance between banks. Our example is a weighted

directed cuboctahedral network. The financial obligations for this network are given by

$$\Lambda = \begin{bmatrix} 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \end{bmatrix}$$

The implied network is shown graphically in Figure A1. The distinguishing feature of the cuboctahedral network is that each node belongs to exactly two triangular groups, as evident in Figure A1. None of the circulant networks with 12 nodes possess this feature. Essentially, the obligations Λ_{ij} depend not only on distance but also on whether bank i is even or odd.

Example 2: An asymmetric network that is symmetrically vulnerable to contagion

To show that symmetry is not necessary to satisfy symmetric vulnerability to contagion, we construct an example that uses a 4-regular asymmetric undirected graph, i.e. a graph where each node has exactly four vertices (4-regular) and whose automorphism group size is 1 (asymmetric). Gewirtz, Hill, and Quintas (1969) establish that the smallest such network involves 10 nodes. Starting with such a network, which we obtain using the algorithm by Meringer (1999) to compute automorphism group size, we impose equal directional flows of λ so that each node receives 2λ and pays 2λ. The asymmetry of the undirected graph must carry over to the direct graph. The financial obligations for this network are given by

$$\Lambda = \begin{bmatrix} 0 & \lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & \lambda \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & \lambda \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \end{bmatrix}$$

The implied network is shown graphically in Figure A2.

Consider the case where $b = 1$, $\pi < \phi < 3\pi$, and $\lambda > \phi - \pi$. Although the network is asymmetric, we can easily confirm that e_j can only assume 3 values for each j: 0, $\pi - \frac{\phi-\pi}{2}$, and π with probabilities $\frac{1}{10}$, $\frac{2}{10}$, and $\frac{7}{10}$, respectively.

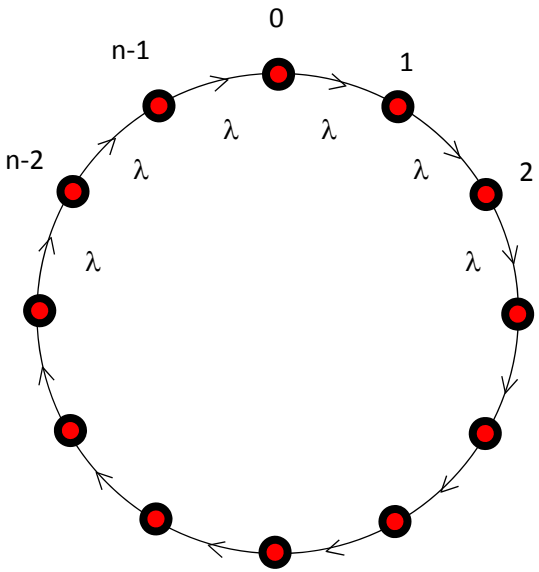
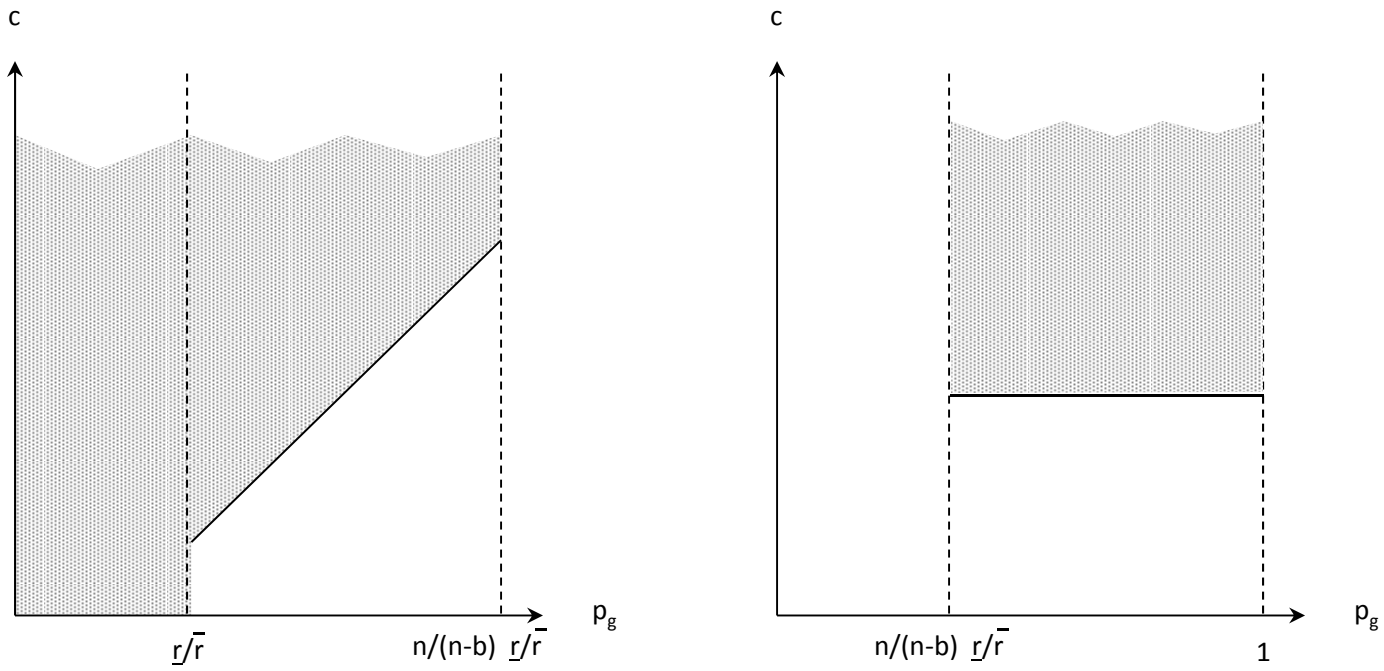


Figure 1: A Circular Network



a) $p_g < n/(n-b)\ \underline{r} / \bar{r}$

b) $p_g > n/(n-b)\ \underline{r} / \bar{r}$

Figure 2: Region where a non-disclosure equilibrium exists

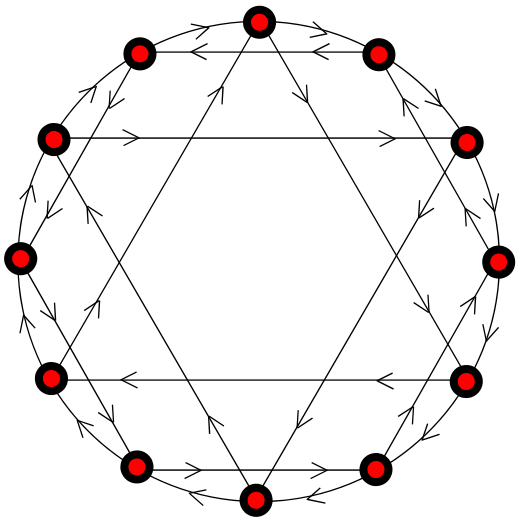


Figure A1: A Directed Cuboctahedral Network

An example of a symmetric network that cannot be represented as a circulant network

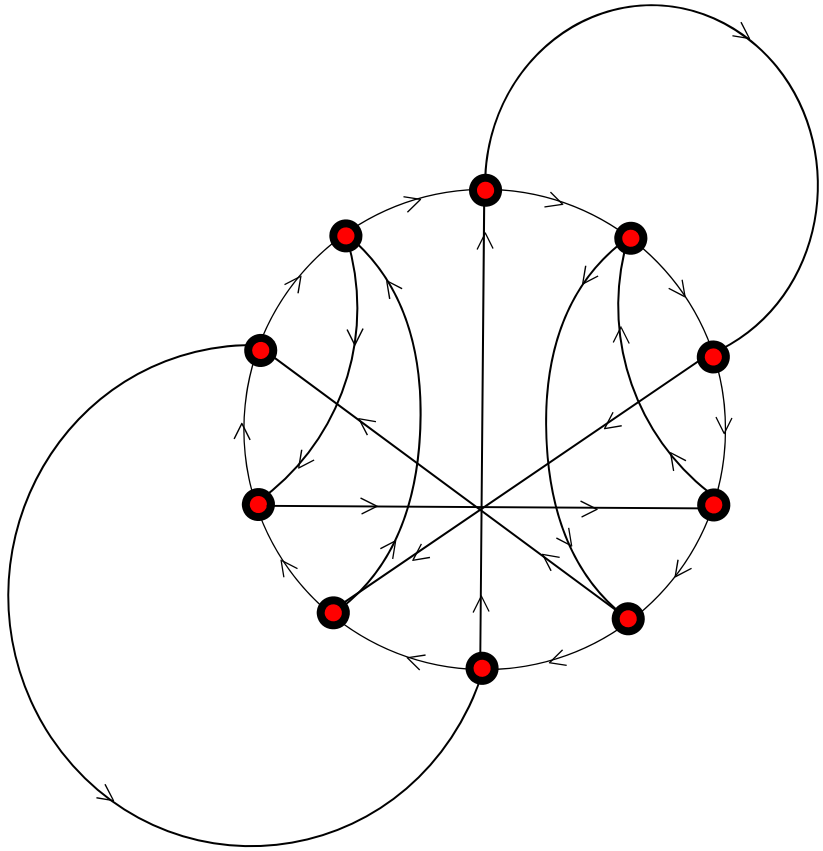


Figure A2: An asymmetric network that can satisfy Symmetric Vulnerability to Contagion