Federal Reserve Bank of Minneapolis
Research Department Staff Report

November 2009

Bailouts, Time Inconsistency and Optimal Regulation*

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ABSTRACT

We make three points. First, ex ante efficient contracts often require ex post inefficiency. Second, the time inconsistency problem for the government is more severe than for private agents because fire sale effects give governments stronger incentives to renegotiate contracts than private agents. Third, given that the government cannot commit itself to not bailing out firms ex post, ex ante regulation of firms is desirable.

*Chari, and Kehoe thank the National Science Foundation for financial support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Recent experience has shown that governments can, will, and perhaps should intervene during financial crises. Such interventions typically occur because governments seek to minimize the spillover effects of bankruptcy and liquidation upon the broader economy. Such interventions during financial crises alter the incentives for firms and financial intermediaries ex ante. In this paper we ask how optimal regulation should be designed to maximize ex ante welfare taking into account the temptation for the government to intervene ex post.

The theme that we explore in this paper is that, by altering private contracts, the prospect of bailouts reduces ex ante welfare. We view the prescription that governments should refrain from bailing out potentially bankrupt firms as unrealistic in practice. Benevolent governments simply do not have the power to commit themselves to such a prescription. A pragmatic approach to policy dictates that we take as given the incentives of governments to undertake bailouts and design ex ante regulation to minimize the ex ante costs of these ex post bailouts.

In thinking about bailouts by governments, a central question is why would the government find it optimal to bail out firms ex post. We argue that confronted with an ex post situation in which many firms are about to undergo costly bankruptcies, a benevolent government has a strong incentive to bail out firms. These ex post bailouts, however, may distort the ex ante incentives of managers and firms and reduce ex ante welfare. In such a situation, a government with commitment would commit itself not to undertake bailouts. If the government lacks such commitment, it will bail out firms ex post and the expectation of such bailouts will reduce ex ante welfare. In this sense, the government has a time inconsistency problem in bailout policy. We show that this time inconsistency problem creates a role for ex ante regulation. Such regulation can reduce the temptation of governments to bail out firms ex post and thereby raise ex ante welfare.

In analyzing the incentives of benevolent governments to intervene and prevent costly bankruptcies ex post, the obvious question arises, why would firms ex ante enter into contracts which impose ex post costs? More generally, why would firms design contracts that feature ex post inefficient outcomes? Here we develop a model in which the optimal contract between a firm and a manager specifies bankruptcy when outcomes are bad in order to provide proper incentives to managers to engage in effort. Bankruptcy is costly in two ways: it reduces the output of the firm and it imposes nonpecuniary costs on the manager. We think of these nonpecuniary costs as arising both from stigma-like effects on the manager's career as well as loss of private benefits from operating the firm. In the model the optimal contract is ex post inefficient in the sense that, once the manager has exerted effort, bankruptcy imposes costs on the owners of the firm and the manager.

While these ex post inefficiencies create a time inconsistency problem for the government by giving it an incentive to bailout firms ex post, they also create a time inconsistency problem for private agents by giving them an incentive to avoid costly bankruptcy by renegotiating their contracts ex post. Analyzing these incentives requires modeling the benefits and costs of both renegotiation and bailouts. The benefits are the reduction in costly bankruptcies. We assume that the costs arise from changes in the beliefs of private agents about future outcomes. In particular, if a firm ever agrees to renegotiate, private agents will believe that firm will always renegotiate in the future. Expectations of such renegotiations constrain future contracts and thereby reduce future welfare. Likewise, if a government ever bails out firms, private agents believe that the government will always bailout firms in the future. Expectations of such bailouts constrain future contracts and reduce future welfare.

In an environment without commitment, private agents and governments balance these benefits and costs in designing their interventions. For the private agents, this balance implies that ex ante optimal contracts must satisfy a *private sustainability constraint.* For the government, this balance implies that ex ante optimal contracts must, in equilibrium, satisfy a *sustainability to bailouts constraint.*

The parallel way we have modeled benefits and costs for governments and private agents leads us to ask, Given that a contract has already been designed to be privately sustainable, why would it not be sustainable to bailouts? When deciding whether to renegotiate a given contract, the private agents involved in that contract consider the benefits from eliminating bankruptcy of their firm at given prices. When the government decides to bail out firms, it takes into account the private benefits per firm in the same way that private agents do, but, in addition, it also takes into account the benefits to other firms from its intervention. These benefits arise because by bailing out firms the government can reduce the aggregate amount of assets sold in the market place and thereby raise the prices of these assets. The idea is that bankruptcy is socially costly because it forces firms to sell their assets and these *fire sales* reduce the value of assets in otherwise healthy firms. Bailouts help reduce fire sales and the resulting negative price effects that give rise to the social cost. Since governments take into account fire sale effects and private agents do not, the sustainability to bailouts constraint is tighter than the private sustainability constraint. Thus, a contract that is privately sustainable is not necessarily sustainable to bailouts. In this sense, the time inconsistency problem is for the government is more severe than it is for private agents.

The greater severity of the time inconsistency problem for the government implies that the equilibrium in an economy with bailouts has lower welfare than in an economy without bailouts. It also implies that ex-ante regulation can be desirable. Such regulation must be designed so that ex-post the government does not have an incentive to engage in bailouts. The incentive to bail out firms is large when the aggregate amount of assets in bankrupt firms is large. We show that the optimal ex-ante regulation is to impose a cap on quantity of assets used by each manager and a cap on the probability of bankruptcy. This cap on assets limits the size of individual firms and thus can be interpreted as a regulation that prevents firms from becoming too big. We refer to this regulation as a *too-big-to-fail-cap*.

The cap on the probability of bankruptcy can be implemented by a cap on the debt to value ratio of the firm. The reason is that this ratio is an increasing function of the probability of bankruptcy so that a cap on the probability of bankruptcy is equivalent to a cap on the debt to value ratio.

1. A simple economy

We begin with a simple static version of our benchmark economy. We use this version to show that, in order to provide incentives, optimal contracts often require *ex post inefficiency*, in the sense that ex post all agents can benefit by altering the terms of the contract. This feature of the model makes optimal contracts time inconsistent, in the sense that optimal contracts without commitment differ from those with commitment, and, in particular, give lower welfare.

Consider a model in which decisions are made at two stages: a first stage, called the beginning of the period, and a second stage called the end of the period. There are two types of agents, called lenders and managers both of whom are risk neutral and consume at the end of the period. There is a measure 1 of managers and a measure 1 of lenders.

The economy has two production technologies. The *storage* technology is available to all agents, which transforms one unit of endowments at the first stage into one unit of consumption goods at the second stage. The *corporate* technology specifies projects that require two inputs at the first stage: effort a of managers and an investment of 1 of goods. This technology transforms these inputs into capital goods. The capital goods then can be used to make stage two consumption goods. Effort a of managers is unobserved by lenders.

If the corporate technology is used the amount of capital goods produced in the second stage stochastically depends on the effort level a of the manager as well as an idiosyncratic exogenous shock representing the manager's current draw of ability. In particular, given effort level a and a draw of ε with probability $p_H(a)$ the high state is realized and $A_H(1+\varepsilon)$ units of capital goods are produced and with probability $p_L(a) = 1 - p_H(a)$ the low state is realized and $A_L(1+\varepsilon)$ units of capital goods are produced where $A_L < A_H$. We assume that higher effort levels increase the probability of the high state. Specifically, we assume that $p_H(a)$ is an increasing, strictly concave function of a. Notice that since $p_H(a)$ is increasing this technology satisfies that monotone likelihood ratio property and since $p_H(a)$ is strictly concave it satisifies the convexity of distribution function property[1]. These assumptions guarantee that the first order approach is valid. (See Rogerson 19?? for details.) We assume that ε has mean zero, support $[\underline{\varepsilon}, \bar{\varepsilon}]$, and distribution H.

We think of the project as being undertaken by a firm. We think of managers as

[1]Recall that the monotone likelihood ratio property is that if $a > \hat{a}$

$$\frac{p_H(a)}{p_H(\hat{a})} > \frac{1 - p_H(a)}{1 - p_H(\hat{a})}$$

while the convexity of distribution property is that the cdf induced by $p_H(a)$, namely $F_L(a) = 1 - p_H(a)$, has a strictly positive second derivative.

performing two tasks. The first task is to exert effort a and transform consumption goods from stage 1 into capital goods at stage 2. The second task is to transform capital goods stage 2 into final consumption goods.

After the manager has completed the first task and a certain amount of capital has been produced the firm can choose to continue the project under the incumbent manager or it can declare bankruptcy. If it continues then the project produces one unit of output for every unit of capital, so that the firm's output is

$$(1) \quad Y_{ci}(\varepsilon) = A_i(1+\varepsilon) \text{ for } i \in \{H, L\}$$

where c denotes *continue*. If the firm declares bankruptcy, the incumbent manager is removed and suffers a nonpecuniary cost. The replacement manager is less-efficient produces consumption goods from the given capital $A_i(1+\varepsilon)$ is according to

$$(2) \quad Y_{bi}(\varepsilon) = RA_i(1+\varepsilon)$$

where b denotes *bankruptcy* and $R \leq 1$. In the event of bankruptcy the incumbent manager suffers a nonpecuniary loss $-B$. This nonpecuniary cost is supposed to represent extra costs to the incumbent manager, such as a loss in reputation or a loss in nonpecuniary benefits from being employed as a manager that are incurred from a liquidation.

We think of replacement managers as being chosen from the pool of managers who have been replaced due to bankruptcy and randomly assigned to manage capital in a firm that has undergone bankruptcy. We think of incumbent managers as having developed specialized expertise in particular firms and, therefore, as being more productive than replacement managers who have not developed specialized expertise.

A critical assumption we make is that bankruptcy necessarily requires both that the incumbent manager suffer a loss and that the manager be replaced. In particular, we do not allow the manager to suffer a nonpecuniary loss and continue operating the project. While we do not explicitly model the underlying reputational story or the loss in nonpecuniary benefits from employment, we think of these costs as being incurred only if the manager is dismissed.

Lenders are endowed with e units of a consumption good in the first stage but cannot operate the corporate technology. Managers have no endowments of goods but can operate the corporate technology. Lenders choose whether to lend to firms that operate the corporate technology or to store their endowments.

We assume that $e > 1$. Since the economy has an equal measure of managers and lenders and since the corporate technology uses 1 unit of the endowment per manager the storage technology is always active and the rate of return to lending to the corporate technology is 1.

Let $c_i(\varepsilon)$ denote the consumption of the managers in state i given the realization ε and $d_i(\varepsilon)$ the return to the investor in a project when the state is i and the idiosyncratic shock is given by ε. Let B_i denote the set of idiosyncratic shocks ε such that the firms declares bankruptcy in state $i \in \{H, L\}$ and C_i denote the complementary sent in which the project is continued.

We assume that a large number of *financial intermediaries*, operate a continuum of firms, each of which has one project. Given the symmetry of the expected returns across projects, financial intermediaries will choose the same effort level for all managers. The profits generated by a financial intermediary which finds it optimal to operate the corporate technology at a positive level are

$$(3) \quad \sum_i p_i(a) \left[\int_{C_i} Y_{ci}(\varepsilon) dH(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon) dH(\varepsilon) - \int [c_i(\varepsilon) + d_i(\varepsilon)] dH(\varepsilon) \right]$$

financial intermediaries compete in offering contracts to managers and lenders. These contracts must attract investment by lenders so that they must offer a return to lenders of at least one. Thus, a contract must meet the following participation constraint for lenders

$$(4) \quad \sum_i p_i(a) \left[\int d_i(\varepsilon) dH(\varepsilon) \right] \geq 1$$

The contracts must also attract managers. Let $\bar{U}$ denote the value of the best alternative contract offered to a managers. Thus, a contract must meet a participation constraint for managers

$$(5) \quad \sum_i p_i(a) \left[\int c_i(\varepsilon) dH(\varepsilon) - B \int_{B_i} dH(\varepsilon) \right] - a \geq \bar{U}.$$

Since the effort choice a of managers is unobservable a contract must satisfy an incentive constraint given by

$$(6) \quad a \in \arg\max_a \sum_i p_i(a) \left[\int c_i(\varepsilon) dH(\varepsilon) - B \int_{B_i} dH(\varepsilon) \right] - a.$$

Finally, the consumption of managers must satisfy a nonnegativity constraint

(7) $c_i(\varepsilon) \geq 0$

A. With commitment

Suppose now that financial intermediaries and managers can commit to contracts. Under this assumption the financial intermediaries' contracting problem is to choose a recommended action a, compensation schemes $c_i(\cdot)$, $d_i(\cdot)$ and bankruptcy and continuation sets B_i and C_i to maximize profits (3) subject to (4), (5), (6), and (7).

Clearly the consumption level of a lender that lends 1 to financial intermediaries and invests the rest in the storage technology is given by

(8) $$c^I = \sum_i p_i(a) \left[\int d_i(\varepsilon) dH(\varepsilon) \right] + e - 1$$

The resource constraint is

(9) $$\sum_i p_i(a) \left[\int c_i(\varepsilon) dH(\varepsilon) \right] + c^I \leq$$

$$\sum_i p_i(a) \left[\int_{C_i} Y_{ci}(\varepsilon) dH(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon) dH(\varepsilon) \right] + e - 1$$

An *allocation* is a collection $a, c_i(\cdot)$, $d_i(\cdot)$, c^I, C_i, B_i. A *competitive equilibrium* is an allocation together with a minimum utility level $\bar{U}$ such that

i) the allocations $a, c_i(\cdot)$, $d_i(\cdot)$, and sets C_i, B_i solve the contracting problem.

ii) the minimum utility level $\bar{U}$ is such that firm profits are zero.

iii) the consumption of lenders satisfies (8).

iv) the resource constraint (9) holds.

Note here that $\bar{U}$ plays the role of a price and that by Walras' Law the resource constraint is implied by zero profits of financial intermediaries and the consumption of lenders (8).

Throughout we will restrict attention to environments in which the competitive equilibrium has an active corporate technology. A sufficient condition for such an equilibrium to exist is that A_H and $p'(0)$ are sufficiently large.

We turn the efficiency of a competitive equilibrium. Given a utility level of lenders $\bar{c}^I$, an allocation is *efficient* if it satisfies the following planning problem, namely to maximize the welfare of managers subject to (6), (7), (8), and

(10) $c^I \geq \bar{c}^I$.

Proposition 1. The competitive equilibrium is efficient.

Proof: Since profits are zero in a competitive equilibrium, we can use duality to rewrite the contracting problem as one of maximizing the utility of managers subject to the constraint the firm profits be nonnegative. Substituting for the consumption of lenders from (8) into financial intermediaries' profits (3) yields the resource constraint. Clearly, the rewritten contracting problem coincides with the planning problem. *Q.E.D.*

Consider the following assumption. Let a^O be the solution to the version of the problem with publicly observed effort, namely the value of a that solves

(11) $p_H'(a)A_H - A_L = 1.$

Assume that

(12) $p_H(a^O) < 1$

Proposition 2. If $A_L < 1$ and (12) holds, then the competitive equilibrium with privately observed effort information has strictly lower effort level a and welfare than the competitive equilibrium with publicly observed effort.

Proof. In the competitive equilibrium with publicly observed effort it is straightforward to show that the optimal effort level solves (11) and the liquidation sets B_H and B_L are empty. The first order condition for effort in the private information economy is

$$\sum_i p_i'(a)\left[\int c_i(\varepsilon)dG(\varepsilon) - B\int_{B_i} dG(\varepsilon)\right] = 1$$

A moment's reflection makes clear that the only way to support the allocations with publicly observed effort in the economy with privately observed effort is to pay the manager an expected compensation of

(13) $\int c_H(\varepsilon)dH(\varepsilon) = A_H - A_L$

in the high state and zero in the low state. But, since $A_L < 1$ if financial intermediaries pay managers this much and pay the lenders 1 unit then profits are negative. To establish this result substitute (1), (2), (4) with equality and (13) into the expression for firm's profits (3) and using the assumption that the expected value of ε is zero, to obtain

$$p_H(a)\left[A_H - (A_H - A_L)\right] + p_L(a)A_L - 1 = A_L - 1$$

which is negative since $A_L < 1$. *Q.E.D.*

From here onwards the term competitive equilibrium refers to competitive equilibrium with privately observed effort.

We now show that the contracting problem reduces to a simpler one under the condition that $A_L < 1$. We will show that in any competitive equilibrium the optimal contracting problem can be reduced to the following: Choose c_H, a, and ε^* to solve

$$(14) \quad \max p_H(a)c_H - p_L(a)BH(\varepsilon^*) - a$$

subject to

$$(15) \quad a \in \arg\max_a p_H(a)c_H - p_L(a)BH(\varepsilon^*) - a.$$

$$(16) \quad p_H(a)c_H + 1 \leq p_H(a)A_H + p_L(a)A_L\left[\int_{\varepsilon^*}^{\bar{\varepsilon}}(1+\varepsilon)dH(\varepsilon) + R\int_{\underline{\varepsilon}}^{\varepsilon^*}(1+\varepsilon)dH(\varepsilon)\right]$$

We refer to $x = (c_H, a, \varepsilon^*)$ as the *contract*.

To establish this result we first note that if $A_L < 1$ the incentive constraint is always binding. Hence an optimal contract must reward the manager only in the high state and set the consumption of managers in the low state to be zero for all ε, that is, $c_L(\varepsilon) = 0$. The intuition for this result is that as long as consumption is positive in the low state, manager's incentives can be improved by shifting consumption from the low state to the high state. Since the manager cares only about expected consumption the optimum can be achieved by setting consumption in the high state to be a constant so that $c_H(\varepsilon) = c_H$.

Second, note the only role of bankruptcy is to improve incentives so that it is never optimal to declare bankruptcy in the high state. In the low state, the optimal bankruptcy rule has a cutoff form: declare bankruptcy for $\varepsilon \leq \varepsilon^*$ and continue otherwise. This result follows because the output loss from bankruptcy, $(1 - R)A_L(1 + \varepsilon)$, is smaller the lower is ε and the manager only cares about the probability of bankruptcy in the low state. More formally, if the optimal contract had bankruptcy for a high realization ε and continuation for a low realization of ε, then the output loss could be reduced by rearranging the set of realizations for which there is bankruptcy while maintaining the manager's incentives.

Third, in any competitive equilibrium profits are zero. Hence, we can use duality to write the optimal contracting problem as maximizing the utility of the manager subject to a nonnegativity constraint on profits. Note that we write the nonnegativity constraint on profits as (16) using the assumption that the expected value of ε is zero along with the other features of the optimal contract derived above.

We summarize this discussion in a proposition.

Proposition 3. If $A_L < 1$ the optimal contracting problem in a competitive equilibrium can be written as (14).

Next, we will say that allocations are *ex post inefficient* if $\varepsilon^* > \underline{\varepsilon}$. If this inequality holds, then clearly all agents economy can be made better off ex post by continuing the project in the states $[\underline{\varepsilon}, \varepsilon^*]$. Nonetheless, committing to ex post inefficient allocations may be desirable as a way of providing the manager with stronger incentives for providing high effort and thereby raising ex ante welfare.

We now give sufficient conditions so that the optimal allocations with commitment require ex post inefficiency. In providing these conditions, it is convenient to adopt a change of variables so that the manager can be thought of as choosing the probability of success p and incurring an effort cost $a(p)$. Formally, let $a(p)$ be the inverse of the function p_H so that $a(p) = p_H^{-1}(p)$. Consider the allocations that arise when ε^* is restricted to equal $\underline{\varepsilon}$, so that there is no ex post inefficiency (no bankruptcy). Let p_H^E denote the optimal probabilities under this restriction.

Proposition 4. If R is sufficiently close to 1 and $a''(p_H^E)$ is sufficiently small then $\varepsilon^* > \underline{\varepsilon}$. That is, supporting ex ante efficient allocations requires ex post inefficiency.

The proof of this proposition is in the appendix. The basic idea is that the incentive effects associated with bankruptcy are large when $a''(p)$ is small. To see the role of these incentive effects consider the first order condition associated with the incentive constraint, given by

$$(17) \quad c_H + BH(\varepsilon^*) = a'(p_H)$$

Consider the incentive gains from a small increase in the probability of bankruptcy resulting from an increase in ε^*, holding fixed c_H. Differentiating (17) gives

$$\frac{dp_H}{d\varepsilon^*} = \frac{Bh(\varepsilon^*)}{a''(p)}$$

Thus, when $a''(p)$ is small the incentive gains from increasing the probability of bankruptcy are large. If R is sufficiently close to 1, the resource costs of increasing the probability of bankruptcy are small. Hence, when these conditions are met, supporting efficient allocations requires a positive probability of bankruptcy.

B. Implementing the competitive equilibrium

Here we argue that the equilibrium outcomes can be interpreted as outcomes that arise with financial contracts that resemble debt, equity, and managerial compensation combined with an institutional arrangement that resembles bankruptcy. Under our interpretation the model implies a unique level of debt and equity. In this sense, the agency problems in our model make the Modigliani-Miller theorem not apply.

Our model implies a unique level of compensation for managers and a unique level of state-contingent payments to investors. Consider the following interpretation of these state contingent payments. Under this interpretation a firm operated by a manager issues the following financial claims. The firm issues (risky) debt and equity and enters into a compensation contract with the manager. The debt promises a face value of $A_L(1 + \varepsilon^*)$. The nature of the debt contract is that if the firm is unable to meet the face value of its debt payments, the firm is forced into bankruptcy, equity holders lose their claims and debt holders receive the liquidation value of the firm, so that for each $\varepsilon \leq \varepsilon^*$ debt-holders receive $RA_L(1 + \varepsilon)g(k_c)$. The manager's compensation contract specifies a payment of c_H if the manager retains his managerial capability and if the firm is successful and zero otherwise. Outside equity is the residual claimant.

Suppose that the equilibrium allocation satisfies

$$(18) \quad A_H(1+\underline{\varepsilon}) - c_H \geq A_L(1+\varepsilon^*)$$

and

$$(19) \quad R \sum p_i(a) A_i \geq A_L(1+\varepsilon^*)$$

Note that (18) guarantees that in the high state when the manager keeps the ability to manage the project, the firm can pay the face value of the debt, while (19) guarantees under the event that the manager loses the ability to manage the project, the firm can pay the face value of the debt by selling its assets.

C. Without commitment

Suppose now that the agents in this economy cannot commit to contracts. We show that equilibrium allocations without commitment give lower welfare than those with commitment.

Specifically, suppose that after the action a has been taken and the first stage investments have been made, but before the state and the realization of ε have occurred, financial intermediaries and managers can renegotiate their contracts if both parties agree. Clearly, all projects will be continued in order to avoid the output and the nonpecuniary costs of bankruptcy.

To see this result more formally, suppose now that a manager has taken an action a and first stage investment decisions have been made, but uncertainty has not yet been realized.

Consider the outcomes if the financial intermediary and the manager agree to renegotiate. We model the renegotiation as follows. The financial intermediary makes a take it or leave it offer to the manager. If the manager takes the offer that offer is implemented, while if the manager rejects the offer the existing contract is implemented. Clearly, the manager will accept any offer which makes the manager at least as well off as under the existing contract. Since the action a has already been taken, it is optimal for the financial intermediary to set $\varepsilon^* = \underline{\varepsilon}$ and avoid bankruptcy.

In sum, in this static model without commitment the incentive to renegotiate is so strong that the equilibrium has no bankruptcy and, hence, no ex post inefficiency. Thus, without commitment the optimal contracting problem solves (14) subject to the additional constraint that $\varepsilon^* = \underline{\varepsilon}$. Clearly, welfare in an equilibrium without commitment is lower than that with commitment.

2. The Dynamic Contracting Model

In the static model without commitment, the equilibrium has no bankruptcy because there because there are no costs to renegotiation. Here we develop a dynamic contracting model without commitment in which these costs arise because of reputational considerations which make the nature of future contracts depend on whether there has been renegotiation in the past.

Our dynamic model is an infinite repetition of a modified version of our simple model. The infinite repetition allows for trigger strategies in which contracts depend on the history of past renegotiation while the modifications to the simple model allow for fire sale effects in which changes in the aggregate incidence of bankruptcy alter the prices at which assets are sold.

In later sections when we turn to optimal policy these fire sale effects will play a prominent role.

A. The benchmark economy

The benchmark economy we consider is an infinitely-repeated version of a static model. Our benchmark economy has no technology to transform goods from period t to period $t+1$, so that agents cannot save across periods. The economy has an equal measure of managers and agents called consumers. (This nomenclature while convenient is not quite precise because managers both manage and consume while consumers work, lend, and consume.)

The static model is a version of the simple economy with two modifications. These two modifications allows for fire sale effects. First, we assume that the task of transforming capital goods into consumption goods requires skilled labor, in addition to capital and managers. Specifically, we assume that the production function for transforming capital goods into consumption goods is given by

$$F(k_c, l_c) + bF(k_b, l_b)$$

where F is a constant returns to scale function, k_c and l_c denote the amount of capital goods and labor managed by *incumbent* (or continuing) managers and k_b and l_b denote the amount of capital goods and labor managed by *replacement* managers, that is in firms that have declared bankruptcy. We assume that $b < 1$. This assumption is meant to capture the idea that replacement managers are less effective than incumbents at managing firms.

Second we assume that supplying skilled labor to the task of transforming capital into consumption goods requires an upfront investment of w units of goods per unit of labor. The amount of labor can be intepreted either as the amount of time that the representative consumer works or as the number of skilled workers.

With these modifications, the timing of decisions in each period is as follows. At the first stage, consumers choose, how much skilled labor to acquire, how much to invest in the corporate technology, and how much to store. Also at this stage, financial intermediaries and managers choose contracts, and after the contracts are chosen, managers choose actions. At the second stage, before shocks are realized, financial intermediaries and managers can renegotiate their contracts if they so desire. The state and the idiosyncratic shocks are then realized, the firm is allowed to continue or to declare bankruptcy according to the terms of the (possibly renegotiated) contract, firms hire labor and output is produced and consumed.

The resource constraint for this economy at the second stage is

$$(20) \quad p_H(a)c_H + c_l \leq F(k_c, l_c) + bF(k_b, l_b)$$

with

$$(21) \quad l_c + l_b \leq l$$

where c_l denotes the consumption of the consumers and l denotes the total amount of skilled labor. Note that we have assumed that the manager is compensated only in the high state, as will be true in the equilibria analyzed below.

The resource constraint for this economy at the first stage is

(22) $wl + k + k_s \leq e, k \leq 1.$

where k denotes investment in the corporate technology and k_s denotes investment in storage. Note that wl denotes the goods required to acquire l units of skilled labor, where w is a parameter of the labor investment technology and that the the corporate technology requires one unit of investment of goods per manager, and the measure of managers is 1.

The consumers in this economy choose how much of their endowment e to invest in acquiring labor skills rewarded at wage w_c, how much to invest in the corporate technology, k at rate R, and how much to store, k_s at rate 1. That is, consumers solve

(23) $c_I = \max w_c l + Rk + k_s$

subject to

(24) $wl + k + k_s \leq e.$

We will assume that all three technologies are used in equilibrium. A set of sufficient conditions is the following. First, e is sufficiently large, so that the storage technology is always used. Second, that the corporate technology is sufficiently productive in that A_H is large enough and that $p'_H(0)$ is sufficiently large, so that it is always used. Finally, that $F_2(k_I, 0) > w$ for all $k_I > 0$, so that the skilled labor is always used. Under these assumptions we have that the wage rate in the corporate technology w_c is pinned down to equal w, the parameter of the labor investment technology.

(25) $w_c = w$

and that the rate of return in the corporate technology R_c must equal the rate of return to storage, so that

(26) $R = 1$.

With Commitment

To set the stage for our environment without commitment by private agents, we briefly describe the dynamic model with commitment by private agents. In our model, financial intermediaries live for only one period and financial intermediaries in any period t cannot observe the output of firms in earlier periods. Hence, managers cannot enter into contracts that condition on their past output levels. This assumption ensures that the manager's incentive problem is static and that equilibrium in the dynamic model reduces to an infinite-repetition of that in the static model.

Recall that in the simple economy, the incentive constraint for the manager is binding if $A_L < 1$. It is straightforward to check that the incentive constraint in the benchmark economy is binding if A_L, is sufficiently small. We will assume that the incentive constraint is binding in the benchmark economy from now on.

We now set up the contracting problem for this economy. Following the logic of Proposition 3, we focus on the dual form of the contracting problem, namely to maximize the manager's utility subject the manager's incentive constraint and a budget constraint (which corresponds to the zero profit condition of a competitive equilibrium) and impose that the bankruptcy sets have a cutoff form. Let $R_c k_c$ denote the return to capital in a firm operated by an incumbent manager with capital k_c. This return is given by

$$(27) \quad R_c k_c = \max_l F(k_c, l) - wl.$$

The return $R_b k_b$ generated by a replacement manager is given by

$$(28) \quad R_b k_b = \max_l bF(k_b, l) - wl.$$

Let l_c and l_b denote the solutions to (27) and (28).

We then have that given R_c and R_b the optimal contract $\{c_H, a, \varepsilon^*, k_c, k_b\}$ solves

$$(29) \quad \max p_H(a)c_H - p_L(a)BH(\varepsilon^*) - a$$

subject to

$$(30) \quad a \in \arg\max_a p_H(a)c_H - p_L(a)BH(\varepsilon^*) - a.$$

$$(31) \quad p_H(a)c_H + 1 \leq R_c k_c + R_b k_b$$

where

$$k_c = p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dH(\varepsilon)$$

and

$$(32) \quad k_b = p_L(a)A_L g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon)dH(\varepsilon).$$

Recalling that in any equilibrium (25) and (26) hold, we have the following definition.

A *competitive equilibrium with commitment* is an allocation $c_H, c_l, a,\ \varepsilon^*, k_c, k_b, l_c, l_b, l$ and prices R_c, R_b,such that *i)* given R_c and R_b, the allocations solve the contracting problem (29) *ii)* l_c and l_b solve (27) and (28), *iii)* the consumption of lenders satisfies (23), and *iv)* the resource constraints (20)–(22) hold.

Without Commitment by Private Agents

We now characterize the best equilibrium outcomes without commitment. We show that these outcomes solve the programming problem (29) with an additional constraint, called the *private sustainability* constraint.

Without such commitment, we require that the contracts managers and financial intermediaries enter into must be self enforcing. We say that a contract is *self-enforcing* if, after the manager has chosen the effort level, the payoff from continuing with the contract is at least as large as the payoff from deviating.

In order to construct these payoffs from deviating we make a key assumption, namely that lenders, financial intermediaries, and other other managers only observe whether a given manager has renegotiated in the past. This assumption keeps the manager's incentive constraint static and allows us to focus the dynamic analysis on the incentives to renegotiate.

We assume the payoffs from deviating are given by the worst equilibrium payoffs. We do so because we are interested in the best equilibrium outcomes and a standard result in game theory is that the best equilibrium can be supported by a trigger strategy which prescribes the worst equilibrium continuation payoff following any deviation. In our economy, the worst equilibrium is the infinite repetition of the static equilibrium without commitment, namely the solution to the optimal contracting problem (29) with $\varepsilon^* = 0$. Let U^N denote the value of the contracting problem with this restriction.

Here we focus on the best equilibrium outcomes. In the appendix we formally describe a sustainable equilibrium, characterize the entire set of equilibrium outcomes, and show that the best outcome is stationary. Given this result here we develop notation only for stationary outcomes.

We turn now to developing the optimal contracting problem. Given that any deviation triggers the same continuation equilibrium, clearly, if the manager and the firm agree to renegotiation, they should choose a renegotiated contract that maximizes the sum of their current payoffs. As in the simple economy without commitment, the best renegotiated contract is clearly to set ε^* to zero to avoid the output and nonpecuniary costs of bankruptcy.

Under the best one shot renegotiated contract the sum of the current period expected payoffs to the manager and the financial intermediary are

$$(33) \quad \hat{U}(a, k_c) = R_c(k_c + k_b) - 1 - a.$$

For some given contract a, k_c, ε^* if there is not a renegotiation, then the sum of the current period expected payoffs to the manager and the financial intermediary are $U(a, \varepsilon^*, k_c)$

(34) $= R_c k_c + R_b k_b - 1 - \alpha_1 B H(\varepsilon^*) - a$

A contract (a, ε^*, k_c) is *privately sustainable* if

(35) $U(a, \varepsilon^*, k_c) + \frac{\beta}{1-\beta} U(a, \varepsilon^*, k_c) \geq \hat{U}(a, k_c) + \frac{\beta}{1-\beta} U^N.$

The *optimal contracting problem without commitment* is to maximize the manager's utility (29) subject to (30), (31), (32) and (35).

The *best privately sustainable equilibrium* is an allocation $c_H, c_l, a, \varepsilon^*, k_c, k_b, l_c, l_b, l$ and prices R_c, R_b,such that *i*) given R_c and R_b,the allocations solve the optimal contracting problem without commitment. *ii*) l_c and l_b solve (27) and (28), *iii*) the consumption of lenders satisfies (23), and *iv*) the resource constraints (20)–(22) hold.

Note that our notion of a best equilibrium does not depend on the game theoretic rationalization in the appendix. Formally, our optimal contracting problem is analogous to that in the literature on models with enforcement constraints, in that we replace the enforcement constraints by sustainability constraints.

We now turn to welfare with and without commitment. We begin by showing that the equilibrium value of R_2 is the same in the economies with and without commitment. To show this result note that in both economies $F_1(k_1, k_2) = 1$ and hence since F has constant returns to scale, this implies that $F_1(k_1/k_2, 1) = 1$ so that k_1/k_2 is the same value, say $\tilde{k}$

in both economies. Since $R_2 = F_2(k_1, k_2) = F_2(\tilde{k}, 1)$ we know R_2 is also the same in both economies. We record this result in the following lemma.

Lemma 1. The equilibrium values of R_1 and R_2 are the same in the economies with and without commitment. Furthermore, the value of $R_1 = 1$.

Since market prices are the same in the economies with and without commitment, the only difference between the associated contracting problems is the private sustainability constraint. If this constraint is binding in the contracting problem, the privately sustainable equilibrium yields lower welfare than the competitive equilibrium under commitment. The private sustainability constraint is binding if the discount factor β is not too large. We denote by β_p the critical value of the discount factor such that the the private sustainability constraint just binds at the commitment allocations. That is β_p satisfies

$$\text{(36a)}\quad U(a^c, \varepsilon^{*c}, k_c^c) + \frac{\beta_p}{1-\beta_p} U(a^c, \varepsilon^{*c}, k_c^c) = \hat{U}(a^c, k_c^c) + \frac{\beta_p}{1-\beta_p} U^N$$

where a^c, ε^{*c} denote the contract in a competitive equilibrium with commitment. Clearly, if $\beta \geq \beta_p$, the commitment outcomes are privately sustainable, and if $\beta < \beta_p$, the commitment outcomes are not privately sustainable.

3. Adding Government Policies

We now allow for the possibility of intervention by benevolent government authorities without commitment.

We begin with a bailout authority which uses transfers to alter bankruptcy decisions financed by a tax on output. After managers have chosen their actions, the bailout authority has an incentive to use transfers to reduce ex post inefficiency. In using these instruments, we assume that the bailout authority faces a trade off parallel to that faced by private agents: if the authority deviates from some given equilibrium policy, private agents believe that the bailout authority will choose future policies so as to eliminate ex post inefficiency.

These beliefs induce a government sustainability constraint which is similar to the private sustainability constraint with one important difference. This difference is that the government sustainability constraint is tighter because it takes into account fire sales effects. That is, when a bailout authority intervenes to prevent bankruptcies ex post it recognizes it recognizes that its action raise the price of liquidated assets. In contrast, the actions of individual private agents do not affect prices. In our model a rise in the price of liquidated assets raises welfare and therefore makes the government sustainability constraint tighter and hence makes the equilibrium outcomes with a bailout authority worse than without such an authority.

We then ask, Can a regulator armed with the ability to limit the terms of private contracts improve on these outcomes? We find that it can. We show that the optimal regulation imposes a distorting tax on investment in the corporate sector and a cap on the liquidation level, a *bankruptcy cap.* Such a regulator takes into account the incentives of the bailout authority to intervene and structures the terms of private contracts so as to reduce the incentives of the bailout authority to intervene. We show that the regulator can improve upon the equilibrium outcomes with a bailout authority.

A. A Bailout Authority

Consider a bailout authority that can make transfers or levy taxes on financial intermediaries contingent on the state and the realization of the idiosyncratic shock ε. Suppose now that the bailout authority, as well as private agents, cannot commit to their future actions. The bailout authority's per period payoff is given by the sum of the consumption of all agents in the economy.

The bailout authority makes its bailout offers after the managers have chosen their actions but before the realization of either the state, H or L or the shocks ε. The instruments available to the bailout authority are lump sum taxes T levied on the financial intermediary and bailout offers, consisting of promise to give lump sum transfers $T_i(\varepsilon)$ conditional on the firm continuing in state i with idiosyncratic shock ε. Let $I_i(\varepsilon) = 1$ denote that the bailout offer is accepted by the representative financial intermediary in the state i with shock ε and $I_i(\varepsilon) = 0$ denote that the bailout offer is rejected. The bailout authority's budget constraint is

$$(37) \quad \sum_i P_i \int I_i(\varepsilon) T_i(\varepsilon) dH(\varepsilon) = T$$

where P_i is the probability of state i for the representative intermediary. (Of course, in equilibrium $P_i = p_i(a)$, but our notation helps keep clear that an individual financial intermediary does not perceive its actions as affecting P_i.)

In terms of the information structure, we assume that future private agents observe the aggregate amount of transfers given by the left side of (37), but not the project-by-project transfers. This assumption ensures that future private agents cannot infer the actions of past managers and serves the same role as our assumption that the outcomes of individual projects are not observed.

The payoffs to the government are given by the sum of utility of managers and lenders is given by

$$(38)\quad \alpha_1 \left[\sum P_i \int_{C_i} A_i(1+\varepsilon)dH(\varepsilon)\right] g(k_c) - \alpha_1 B \left[\sum P_i \int_{B_i} A_i(1+\varepsilon)dH(\varepsilon)\right]$$

$$+F(k_1, k_2) + e - k_c - k_1 - a$$

where

$$(39)\quad k_2 = \left[\alpha_0 \sum P_i A_i + \alpha_1 \sum P_i \int_{B_i} A_i(1+\varepsilon)dH(\varepsilon)\right] g(k_c)$$

We let $U^G(a, C, k_c)$ denote this payoff where $C = (C_i)$ and the bankruptcy sets B_i are the complements of the continuation sets C_i.

We formally define a sustainable equilibrium with bailouts in the appendix. This equilibrium is very similar to the private sustainable equilibrium. In the appendix we show that the best bailout equilibrium is stationary. Since we focus on the best equilibrium, for simplicity here we develop a definition for a stationary bailout equilibrium.

The key condition that a bailout condition must satisfy is a government sustainability constraint which is analogous to the private sustainability constraint. As is standard any equilibrium outcome can be supported by reverting to the worst equilibrium after a deviation. Any equilibrium must therefore yield a discounted payoff at least as large as the payoff from a one-period deviation that maximizes current utility followed by the payoffs associated with reversion to the worst equilibrium thereafter.

Clearly, the payoff from a one-period deviation that maximizes current utility is that associated with no bankruptcy and is given by

$$(40)\quad \hat{U}^G(a, k_c) = \alpha_1 \left[p_H(a)A_H + p_L(a)A_L\right] g(k_c) + F(k_1, \hat{k}_2) + e - k_c - k_1 - a$$

junk

$$k_1 + R_2(\hat{k}_2/k_2)\hat{k}_2$$

$$F(k_1, 0) - k_1$$

where $\hat{k}_2 = \alpha_0 \sum p_i(a)A_i g(k_c)$. The worst equilibrium is that associated with no bankruptcy in any period and yields period utility U^N. Thus, any stationary equilibrium payoff must satisfy the *government sustainability constraint*

$$(41)\quad \frac{U^G(a, C, k_c)}{1-\beta} \geq \hat{U}^G(a, k_c) + \frac{\beta}{1-\beta} U^N.$$

Given R_2, and the government policy $T_i(\varepsilon), T$, the *optimal contracting problem with a bailout policy* is to choose a contract c_i, a, k_c, I_i and C_i to maximize the utility of the manager

$$(42)\quad \alpha_1 \left[\sum p_i(a) \left[c_i - B \int_{B_i} dH(\varepsilon)\right]\right] - a$$

subject to the incentive constraint for the manager, namely that a maximizes (42), given c_i

and C_i, the budget constraint of the financial intermediary

$$(43) \quad \alpha_1 \sum p_i(a)c_i - k_c \leq$$

$$\alpha_1 \left[\sum p_i(a) \int_{C_i} A_i(1 + \varepsilon + I_i(\varepsilon)T_i(\varepsilon))dH(\varepsilon) \right] g(k_c) + R_2 k_2 - T$$

where k_2 is given by (39), the private sustainability constraint

$$\frac{U(a, C, k_c)}{1 - \beta} \geq \hat{U}(a, k_c) + \frac{\beta}{1 - \beta} U^N$$

where $U(a, C, k_c)$ is given by (42) and

$$\hat{U}(a, k_c) = \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \right] g(k_c) + R_2 \hat{k}_2 - k_c - a$$

where $\hat{k}_2 = \alpha_0 \sum p_i(a)A_i g(k_c)$.

A stationary *sustainable equilibrium with a bailout policy* consists of an allocation $c_i, a, \ C_i, k_1, k_2, k_c, R_2$ and a policy $T_i(\varepsilon), T$ such that $i)$ the allocations solve the optimal contracting problem with a bailout policy, $ii)$ given R_2, k_1 and k_2 satisfy (**??**) and (**??**), $iii)$ the consumption of lenders satisfies (23) with $R_c = R_1 = 1$, $iv)$ the resource constraints

$$(44) \quad \alpha_1 \sum p_i(a)c_i + k_c + k_1 \leq$$

$$\alpha_1 \left[\sum p_i(a) \int_{C_i} A_i(1 + \varepsilon)dH(\varepsilon) \right] g(k_c) + F(k_1, k_2) + e$$

where k_2 is given by (39), and $v)$ the government's sustainability constraint (41).

We then have the following proposition.

Proposition 5. Consider any contract (a, C_i, k_c) with B_i nonempty and suppose that $F(k_1, k_2)$ is strictly concave in k_2. The government sustainability constraint (41) is tighter than the private sustainability constraint (35), in the sense that if any such contract satisfies (41) it also satisfies (35). Furthermore, if any such contract satisfies (35) with equality, it violates (41).

Proof. From Euler's theorem $F(k_1, k_2) = F_1 k_1 + F_2 k_2$. Since $F_1 = 1$ in any equilibrium and $F_2 = R_2$ it follows that

$$(45) \quad F(k_1, k_2) - k_1 = R_2 k_2$$

Using (45) it follows that $U^G(a, \varepsilon^*, k_c) = U(a, \varepsilon^*, k_c) + e$ and $U^{GN} = U^N + e$. Using this result and canceling terms in (**??**) gives that (**??**) holds if and only if

$$(46) \quad F(k_1, \hat{k}_2) - k_1 > R_2 \hat{k}_2$$

Adding $R_2 k_2$ to both sides, using Euler's theorem and rearranging terms, (46) can be written as

$$(47) \quad R_2(k_2 - \hat{k}_2) > F(k_1, k_2) - F(k_1, \hat{k}_2)$$

Since $\varepsilon^* > \underline{\varepsilon}$ it follows that $k_2 > \hat{k}_2$. Hence, (47) must hold because F is a strictly concave function of k_2, . This result proves that (41) is tighter than (35). *Q.E.D.*

If the production function satisfies (47) we say that the economy has fire sale effects.

The key idea in the proof of Proposition 8 is that when the bailout authority contemplates a deviation it realizes that by lowering the measure of bankruptcies, it recognizes the effects of fire sales. That is, it recognizes that lowering the measure of bankruptcies raises the value R_2 of the capital that is transferred from the corporate sector to the traditional sector. In contrast, when a private firm contemplates a deviation it takes the value R_2 as given. Thus, the right side of the private sustainability constraint is lower than the right side of the sustainability to bailout constraint.

Note that if there are no fire sale effects the private sustainability constraint and the government sustainability constraint coincide. To see this suppose that F is linear in k_1 and k_2 so that it can be written as $F(k_1, k_2) = \rho_1 k_1 + \rho_2 k_2$ where ρ_1 and ρ_2 are constants. Then it is easy to show that

$$\hat{U}^G(a, k_c) - U^G(a, \varepsilon^*, k_c) = \hat{U}(a, k_c) - U(a, \varepsilon^*, k_c)$$

so that the two constraints coincide.

We use Proposition 5 to show that the sustainable equilibrium with bailouts yields lower welfare than the privately sustainable equilibrium.

Proposition 6. Suppose the discount factor β is strictly less than the threshold $\bar{\beta}$ given by (36a) at which the private sustainability constraint is binding. Any sustainable equilibrium with bailouts yields strictly lower welfare than the privately sustainable equilibrium. Furthermore, any sustainable equilibrium with bailout policy has bailouts in equilibrium, in the sense that $\tau > 0$.

Proof. Since $\beta < \bar{\beta}$, the private sustainability constraint is binding in a privately sustainable equilibrium. From Proposition 5 it follows that the privately sustainable equilibrium allocations violate the government sustainability constraint. Clearly, any sustainable equilibrium with bailout policy is a feasible allocation for the dynamic contracting problem since it satisfies the budget constraint of the financial intermediary, the incentive constraint of the manager, and the private sustainability constraint. Thus, it must yield lower welfare than the optimal allocation from the dynamic contracting problem. It follows that welfare is strictly lower in the bailout equilibrium.

We prove that any sustainable equilibrium with bailout policy has $\tau > 0$ by way of contradiction. Suppose that $\tau = 0$. Then, using Lemma 1 it follows that the solution to the dynamic contracting problem coincides with that of the privately sustainable equilibrium. This allocation violates the government sustainability constraint. Thus, any sustainable equilibrium with bailout policy must have $\tau > 0$. $Q.E.D.$

Characterization of the best bailout equilibrium: Let

$$V(\varepsilon^b) = \max U(a, \varepsilon^*, k_c)$$

subject to (30), (31), (32) and the additional constraint

$$\varepsilon^* = \varepsilon^b.$$

Now consider the maximization problem

$$\max V(\varepsilon^b)$$

subject to (41). The solution to this problem consists of the best baiout equilibrium allocations. (INSERT PROOF)

B. Can an ex ante regulator improve welfare?

Consider the situation described in the previous section in which neither the bailout authority nor the private agents can commit to their actions. We show that a regulatory authority armed with the ability the dictate the terms of the private contract, namely the compensation contract c_H^R , the scale of the corporate technology k_c^R, and the liquidation level ε^R, can improve ex ante welfare. Such a regulator must take into account the incentives of the bailout authority to intervene.

To see how a regulator can improve upon equilibrium allocations, we need to define a competitive equilibrium with regulation. We begin with an extreme form of regulation in which the regulator specifies the exact size of the firm and the exact set of states in which the firm can declare bankruptcy, and then show that less extreme regulations can achieve desired outcomes. Under the extreme form of regulation, the regulator chooses taxes, transfers and specifies the following constraints on contracts.

(48) $k_c = k^r$ and $\varepsilon^* = \varepsilon^r$.

The *optimal contracting problem with regulation* is now to choose a contract c_H and ε^* to maximize the utility of the manager (29) subject to the incentive constraint for the manager (30), the private sustainability constraint (35), the budget constraint of the financial intermediary (**??**) where k_2 is given by (32) and subject to the policy constraints (48).

A *sustainable equilibrium with regulation* consists of an allocation $c_H, a,\ \varepsilon^*, k_1, k_2, k_c$ R_2,U and a regulatory policy $k^r, c^r_H,\ \varepsilon^r,\ \tau,\ T_L(\varepsilon)$ is defined is defined in the same way as a sustainable equilibrium with bailout policy with one important difference. That difference, of course, is that the contracting problem now has additional constraints.

The regulator's problem is to structure policies so as to maximize the manager's welfare given that the allocations associated with a given policy must be part of a sustainable equilibrium.

Consider the *regulator's problem* given utility level e for lenders is to choose $c_H, a, \varepsilon^*, k_c, k_1, k_2, k_s$ to solve

$$\text{(49)} \quad \max \alpha_1 \left[p_H(a)c_H - p_L(a)BH(\varepsilon^*)\right] - a$$

subject to the manager's incentive constraint

$$\text{(50)} \quad a \in \arg\max_a \alpha_1 \left[p_H(a)c_H - p_L(a)BH(\varepsilon^*)\right] - a$$

the resource constraint

$$\text{(51)} \quad \alpha_1 p_H(a)c_H + c^I \leq \alpha_1 \left[p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon)dH(\varepsilon)\right] g(k_c) + F(k_1, k_2) + k_s$$

where k_2 is given by

$$(52)\quad k_2 = \alpha_0 \left[\sum p_i(a) A_i\right] g(k_c) + \alpha_1 p_L(a) g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon)$$

voluntary savings by lenders

$$(53)\quad F_1(k_1, k_2) = 1$$

and the bailout authority's sustainability constraint

$$(54)\quad \frac{U(a, \varepsilon^*, k_c)}{1-\beta} \geq \hat{U}^G(a, k_c) + \frac{\beta}{1-\beta} U^N$$

minimum utility level for managers

$$(55)\quad c^I \geq e$$

the stage 1 investment constraint

$$(56)\quad k_c + k_s + k_1 \leq e.$$

Note that the voluntary savings by lenders constraint (53) arises because the regulator has no instruments that can affect the return to investment k_1 in the traditional technology.

The regulatory problem is equivalent to the *simplified regulatory problem* of maximizing

(49) subject to (50),

$$(57) \quad \alpha_1 p_H(a) c_H + k_c \leq \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right] g(k_c) + R_2^* k_2$$

where k_2 is given by (52), and (54).

To see these two problems are equivalent, we show that the constraint sets of the two problems are the same. Consider first showing that (57) is implied by the constraints of the original problem. To do so, substitute $c^I = e = k_c + k_s + k_1$ into (51), use $F_1 = 1$ and Euler's theorem to obtain (57). Since the rest of the constraints are the same, it follows that if an allocation satisifies the constraints of the regulatory problem then it satisfies the constraints of the restated problem. Now consider an allocation that satisfies the constraints of the restated problem. We need to show that k_1 and k_s can be chosen to satisfy the original problem. To do so define

$$k_1 = \kappa k_2$$

where κ is such that $F_1(k_1, k_2) = F_1(\kappa, 1) = 1$ and choose $k_s = e - k_c - k_1$. Since we have assumed that e is large, a nonnegative k_s can be so chosen. Thus, the constraint sets are equivalent.

Proposition 7. The solution to the simplified regulator's problem coincides with the best sustainable equilibrium with regulation and has taxes equal to zero.

Proof. Inspection of the problems and constraints that define the sustainable equilibrium make it clear that any such equilibrium must satisfy (50)-(56). Clearly, the regulatory equilibrium must maximize manager's welfare subject to these constraints.

Next we show that any solution to the simplified regulator's problem can be supported as a sustainble equilibrium with regulation. To do so in the sustainable equilibrium let the policies be defined by (48) and let $\tau = T_L(\varepsilon) = 0$. Clearly, the simplified regulator's problem is equivalent to one in which we replace the government sustainability constraint (54) with the (48). From Proposition 5 the private sustainability constraint is necessarily satisified at these allocations. Thus, the equivalent problem coincides with the optimal contracting problem without commitment. *Q.E.D.*

Next, we have

Proposition 8. Suppose $\beta < \bar{\beta}$. The regulatory equilibrium yields higher welfare than any sustainable equilibrium with bailout policy.

Proof. The proof is by contradiction. Suppose that the bailout authority could achieve the same allocations as the regulator. Since the sustainability constraint for the government is tighter than it is for private agents, then at the regulator's allocations the private sustainability constraint in the contracting problem must be slack. Now consider the first order conditions with respect to k_c in the regulator's problem and in the contracting problem.

To derive these first order conditions for the regulator's problem we first rewrite the resource constraint (51). To do so we note from Euler's theorem that $F = F_1 k_1 + F_2 k_2$, so that using (53) we have that $F = k_1 + R_2 k_2$, where, as before, R_2 is uniquely pinned down by the condition that $F_1 = 1$. Substituting $F = k_1 + R_2 k_2$ and using that (55) and (56) hold with equality we can rewrite this constraint as

$$\alpha_1 p_H(a) c_H \le \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right] g(k_c) + R_2 k_2 - k_c$$

In the rewritten regulator's problem the first order condition for k_c is given by

$$(58)\quad \mu \left\{ g'(k_c) \left[Y_c + R_2 Y_l \right] - 1 \right\} = \theta \left[\hat{U}_k^G(a, k_c) - U_k(a, \varepsilon^*, k_c) \right]$$

where μ and θ are the multipliers on (51) and (54),

$$Y_c = \alpha_1 \left[p_H(a) A_H + p_L(a) A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon) \right]$$

$$Y_l = \alpha_0 \left[p_H(a) A_H + p_L(a) A_L \right] + \alpha_1 p_L(a) A_L \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon)$$

Consider next the first order conditions for the dynamic contracting problem in dual form with a slack private sustainability constraint. The budget constraint for this problem is given by

$$\alpha_1 p_H(a) c_H \leq$$

$$\alpha_1 \left[p_H(a)(A_H - \tau) + p_L(a) \int_{\varepsilon^*}^{\bar{\varepsilon}} \left[A_L(1+\varepsilon) + T_L(\varepsilon) \right] dH(\varepsilon) \right] g(k_c) + R_2 k_2 - k_c$$

The first order condition for k_c for this problem is given by

$$(59)\quad \tilde{\mu} \left\{ g'(k_c) \left[Y_c(\tau, T_L) + R_2 Y_l \right] - 1 \right\} = 0$$

where

$$Y_c(\tau, T_L) = \alpha_1 \left[p_H(A_H - \tau) + p_L \int_{\varepsilon^*}^{\bar{\varepsilon}} \left[A_L(1+\varepsilon) + T_L(\varepsilon) \right] dH(\varepsilon) \right].$$

In equilibrium, the government's budget constraint implies the value of taxes equal the value of transfers so that

(60) $Y_c = Y_c(\tau, T_L)$.

Combining (59) and (60) and using that the multiplier on the budget constraint is nonzero gives that in bailout equilibrium the allocations satisfy

(61) $\{g'(k_c)\left[Y_c + R_2 Y_l\right] - 1\} = 0$

The bailout allocations do not satisfy the first order condition for the regulatory equilibrium (58) because the right side of (58) is nonzero. Hence, the allocations in the regulator's problem and the contracting problem must differ. Since the bailout allocations are feasible for the regulator's problem, the bailout equilibrium must yield lower welfare than the regulatory equilibrium. *Q.E.D.*

The idea behind this proposition is that since the bailout authority has a balanced budget, in equilibrium, the tax-transfer scheme can only indirectly influence the choice of k_c. The key idea is that the regulator has a richer set of policy instruments than does the bailout authority.

C. A simple implementation

We consider how we can implement the regulatory equilibrium as a competitive equilibrium in which the goverment imposes constraints on the type of contracts that firms sign with managers. In the next proposition we show that the solution to the regulator's problem can be implemented a too-big-to-fail cap of the form $k_c \leq k^r$ and a liquidation constraint of the form $\varepsilon^* \leq \varepsilon^r$, with no taxes or transfers.

Proposition 9. The solution to the regulator's problem can be implemented with a too-big-to-fail cap of the form

$$(62) \quad k_c \leq k^r$$

and a liquidation cap of the form

$$(63) \quad \varepsilon^* \leq \varepsilon^r$$

where ε^r is the optimal level of ε^* in the solution to regulator's problem.

The proof is in the appendix.

Next we show that the regulatory equilibrium can be implemented by a cap on the size of the firm k_c and a cap on the firm's debt to value ratio. From Proposition 9, it follows that to show this result we need only show that a cap on the firm's debt to value ratio is equivalent to a cap on ε^*. We start by calculating the firm's debt to value ratio under this decentralization. To do so we calculate the expected present value of debt payments. With probability α_0, the manager loses the ability to manage the firm and the debt holders receive the face value of their debt. With probability $\alpha_1(p_H(a) + p_L(a)(1 - H(\varepsilon^*)) + \alpha_0$, the firm's cash flows exceed the required debt payment. In the event of bankruptcy, debt holders receive the liquidation value of the debt. The present value of debt payments is then given by

(64)

$$\{\alpha_1 [p_H(a) + p_L(a)(1 - H(\varepsilon^*))] + \alpha_0\} A_L(1+\varepsilon^*)g(k_c) + \alpha_1 p_L(a) R_2 A_L g(k_c) \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon).$$

The value of the firm is simply k_c. We now argue that if R_2 is sufficiently close to 1 then debt to value ratio is increasing in ε^*. To see this result we note that the derivative of (64) with respect to ε^* is proportional to

$$\{\alpha_1 [p_H(a) + p_L(a)(1 - H(\varepsilon^*))] + \alpha_0\} - \alpha_1(1 - R_2)p_L(a)h(\varepsilon^*)(1 + \varepsilon^*)$$

Clearly, if R_2 is close enough to 1 then the debt is increasing in ε^*.

Letting D denote the value of debt under any contract and D^r denote the value of debt (64) in a regulatory equilibrium, we summarize this discussion in a proposition.

Proposition 10. If R_2 is sufficiently close to 1 then the solution to the regulator's problem can be implemented with a too-big-to-fail cap of the form $k_c \leq k^r$ and a cap on the debt to value ratio of the form

$$\frac{D}{k_c} \leq \frac{D^r}{k^r}.$$

4. Conclusion

We have made three points in this paper. First, ex ante efficient contracts often require ex post inefficiency. Second, the time inconsistency problem for the government is more severe than for private agents because fire sale effects give governments stronger incentives to renegotiate contracts than private agents. Third, given that the government cannot commit itself to not bailing out firms ex post, ex ante regulation of firms is desirable.

5. Appendix

Proposition 4. If R is sufficiently close to 1 and $a''(p_H^E)$ is sufficiently small then $\varepsilon^* > \underline{\varepsilon}$. That is, supporting ex ante efficient allocations requires ex post inefficiency.

Proof: The proof is by contradiction. Suppose that $\varepsilon^* = \underline{\varepsilon}$. We will show that a small increase in ε^* from $\underline{\varepsilon}$ raises welfare. To show this result, we totally differentiate the budget constraint (16) and the incentive constraint (17) and evaluate these derivatives at $\varepsilon^* = \underline{\varepsilon}$. We obtain the following relationships between dc_H, dp and $d\varepsilon^*$

$$(65)\quad [A_H - A_L - c_H]\, dp - p dc_H - (1-p)A_L(1-R_2)h(\underline{\varepsilon})d\varepsilon^* = 0$$

$$(66)\quad dc_H + Bh(\underline{\varepsilon})d\varepsilon^* = a''(p)dp$$

where $p = p_H$. Substituting for dp from (66) into (65) and rearranging terms we obtain

$$(67)\quad \left(p - \frac{[A_H - A_L - c_H]}{a''(p)}\right) dc_H = \left[[A_H - A_L - c_H]\,\frac{B}{a''(p)} - (1-p)A_L(1-R_2)\right] h(\underline{\varepsilon})d\varepsilon^*.$$

The budget constraint evaluated at $\varepsilon^* = \underline{\varepsilon}$ implies that $1 - A_L = p(A_H - A_L - c_H)$, so that (67) can be rewritten as

$$(68)\quad \left(p - \frac{1-A_L}{pa''(p)}\right) dc_H = \left[\frac{1-A_L}{pa''(p)}B - (1-p)A_L(1-R_2)\right] h(\underline{\varepsilon})d\varepsilon^*$$

Totally differentiating the objective function, we obtain that the change in the utility of the manager dU is given by

$$dU = pdc_H - (1-p)Bh(\underline{\varepsilon})d\varepsilon^*$$

and from (68) we have that $dU > 0$ if and only if

$$(69) \quad dU = p\frac{\left[\frac{1-A_L}{pa''(p)}B - (1-p)A_L(1-R_2)\right]}{\left(p - \frac{1-A_L}{pa''(p)}\right)} - (1-p)B > 0.$$

Next, we show that the denominator of the first term in (69) is positive. To do so, consider the solutions to c_H and p obtained from the incentive constraint (15) and the budget constraint (16). Typically, these conditions yield multiple solutions. The solution that maximizes the manager's welfare is the largest value of p that satisfies these conditions. Substituting for c_H from (15) into (16) we obtain

$$(70) \quad pa'(p) + 1 = pA_H + (1-p)A_L.$$

At the largest value of p that satisfies (70), we must have that the derivative of the left side of (70) must be greater than the derivative of the right side of (70) so that

$$(71) \quad pa'' + a'(p) > A_H - A_L$$

Since the incentive constraint requires that $a'(p_H) = c_H$ and the budget constraint implies that $1 - A_L = p\left[A_H - A_L - c_H\right]$, (71) can be written as

$$pa'' > \frac{1 - A_L}{p}.$$

Thus, the denominator of the first term in (69) is positive.

Next we rewrite (69) as

$$dU = \left[p \frac{\left[\frac{1-A_L}{pa''(p)}\right] B}{\left(p - \frac{1-A_L}{pa''(p)}\right)} - (1-p) \right] B - p(1-p) \frac{A_L(1-R_2)}{\left(p - \frac{1-A_L}{pa''(p)}\right)} > 0$$

which, in turn can be rewritten as

$$(72) \quad dU = \left[\frac{\left[\frac{1-A_L}{pa''(p)}\right] - p(1-p)}{\left(p - \frac{1-A_L}{pa''(p)}\right)} \right] B - p(1-p) \frac{A_L(1-R_2)}{\left(p - \frac{1-A_L}{pa''(p)}\right)} > 0$$

Since $p < 1$, $p(1-p) \leq 1/4$ so that $(1-A_L)/pa''(p) - p(1-p) > 0$ if $a''(p) < 4(1-A_L)$. Thus if $a''(p)$ is sufficiently small, the first term in (72) is positive and if R_2 is sufficiently close to 1 the second term is small, so that, under these conditions, the change in utility given in (72) is positive. *Q.E.D.*

Proposition 9. The solution to the simplified regulator's problem can be implemented with a too-big-to-fail cap of the form $k_c \leq k^r$ and a liquidation cap of the form $\varepsilon^* \leq \varepsilon^r$ where k^r and ε^r are part of the solution to the regulator's problem.

Proof. We will show that the simplified regulatory problem is equivalent to a *constrained regulatory problem*, is which the sustainability constraint in the simplified regulatory problem are replaced by two constraints of the form

$$(73) \quad k_c \leq k^r \text{ and } \varepsilon^* \leq \varepsilon^r.$$

To do so we consider an intermediate problem, call the *constrained commitment problem* which is competitive contracting problem (29) with the extra constraint

$$(74) \quad k_c \leq k^r.$$

Note that the constraint (74) binds.(prove it binds) Let $(\varepsilon^c, a^c, k^r)$ denote the solution to this problem. We claim that

$$(75) \quad \varepsilon^r \leq \varepsilon^c.$$

If the sustainability constraint (54) does not bind then clearly $\varepsilon^r = \varepsilon^c$. Suppose next that the sustainability constraint does bind. Then

$$(76) \quad U(a^c, \varepsilon^c, k^r) \leq (1-\beta)\hat{U}(a^c, k^r) + \beta U^N$$

$$(77) \quad U(a^r, \varepsilon^r, k^r) = (1-\beta)\hat{U}(a^r, k^r) + \beta U^N$$

Next, since the regulatory solution is feasible for the constrained commitment problem

$$(78) \quad U(a^r, \varepsilon^r, k^r) \leq U(a^c, \varepsilon^c, k^r).$$

Thus, using (76)- (78) gives

(79) $\hat{U}(a^r, k^r) \leq \hat{U}(a^c, k^r)$.

From (33) we have that

(80) $\hat{U}(a, k_c) = (\alpha_1 + \alpha_0 R_2) \left[p_H(a) A_H + p_L(a) A_L \right] g(k_c) - k_c - a$

Note that the $\hat{U}(a, k_c)$ is a strictly concave function of effort which is increasing below its maximum and decreasing above its maximum. This function reaches its maximum at the full information optimum level of effort a^{FI}. We claim that both a^r and a^c are below a^{FI}. To see this claim, suppose for example that a^r is above a^{FI}, then it is feasible to reduce the costs for sustaining this effort by reducing ε^r and achieve a higher level of utility. A similar argument holds for a^c. Since a^r and a^c are both below a^{FI} then $\hat{U}(a, k_c)$ is increasing in a in this region and it follows from (79) that

(81) $a^r \leq a^c$.

We next show that (81) implies that $\varepsilon^r < \varepsilon^c$. To do consider a version of the constrained commitment problem in which we replace (73) with

$k_c \leq k^r$ and $\varepsilon^* \leq \varepsilon$

for an arbitrary $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$. Clearly, in any solution to this problem $k_c = k^r$. Denote the solution to this problem by $a(\varepsilon)$. Since $p_H(a)$ satisfies the monotone likelihood ratio property as well as convexity of distribution function assumption the first order approach is valid. It follows immediately that the maximizer $a(\varepsilon)$ is a continuous function of ε.

We next claim that $a(\varepsilon^c) \geq a(\varepsilon)$ for all $\varepsilon \in [\underline{\varepsilon}, \varepsilon^c]$. To show this, suppose by way of contradiction that for some $\varepsilon \in [\underline{\varepsilon}, \varepsilon^c]$, $a(\varepsilon) > a(\varepsilon^c)$. This choice of ε leads to higher output and higher utility than does $(\varepsilon^c, a(\varepsilon^c))$, which contradicts that the choice of ε^c maximizes utility.

Next, note that $a(\varepsilon^r) \geq a(\underline{\varepsilon})$. To show this, suppose by way of contradiction that $a(\varepsilon^r) < a(\underline{\varepsilon})$. Then the choice $(\underline{\varepsilon}, a(\underline{\varepsilon}))$ by the regulator yields higher utlity than $(\varepsilon^r, a(\varepsilon^r))$ and satisfies the sustainability constraint. This is a contradiction that $(\varepsilon^r, a(\varepsilon^r))$ solves the regulator's problem.

Since $a(\varepsilon)$ is continuous and $a(\varepsilon^r) \in [a(\underline{\varepsilon}),\ a(\varepsilon^c)]$ then there exists some value of $\varepsilon \in [\underline{\varepsilon}, \varepsilon^c]$ such that $a(\varepsilon^r) = a(\varepsilon)$. We claim that ε^r is the smallest value of ε with this property. To see this claim note that any higher value of ε with this property has lower output and lower utility than the smallest value does. We have therefore shown that $\varepsilon^r \leq \varepsilon^c$. Note, for later use, that this argument also implies that utility is lower than the regulatory value for any value of ε less than ε^r.

We now establish our main result. To that end consider a contracting problem with a too-big-too-fail cap $k_c \leq k^r$ and a liquidation cap $\varepsilon^* \leq \varepsilon^r$. Clearly, the constraint $k_c \leq k^r$ binds. Since, as we have just remarked, utility is lower than the regulatory value for any value of ε less than ε^r is follows that the solution to the contracting problem has $\varepsilon^* = \varepsilon^r$. *Q.E.D.*

Appendix B: Setup and definition of a privately sustainable equilibrium

The timing of events within a period are that first financial intermediaries simultaneously offer contracts to managers. The managers then decide whether to accept or decline the contracts. A manager who has accepted a contract with a financial intermediary then chooses a privately observed effort level. After the effort level has been chosen the financial intermediary then makes a take it or leave it offer to the manager. If the manager rejects renegotiated contract, the original contract is implemented. If the manager accepts then renegotiated contract then the renegotiated contract is implemented. The idiosyncratic state is then realized and payments occur according to the implemented contract. (Note that we have abstracted from renegotiation with lenders. Adding in such renegotiation simply adds notation with no substantive change in the results.)

Lenders and the financial intermediary in period t observe the past history of a managers renegotiation choices, $H_t = (H_{t-1}, \delta_t)$, but do not observe the past realizations of output[2]. We refer to H_t as the *public* history of a manager. Let $\sigma_{xt}(H_{t-1})$ denote the period t contract $x_t = (c_{Ht}, \varepsilon_t^*, k_{ct})$ offered by the financial intermediary to a manager at t[3].

Let $h_{ct} = (H_{t-1}, c_{Ht}, \varepsilon_t^*, k_{ct})$ be the history given the contract. Let $\sigma_{at}(h_{ct})$ denote the effort decision of the manager. Let $h_{at} = (h_{ct}, a_t)$ denote the history inclusive of the private action. Let $\gamma_t = 1$ signify that a new contract is offered and $\gamma_t = 0$ signify that no new contract is offered. Let $\sigma_{\gamma t}(h_{ct})$ denote the strategy for offering a new contract. Let $\sigma_{rt}(h_{ct})$

[2] It is this latter feature that ensures that the only intertemporal connection in the contract is whether the manager renegotiates. In particular, since the contract cannot depend on realizations of past output, this feature ensures that reputational outcomes in which the manager is induced to supply high effort by a promise of better contracts in the future are not feasible.

[3] Note the here we are focusing on equilibria in which the continuation payoffs depend only on the public history.

denote the terms of the continuation contract $(\hat{c}_{Ht}, \hat{\varepsilon}_t^*)$, if one was offered. Let $\delta_t = 1$ denote the new contract is accpeted and $\delta_t = 0$ denote that either the new contract is rejected or that no new contract is offered, and let $\sigma_{\delta t}(h_{at}, \hat{c}_{Ht}, \hat{\varepsilon}_t^*, \gamma_t)$ denote the associated acceptance strategy.

If $\sigma_{\delta t}(h_{at}, \hat{c}_{Ht}, \hat{\varepsilon}_t^*, \gamma_t) = 1$, then the manager's payoffs for the current period are given by $U(c_{Ht}, \varepsilon_t^*, a_t)$ defined by

$$p_H(a_t)c_{Ht} - BH(\varepsilon_t^*) - a_t$$

and the financial intermediary's payoffs are $\Pi_t(c_{Ht}, \varepsilon_t^*, k_{ct}, a_t)$ defined by

$$[p_H(a_t)A_H + p_L(a_t)A_L \int_{\varepsilon_t^*}^{\bar{\varepsilon}} (1+\varepsilon)dH(\varepsilon) + p_L(a_t)A_L R_{2t} \int_{\underline{\varepsilon}}^{\varepsilon_t^*} (1+\varepsilon)dH(\varepsilon)]g(k_{ct}) - p_H(a_t)c_{Ht} - k_{ct}$$

If $\sigma_{\delta t}(h_{at}, \hat{c}_{Ht}, \hat{\varepsilon}_t^*, \gamma_t) = 0$, then the manager's payoffs for the current period are given by $U(\hat{c}_{Ht}, \hat{\varepsilon}_t^*, a_t)$ and the financial intermediary's payoffs are $\Pi_t(\hat{c}_{Ht}, \hat{\varepsilon}_t^*, k_{ct}, a_t)$.

A strategy profile induces allocations and continuation utilities in the usual fashion. Let $V_t(H_{t-1}(i))$ denote the *continuation* utility of a manager with public history $H_{t-1}(i)$. We will say that a collection of strategies is a *privately sustainable* equilibrium if (i) For any history $(h_{at}, \hat{c}_{Ht}, \hat{\varepsilon}_t^*, \gamma_t)$ with $\gamma_t = 1$, if the equilibrium strategy specifies $\sigma_{\delta t} = 0$ then

$$(82) \quad U(c_{Ht}, \varepsilon_t^*, a_t) + \beta V_{t+1}(H_{t-1}, 0)$$

$$\geq U(\hat{c}_{Ht}, \hat{\varepsilon}_t^*, a_t) + \beta V_{t+1}(H_{t-1}, 1)$$

while if the equilibrium specifies $\sigma_{\delta t} = 1$ then the inequality (82) is reversed,

(iia) for any history h_{ct}, i then the original contract $(c_{Ht}, \varepsilon_t^*)$ must yield higher payoffs than any alternative contract $(\hat{c}_{Ht}, \hat{\varepsilon}_t^*)$ that would be accepted by the manager in that

$$\Pi_t(c_{Ht}, \varepsilon_t^*, k_{ct}, \sigma_{at}(h_{ct})) \geq \Pi_t(\hat{c}_{Ht}, \hat{\varepsilon}_t^*, k_{ct}, \sigma_{at}(h_{ct}))$$

for all $(\hat{c}_{Ht}, \hat{\varepsilon}_t^*)$ such that $\sigma_{\delta t}(h_{ct}, \sigma_{at}(h_{ct}), \hat{c}_{Ht}, \hat{\varepsilon}_t^*, 1) = 1$.

(iib) For any history h_{ct}, if the equilibrium strategy specifies renegotiate in that $\sigma_{\gamma t}(h_{ct}) = 1$ and offer $(\hat{c}_{Ht}, \hat{\varepsilon}_t^*)$ and the offer is accepted in that $\sigma_{\delta t}(h_{ct}, \sigma_{at}(h_{ct}), \hat{c}_{Ht}, \hat{\varepsilon}_t^*, 1) = 1$ then this new contract must yield the highest payoff to the financial intermediary among all offers that will be accepted, in that

$$\Pi_t(\hat{c}_{Ht}, \hat{\varepsilon}_t^*, k_{ct}, \sigma_{at}(h_{ct})) \geq \Pi_t(\tilde{c}_{Ht}, \tilde{\varepsilon}_t^*, k_{ct}, \sigma_{at}(h_{ct}))$$

for all $(\tilde{c}_{Ht}, \tilde{\varepsilon}_t^*)$ such that $\sigma_{\delta t}(h_{at}, \tilde{c}_{Ht}, \tilde{\varepsilon}_t^*, 1) = 1$. Also, if must be optimal for the intermediary to renegotiate in that

$$\Pi_t(\hat{c}_{Ht}, \hat{\varepsilon}_t^*, k_{ct}, \sigma_{at}(h_{ct})) \geq \Pi_t(c_{Ht}, \varepsilon_t^*, k_{ct}, \sigma_{at}(h_{ct})).$$

(Clearly we can ignore strategies that specify renegotation and rejection since the financial intermediary could attain the same payoffs by not renegotiating.)

(iii) For any history h_{ct}, if the equilibrium strategy specifies do not renegotiate in that

$\sigma_{\gamma t}(h_{ct}) = 0$, then the manager's strategy for effort $\sigma_{at}(h_{ct})$ satisfies

$$\sigma_{at}(h_{ct}) \in \arg\max_{a} p_H(a)c_{Ht} - BH(\varepsilon_t^*) - a,$$

while if the equilibrium strategy specifies renegotiate to a new contract $(\hat{c}_{Ht}, \hat{\varepsilon}_t^*)$ then $\sigma_{at}(h_{ct})$ satisfies

$$\sigma_{at}(h_{ct}) \in \arg\max_{a} p_H(a)\hat{c}_{Ht} - BH(\hat{\varepsilon}_t^*) - a.$$

(iv) For any history H_{t-1}, an offered contract x_t together with the strategies $\sigma_{at}, \sigma_{\gamma t}, \sigma_{rt}, \sigma_{\delta t}$ induces an implemented contracted $\tilde{x}_t$, an associated action $\tilde{a}_t$ (where the implemented contract is either the original contract or the renegotiated contract), and an associated accept decision $\tilde{\delta}_t$. Let

$$\left(\tilde{c}_{Ht}(h_{ct}), \tilde{\varepsilon}_t(h_{ct}), k_{ct}, \tilde{a}_t(h_{ct}), \tilde{\delta}_t(h_{ct})\right)$$

denote the induced outcomes. The equilibrium strategy $\sigma_{xt}(H_{t-1})$ must maximize the financial intermediary's profits over all contracts that will be accepted by the manager in that

$$\Pi_t\left(\tilde{c}_{Ht}(h_{ct}), \tilde{\varepsilon}_t(h_{ct}), k_{ct}, \tilde{a}_t(h_{ct})\right) \geq \Pi_t\left(\tilde{c}_{Ht}(\hat{h}_{ct}), \tilde{\varepsilon}_t(\hat{h}_{ct}), \hat{k}_{ct}, \tilde{a}_t(\hat{h}_{ct})\right)$$

where $\hat{h}_{ct} = (H_{t-1}, \hat{x}_t)$ for all alternative contracts $\hat{x}_t$ such that

$$U(\tilde{c}_{Ht}(\hat{h}_{ct}), \tilde{\varepsilon}_t(\hat{h}_{ct}), \tilde{a}_t(\hat{h}_{ct})) + \beta V_{t+1}(H_{t-1}, \tilde{\delta}_t(\hat{h}_{ct})) \geq V_t(H_{t-1}) \tag{83}$$

(v) For any history H_{t-1}, the financial intermediary makes zero profits in that

$$\Pi_t\left(\tilde{c}_{Ht}(h_{ct}), \tilde{\varepsilon}_t(h_{ct}), k_{ct}, \tilde{a}_t(h_{ct})\right) = 0$$

where $h_{ct} = (H_{t-1}, \sigma_{xt}(H_{t-1}))$.

(vi) For any history H_{t-1}, continuation utilities are generated by the equilibrium strategies in that

$$(84) \quad V_t(H_{t-1}) = U(\tilde{c}_{Ht}(h_{ct}), \tilde{\varepsilon}_t(h_{ct}), \tilde{a}_t(h_{ct})) + \beta V_{t+1}(H_{t-1}, \tilde{\delta}_t(h_{ct}))$$

and

$$\lim_{t \to \infty} \sup \beta^t V_t(H_{t-1}(i)) = 0.$$

wlog the induced contract is offered.

any equilibria

Appendix C: Setup and definition of a sustainable equilibrium: add a gov't

Let $x_t(i) = (c_H(i), a(i), \varepsilon^*(i), k_c(i), \delta(i), \hat{c}_H(i), \hat{\varepsilon}^*(i), Y_t)$ and let $h_t(i) = (h_{t-1}(i), x_t(i))$ where $\delta_t(i) = 1$ indicates that manager i renegotiated in period t and $\delta_t(i) = 0$ indicates that manager i did not renegotiate in period t. Lenders and the financial intermediary in period t observe the past history of manager $i's$ renegotiation choices, $H_t(i) = (H_{t-1}(i), \delta_t(i))$, but do not observe the past realizations of output[4]. Let $\sigma_{it}(H_{t-1}(i))$ denote the contract offered by the financial intermediary to a manager i at t. Let

$$\sigma_{it}^{M}(h_{t-1}(i), c_{Ht}(i), a_t(i), \varepsilon_t^*(i), k_{ct}(i))$$

denote the decision of the manager on whether to renegotiate. A strategy profile induces allocations and continuation utilities in the usual fashion. Let $U(a(i), \varepsilon^*(i), k_c(i)))$ denote the period utility associated with a contract $(a(i), \varepsilon^*(i), k_c(i)))$ and let $V(H_t(i))$ denote the present discounted continuation utility of a manager with public history H_t.[5] We will say that a sequence of contracts and renegotiation decisions is a sustainable equilibrium if i) along the equilibrium path $\sigma_{it}^{M}(h_{t-1}(i),\ c_{Ht}(i), a_t(i), \varepsilon_t^*(i), k_{ct}(i))) = 0$, ii) for all histories $H_{t-1}(i)$, the contract $\sigma_{it}(H_{t-1}(i))$ solves the firm's problem, maximize

$$(85)\quad \sum_i p_i(a)\left[\int_{C_i} Y_{ci}(\varepsilon)dH(\varepsilon) + \int_{B_i} Y_{bi}(\varepsilon)dH(\varepsilon) - \int [c_i(\varepsilon) + d_i(\varepsilon)]dH(\varepsilon)\right]$$

subject to the participation constraint for lenders

$$(86)\quad \sum_i p_i(a)\left[\int d_i(\varepsilon)dH(\varepsilon)\right] \geq 1,$$

[4]It is this latter feature that ensures that the only intertemporal connection in the contract is whether the manager renegotiates. In particular, since the contract cannot depend on realizations of past output, this feature ensures that reputational outcomes in which the manager is induced to supply high effort by a promise of better contracts in the future are not feasible.

[5]Note the here we are focusing on equilibria in which the continuation payoffs depend only on the public history

the participation constraint for managers

$$(87)\quad \sum_i p_i(a)\left[\int c_i(\varepsilon)dH(\varepsilon) - B\int_{B_i} dH(\varepsilon)\right] - a \geq \bar{U},$$

the managers' incentive constraint

$$(88)\quad a \in \arg\max_a \sum_i p_i(a)\left[\int c_i(\varepsilon)dH(\varepsilon) - B\int_{B_i} dH(\varepsilon)\right] - a$$

the nonnegativity constraint on consumption of managers

$$(89)\quad c_i(\varepsilon) \geq 0$$

and the private sustainability constraint

$$(90)\quad U(a(i), \varepsilon^*(i), k_c(i))) + \beta V(H_{t-1}(i), 0) \geq \hat{U}(a(i), \hat{\varepsilon}^*(i), k_c(i)) + \beta V(H_{t-1}(i), 1)$$

where we have suppressed the period t subscript on current allocations for convenience.

The sustainability constraint in this contracting problem is needed to ensure that the manager does not have an incentive to renegotiate the contract. To see this result, suppose that a contract does not satisfy this constraint. Then the manager will renegotatiate the contract.

The publicly observed history of manager i is $H_t(i) = (H_{t-1}(i), \delta_t(i))$.

A *sustainable equilibrium with a bailout policy* consists of an allocation $c_H, a, \varepsilon^*, k_1, k_2, k_c, R_2,$

U, U^G and a policy τ, $T_L(\varepsilon)$ such that

i) given R_2, the allocations solve the optimal contracting problem with policy

ii) given R_2, k_1 and k_2 satisfy (**??**) and (**??**)

iii) the consumption of lenders satisfies (23) with $R_c = R_1 = 1$.

iv) the resource constraints (20) and (24) hold.

v) the government's budget constraint (37) holds.

vi) the government's sustainability constraint (41).

vii) the continuation utility $U = U(a, \varepsilon^*, k_c)$ and $U^G = U^G(a, \varepsilon^*, k_c)$

New model:

(91) $R_c k_c = \max_l F(k_c, l) - wl.$

The return $R_b k_b$ generated by a replacement manager is given by

(92) $R_b k_b = \max_l bF(k_b, l) - wl.$

Government sustainability constraint tighter:

Private sustainability: A contract is *privately sustainable* if

(93) $R_c k_c + R_b k_b - 1 - p_L(a)BH(\varepsilon^*) - a + \dfrac{\beta}{1-\beta} U(a, \varepsilon^*, k_c)$

$$\geq R_c(k_c + k_b) - 1 - a + \frac{\beta}{1-\beta} U^N.$$

$$\max_l \left[F(k_c + k_b, l) - wl\right] - 1 - a + \frac{\beta}{1-\beta}U^N$$

Government sustainability

$$(94)\quad F(k_c, l_c) + bF(k_b, l_b) - p_L(a)BH(\varepsilon^*) - a + e - wl - 1 + \frac{\beta}{1-\beta}U^G$$

$$\geq F(k_c + k_b, l_c + l_b) - wl - a + e - 1 + \frac{\beta}{1-\beta}U^{GN}.$$

Using Euler's theorem and $w = F_l$

$$F(k_c, l_c) = R_c k_c + wl_c$$

$$bF(k_b, l_b) = R_b k_b + wl_b$$

Substituting in government sustainability

$$(95)\quad R_c k_c + R_b k_b - p_L(a)BH(\varepsilon^*) - a + e - 1 + \frac{\beta}{1-\beta}U^G$$

$$\geq F(k_c + k_b, l_c + l_b) - a + e - wl - 1 + \frac{\beta}{1-\beta}U^{GN}.$$

Next note $U^G = U + e$ and $U^{GN} = U + e$. Thus, government sustainability becomes

$$(96)\quad R_c k_c + R_b k_b - p_L(a)BH(\varepsilon^*) - a - 1 + \frac{\beta}{1-\beta}U$$

$$\geq F(k_c + k_b, l_c + l_b) - a - wl - 1 + \frac{\beta}{1-\beta}U^N.$$

We need to show

$$F(k_c+k_b, l_c+l_b) - w(l_c+l_b) > R_c(k_c+k_b) = \max_l F(k_c+k_b, l) - wl$$

or

$$F(k_c+k_b, l_c+l_b) > F_1(k_c, l_c)k_c + F_1(k_c, l_c)k_b + w(l_c+l_b)$$

Since $F_1(k_c, l_c) > bF_1(k_b, l_b)$ we need to show

$$F(k_c+k_b, l_c+l_b) > F(k_c, l_c) + bF(k_b, l_b)$$

which is clearly true, since an inferior technology is being used on rhs.

Aggregate Demand Externality

LHS: profits using inferior technology today (statically, but better for incentives)+statically inferior tomorrow but better for incentives

RHS: static profits higher, but essentially banned from using “better” incentive+output technology in future.

Private: static gains for you (one person switch, holding fixed others)

Public gains: all switch to better, get extra boost because all people have higher incomes and hence demand shifts out.

very simple example

$$\int l_i = 1$$

$$y_i = A_i l_i$$

fixed set of goods.

$$Y = (\int y_i^\theta di)^{1/\theta}$$

demand:

$$y_i = \left(\frac{P}{p_i}\right)^{\frac{1}{1-\theta}} Y$$

Monopolist: dropping constants

$$\max_{p_i} p_i \left(\frac{1}{p_i}\right)^{\frac{1}{1-\theta}} - \frac{w}{A_i}\left(\frac{1}{p_i}\right)^{\frac{1}{1-\theta}}$$

$$\max_{p_i} p_i^{1-\frac{1}{1-\theta}} - \frac{w}{A_i} p_i^{-\frac{1}{1-\theta}}$$

$$\max_{p_i} p_i^{-\frac{\theta}{1-\theta}} - \frac{w}{A_i} p_i^{-\frac{1}{1-\theta}}$$

$$-\frac{\theta}{1-\theta} p^{-\frac{1}{1-\theta}} + \frac{1}{1-\theta}\frac{w}{A_i} p_i^{-\frac{1}{1-\theta}-1} = 0$$

$$-\theta p + \frac{w}{A_i} = 0$$

$$p_i = \frac{1}{\theta}\frac{w}{A_i}$$

Profits:

$$\max_{p_i} p_i \left(\frac{P}{p_i}\right)^{\frac{1}{1-\theta}} Y - \frac{w}{A_i}\left(\frac{P}{p_i}\right)^{\frac{1}{1-\theta}} Y$$

$$(\frac{1-\theta}{\theta})\frac{w}{A_i}\left(\frac{\theta A_i}{w}\right)^{\frac{1}{1-\theta}}P^{\frac{1}{1-\theta}}Y$$

Symmetric Equilibrum $A_i = A$ so

$$Y = A$$

$$p_i = P = 1 \text{ so } w = \theta A$$

Equilibrium profits

$$(\frac{1-\theta}{\theta})\theta A = (1-\theta)A$$

Consider an individual guy who has access to a better technology $\hat{A}_i$, how much would his profits go up by:

$$(\frac{1-\theta}{\theta})\frac{\theta A}{\hat{A}_i}\left(\frac{\theta \hat{A}_i}{\theta A}\right)^{\frac{1}{1-\theta}}A-(1-\theta)A$$

$$(1-\theta)\frac{A}{\hat{A}_i}\left(\frac{\hat{A}_i}{A}\right)^{\frac{1}{1-\theta}}A-(1-\theta)A$$

$$(1-\theta)A\left[\left(\frac{\hat{A}_i}{A}\right)^{\frac{1}{1-\theta}-1}-1\right]$$

$$(1-\theta)\left[A^{\frac{1-2\theta}{1-\theta}}\hat{A}_i^{\frac{\theta}{1-\theta}}-A\right]$$

is the increase in one guys profits if he switches

If everyone switches:

$$(1-\theta)(\hat{A}-A)$$

Prove:

$$\hat{A}-A > A^{\frac{1-2\theta}{1-\theta}}\hat{A}_i^{\frac{\theta}{1-\theta}} - A$$

or

$$\hat{A} > A^{\frac{1-2\theta}{1-\theta}}\hat{A}^{\frac{\theta}{1-\theta}}$$

$$\hat{A}^{\frac{1-2\theta}{1-\theta}} > A^{\frac{1-2\theta}{1-\theta}}$$

$$\hat{A} > A$$

Yes, so profits increase more if everyone switches.

Production Technology

$$y_i \leq k_i$$

with $\theta > 0$, profits are increasing as the price falls because output is increasing faster than p is falling so py is increasing as p falls. Thus, it is always optimal to charge a price so that you hit the capacity constraint.

Want

$$y_i = \left(\frac{P}{p_i}\right)^{\frac{1}{1-\theta}} Y = k_i$$

Solve for p_i

$$p_i = P\left(\frac{Y}{k_i}\right)^{1-\theta}$$

so profits are

$$p_i k_i = P\left(\frac{Y}{k_i}\right)^{1-\theta} k_i = PY^{1-\theta}k_i^{\theta}$$

$$P = \left(\int p_i^{\frac{\theta}{\theta-1}} di\right)^{\frac{\theta-1}{\theta}}$$

New Formulation:

Let

$$k_c = p_H(a)A_H + p_L(a)A_L \int_{\varepsilon^*}^{\bar{\varepsilon}} (1+\varepsilon) dH(\varepsilon^*)$$

$$k_d = bp_L(a)A_L \int_{\underline{\varepsilon}}^{\varepsilon^*} (1+\varepsilon) dH(\varepsilon^*) \text{ with } \gamma < 1$$

Output in original equilibrium

$$Y = \left[p_H(a)A_H^{\theta} + p_L(a) \int_{\varepsilon^*}^{\bar{\varepsilon}} \left[A_L(1+\varepsilon)\right]^{\theta} dH(\varepsilon^*) + p_L(a) \int_{\underline{\varepsilon}}^{\varepsilon^*} \left[bA_L(1+\varepsilon)\right]^{\theta} dH(\varepsilon^*)\right]^{1/\theta}$$

where $k = A_H(1+\varepsilon)$ in H, $k = A_L(1+\varepsilon)$ in L.

Output with deviation

$$\hat{Y} = \left[p_H(a)A_H^\theta + p_L(a)A_L^\theta\right]^{1/\theta}$$

Note: our normalization is now

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (1+\varepsilon)^\theta dH(\varepsilon^*) = 1$$

Profits of an individual project:

$$p_i k_i = Y^{1-\theta} k_i^\theta$$

Ex ante (expected) profits Π

$$= \left[p_H(a)A_H^\theta + p_L(a)\int_{\varepsilon^*}^{\bar{\varepsilon}} \left[A_L(1+\varepsilon)\right]^\theta dH(\varepsilon^*) + p_L(a)\int_{\underline{\varepsilon}}^{\varepsilon^*} \left[bA_L(1+\varepsilon)\right]^\theta dH(\varepsilon^*)\right] Y^{1-\theta}$$

$$\hat{\Pi} = \left[p_H(a)A_H^\theta + p_L(a)A_L^\theta dH(\varepsilon^*)\right] Y^{1-\theta}$$

Note: In equilibrium $\Pi = Y^\theta Y^{1-\theta} = Y$ and $\hat{\Pi} = \hat{Y}^\theta Y^{1-\theta}$.

Sustainability constraint for private agents

$$\Pi - B\bar{H}(\varepsilon^*) + \frac{\beta}{1-\beta}\left(\Pi - B\bar{H}(\varepsilon^*)\right) \geq \hat{\Pi} + \frac{\beta}{1-\beta}\Pi^N$$

Sustainability constraint for the government

$$Y - B\bar{H}(\varepsilon^*) + \frac{\beta}{1-\beta}\left(Y - B\bar{H}(\varepsilon^*)\right) \geq \hat{Y} + \frac{\beta}{1-\beta}Y^{GN}$$

Sustainability constraint for private agents can be written

$$Y - B\bar{H}(\varepsilon^*) + \frac{\beta}{1-\beta}\left(Y - B\bar{H}(\varepsilon^*)\right) \geq \hat{Y}^{\theta}Y^{1-\theta} + \frac{\beta}{1-\beta}(Y^N)^{\theta}Y^{1-\theta}$$

$$\Pi^N = (Y^N)^{\theta}Y^{1-\theta}$$

explore Y^{GN}

Simplest formulation:

No first stage where invest in l, just workers with labor l. Instead there is an infinite elastic supply of labor at wage w.

Timing: After see realization A_i and the ε shock, then firms that want to transform capital into output. There are two production functions taking capital into output

$$F(k_c, l_c) \text{ and } bF(k_b, l_b)$$

Firms must pay workers in advance and use a weighted avg of value of k.

(97) $wl_i \leq \gamma q k_i$

where

$$q = \mu R_c + (1 - \mu) R_b$$

μ is $k_c/(k_c + k_d)$, namely the fraction of investment that is continuing and q is interpreted as the price of capital. (mention Kiyotaki and Moore)

Key idea: when other people stop defaulting any single other person's borrowing constraint is relaxed. This is the externality. (Idea?: defaulting capital poisons continuing capital)

Assume that this constraint is binding for the continuing types.

$$wl_c = \gamma q k_c$$

How is $R_c(\mu)$ determined when assume borrowing constraint binding for continuing but not for distressed firms?

$R_c(\mu)$ must satisfy two conditions:

$$wl_i \leq qk_i \tag{98}$$

$$\frac{l}{k} = \gamma \frac{q}{w}$$

where $q = [\mu R_c + (1 - \mu) R_d]$ so

$$\frac{l}{k} = \frac{\gamma \left(\mu R_c(\mu) + (1 - \mu) R_b\right)}{w}$$

Now since $F(k,l) - wl = Rk$,

$$R_c(\mu) = F(1, \frac{l}{k}) - w\frac{l}{k}$$

substituting

$$(99) \quad R_c(\mu) = F(1, \frac{\gamma\,(\mu R_c(\mu) + (1-\mu)R_b)}{w}) - w\frac{\gamma\,(\mu R_c(\mu) + (1-\mu)R_b)}{w}$$

Since the constraint is not binding for the b guy,

$$R_b = R_b(w) = \max_w bF(1,l) - wl$$

Note that $R_c(\mu)$ is the solution to (99). Assume that this solution is unique.

As μ increases, $R_c(\mu)$ increases (see picture).

The reason that the government's rhs static bit is higher:

$$R_c(1)(k_c + k_b) > R_c(\mu)(k_c + k_b)$$

Private sustainability constraint:

$$R_c k_c + R_b k_b - 1 - p_L(a)BH(\varepsilon^*) - a + \frac{\beta}{1-\beta}U(a, \varepsilon^*, k_c)$$

$$\geq R_c(k_c + k_b) - 1 - a + \frac{\beta}{1-\beta}U^N.$$

$$[F(k_c + k_b, l) - wl] - 1 - a + \frac{\beta}{1-\beta} U^N$$

use that after a deviation by an individual to continue with capital k_b rather than declare bankruptcy, the total amount of continuing capital is $k_c + k_b$ and, since, by assumption the borrowing constraint is binding

Government sustainability constraint.

$$(100) \quad F(k_c, l_c) + bF(k_b, l_b) - wl - p_L(a)BH(\varepsilon^*) - a - 1 + \frac{\beta}{1-\beta} U^G$$

$$\geq F(k_c + k_b, l) - wl - a - 1 + \frac{\beta}{1-\beta} U^{GN}.$$

Assuming that the borrowing constraint is still binding if the government stops all bankruptcies. Now the borrowing constraint reads

$$(101) \quad wl_i \leq \gamma \hat{q} k_i$$

where

$$\hat{q} = \hat{R}_c$$

Prove rhs government is larger than rhs private. Clearly, the first part is bigger in that

$$R_c(1)(k_c + k_b) - 1 - a > R_c(\mu) - 1 - a$$

Now show the second part is bigger $U^{GN} > U^G$.

For the private guy, he is confronted with $R_c(\mu)$ and is constrained to choose $\varepsilon^* = \underline{\varepsilon}$ while private agents following a goverment bailout are confronted with $R_c(1)$. So the returns after the government deviated is now higher. So U^{GN} is the solution to

(102) $U^{GN} = \max p_H(a)c_H - a$

subject to

(103) $a \in \arg\max_a p_H(a)c_H - a.$

(104) $p_H(a)c_H + 1 \leq R_c(1)(p_H(a)A_H + p_L(a)A_L)$

while the private payoff U^N is

(105) $U^N = \max p_H(a)c_H - a$

subject to

(106) $a \in \arg\max_a p_H(a)c_H - a.$

(107) $p_H(a)c_H + 1 \leq R_c(\mu)(p_H(a)A_H + p_L(a)A_L)$

Clearly,

$$U^{GN} > U^N$$

since R_c is increasing in μ.

Individual: firms profits i renegotiates

$$f(k_c + k_d, l) - wl$$

where $l = \alpha(k_c + k_d)$, so firm's profits are

$$(k_c + k_d)\left(f(1, \alpha) - w\alpha\right)$$

Planner:

$$f(k_c + k_d, l_c + l_d) - w(l_c + l_d)$$

$$(k_c + k_d)\left[f(1, \frac{l_c + l_d}{k_c + k_d}) - w\frac{l_c + l_d}{k_c + k_d}\right]$$

Need: planner after stop bankruptcy to do better than firm

$$\left[f(1, \frac{l_c + l_d}{k_c + k_d}) - w\frac{l_c + l_d}{k_c + k_d}\right] > f(1, \alpha) - w\alpha$$

If the constraint is binding on the firm, then the firm would benefit (higher profits) from having it relaxed on the margin so

$$f(1, \alpha) - w\alpha$$

is increasing in α. Need

$$\frac{l_c + l_d}{k_c + k_d} > \alpha$$

but this is a problem. Since the high guy has a binding labor constraint $\alpha = l_c/k_c$. So

$$\frac{l_d}{k_d}$$

Akerlof story

continuum of sellers and value car $v \in [0, 1]$ and value of buyers is $v + b$ where $b > 0$. Market price is p. Only sellers with $v \leq p$ will sell. So

$$E(v + b | v \leq p) = p$$

$$\frac{p}{2} + b = p$$

$$p = 2b$$

$$\frac{p}{2} = b$$