Monetary policy and herd behavior in new-tech investment*

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Abstract

This paper studies the interaction between monetary policy and asset prices using a simple general equilibrium model in which asset-price bubbles may form due to herd behavior in investment in a new technology whose productivity is uncertain. The economy is populated with one infinitely lived representative household and overlapping generations of finitely lived entrepreneurs. Entrepreneurs receive private signals about the productivity of the new technology and borrow from the household to publicly invest in the old or the new technology. Monetary policy intervention, by affecting the cost of resources for entrepreneurs, can make the entrepreneurs invest in the new technology if and only if they have received a favourable private signal. In doing so, it reveals this signal and hence prevents herd behavior and the asset-price bubble. We identify conditions under which such a monetary policy intervention is socially desirable.

Key Words : Monetary Policy – Asset Prices – Informational Cascades.

JEL Classification : E52, E32

1 Introduction

Should monetary policy react to perceived asset-price bubbles over and above their short-term effects on the business cycle? This old question has been hotly debated again since the remarkable rise and fall in new-tech equity prices in developed economies in the late 1990s and early 2000s. Today's conventional answer among central bankers is "no". This answer stems from the consideration of the following trade-off. On the one hand, if there is actually a bubble, then such a monetary policy reaction may reduce its size and/or its duration, and hence its welfare costs due to overinvestment in the short to medium term (when the bubble grows) and macroeconomic volatility in the medium term (when the bubble bursts). On the other hand, if alternatively there is actually no bubble, then such a monetary

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policy reaction will reduce welfare in the short term. Given this trade-off, such a monetary policy reaction can be viewed as an insurance-against-bubbles policy, and the conditions most commonly stressed by central bankers for its desirability are the following ones: i) the central bank should be sufficiently certain that there is actually a bubble; ii) the cost of reacting as the bubble grows should be low enough, in particular the bubble should be sufficiently sensitive to modest interest-rate hikes; iii) the cost of reacting only when the bubble bursts, or not reacting at all, should be high enough. Because they commonly view these conditions as unlikely to be met in practice (Greenspan, 2002; Bernanke, 2002; Trichet, 2005; Rudebusch, 2005; Kohn, 2006; Mishkin, 2007), central bankers thus usually conclude that, in most if not all cases, such a monetary policy reaction is not desirable[1].

This paper seeks to challenge this view by considering a simple general-equilibrium model in which, because asset-price bubbles are modeled as (rational) herd behavior, these three conditions can easily be met. We focus on bubbles in new-tech equity prices, as our argument rests on some productivity considerations that are not likely to play a key role in the development of other kinds of asset-price bubbles, *e.g.* bubbles in house prices. More precisely, we assume that a new technology becomes available whose productivity will be known with certainty only in the medium term. Entrepreneurs sequentially choose whether to invest in the old or the new technology, each of them on the basis of both the previous investment decisions that she observes and a private signal that she receives about the productivity of the new technology. Herd behavior may then arise as the result of an informational cascade (Banerjee, 1992; Bikhchandani, Hirshleifer and Welch, 1992) that corresponds to a situation in which, because the first entrepreneurs choose to invest in the new technology as they receive encouraging private signals about its productivity, the following entrepreneurs rationally choose to invest in the new technology too whatever their own private signal.

In this context, monetary policy tightening, by making borrowing dearer for the entrepreneurs, can make them invest in the new technology if and only if they receive an encouraging private signal

[1] Trichet (2005) however points out that, should all three conditions be met at some occasion, its monetary analysis would probably enable the European Central Bank to react to the medium- to long-term effects of the perceived asset-price bubble on the business cycle without departing from the current framework of its monetary policy strategy.

about its productivity. In doing so, it prevents herd behavior and hence the bubble in new-tech equity prices. Under this new explanation of bubbles in new-tech equity prices, the three conditions mentioned above can be met: i) the central bank can detect herd behavior with certainty, even though it then knows less about the productivity of the new technology than each entrepreneur; ii) given the fragility of informational cascades, a modest monetary policy intervention can be enough to interrupt herd behavior, even though it may not interrupt the new-tech investment craze[2]; iii) as we show, under certain conditions, such a monetary policy intervention is *ex ante* preferable, in terms of social welfare, to the laisser-faire policy (itself always preferable to the policy of reacting only when the bubble bursts).

This paper is closely related to a large body of literature on whether the monetary policy rule should make the current short-term nominal interest rate react to some measure of asset-price or credit developments, in addition to standard variables (such as the current or expected future inflation rate, the current output gap and the past short-term nominal interest rate), during an asset-price boom that may correspond to an asset-price bubble. This literature has not reached a consensus on the answer to that question. Indeed, on the one hand, Bernanke and Gertler (1999, 2001) and Gilchrist and Leahy (2002) find that there is no need to include past asset prices in the monetary policy rule, essentially because inflation and asset prices move jointly in their models. Similarly, Tetlow (2006) obtains that, except in particular cases, there is no need to include current asset-price growth in the monetary policy rule. On the other hand, Cecchetti, Genberg, Lipsky and Wadhwani (2000) find that, if the central bank knows the size of the asset-price misalignment (admittedly a big "if"), then it should add the past asset-price misalignment in its monetary policy rule. Moreover, Gilchrist and Saito (2006) obtain that, even when the central bank has no clue about the size of the asset-price misalignement, it is worth adding current asset-price growth in the monetary policy rule, in essence because asset-price growth reflects distortions in the resource allocation more closely than inflation in

[2] The latter outcome would be expected by many a central banker, *e.g.* Greenspan (2002), Bernanke (2002) and Kohn (2006).

their model. Finally, Christiano, Ilut, Motto and Rostagno (2007) find that it is worth adding credit growth in the monetary policy rule because, contrary to inflation, credit growth increases during the asset-price boom in their model.

Our way of modeling bubbles in new-tech equity prices has some important advantages over each of the following three ways in which this literature models asset-price bubbles. First, it may be modeled as an exogenous boom-and-bust term in the asset-price-dynamics equation (Bernanke and Gertler, 1999, 2001; Cecchetti, Genberg, Lipsky and Wadhwani, 2000; Tetlow, 2006). This *ad hoc* modeling makes the bubble by construction insensitive to monetary policy, and tends to cast doubt on the relevance of the welfare function considered. By contrast, our modeling makes the bubble sensitive to monetary policy, and enables us to use a micro-founded welfare function. Second, the asset-price bubble may be modeled as the result of favourable public news about future productivity that eventually fails to materialize (Gilchrist and Leahy, 2002; Christiano, Ilut, Motto and Rostagno, 2007). In this context, given that expectations are assumed to be rational and that the central bank is assumed to have no informational advantage over the private sector and therefore to be as much surprised by the lower-than-expected eventual productivity level as the private sector, a proper unconditional assessment of the desirability of a given monetary policy requires to consider not only the case where the favourable news does not materialize, but also the case where it does, and to assign an occurrence probability to each case − something this branch of the literature usually does not do[3]. Modeling the asset-price bubble as herd behavior enables us to do just that in a micro-founded way. Third and finally, the asset-price bubble may be modeled as the result of a permanent increase in productivity growth that economic agents gradually recognize afterwards (Gilchrist and Saito, 2006). However, in our new-technology context, this late-recognition assumption may be viewed as less relevant than the early-news assumption that we make.

The other side of the coin, though, is that our way of modeling bubbles in new-tech equity prices

[3] Gilchrist and Leahy (2002) do actually consider both cases, without needing to assign an occurrence probability to each of them, because the "strong inflation-targeting" monetary policy that they consider is very close to the optimal monetary policy in both cases.

and our wish to secure some analytical results compel us to consider a highly stylised model that fails to reproduce some basic characteristics of observed new-tech investment crazes, most notably the concomitant steady growth in consumption and asset prices. Indeed, this model predicts that, during a new-tech investment craze, as long as some uncertainty remains about the productivity of the new technology, consumption should initially jump to a lower level and remain at this level thereafter, while asset prices should initially jump to a higher level and, under the laisser-faire policy, remain at that level thereafter. We therefore view our paper as a first step in building an empirically more relevant model of herd behavior in new-tech investment[4].

Our paper is also related to the literature on the role of informational cascades in the business cycle. Within this literature, the paper closest to ours is that of Chamley and Gale (1994), which models investment collapses as the result of herd behavior. A first difference between the two papers is that we consider a general-equilibrium model, which enables us to conduct policy analysis, while they consider a partial-equilibrium model. A second difference is that they consider an endogenous timing of investment decisions, as they are also interested in modelling strategic investment delay, while we consider an exogenous timing of investment decisions. And a third difference is that, in equilibrium, in their model, although an investment collapse may be socially non-optimal, an investment surge is always socially optimal, while in ours, both new-tech and old-tech investment crazes may be socially non-optimal.

The remainder of the paper is structured as follows. Section 2 presents the model. The competitive equilibrium with public information about the productivity of the new technology is described in section 3. We introduce private information, derive the results about the desirability of policy intervention in a simple case and conduct simulations in more complex cases in section 4. Section 5 concludes.

[4]This research agenda would undoubtedly benefit from the works of Beaudry and Portier (2004), Jaimovich and Rebelo (2006) and Christiano, Ilut, Motto and Rostagno (2007), whose models predict an increase in aggregate output, employment, investment and consumption in response to news of future technological improvement.

2 The model

We consider an economy populated with infinitely lived households, overlapping generations of finitely lived entrepreneurs, and a central bank. For simplicity, we limit our analysis to outcomes symmetric across entrepreneurs and across households, *i.e.* there is one representative household and, in each generation, one representative entrepreneur. Time is discrete and there is a single good that is non-storable and can be consumed or invested.

2.1 Technology

A production project requires κ_t units of good in period t, the investment period, and delivers $Y_{t+N} = A_{t+N} L_{t+N}^{\alpha}$ units of good in period $t+N$, where A_{t+N} is a productivity parameter, L_{t+N} is labor services and $0 < \alpha < 1$.

At a given period t, different technologies might be available. A technology $z \in \mathbb{R}$ is defined by a period t investment cost $\kappa_t = \kappa(z)$ and by a period $t+N$ productivity parameter $A_{t+N} = A(z)$. $\mathcal{F}_t$ is the set of technologies available at period t, which includes the particular case $z = 0$ corresponding to the absence of any production project, with $\kappa(0) = 0$ and $A(0) = 0$.

A production project needs a newborn entrepreneur to be undertaken, and an entrepreneur cannot undertake more than one project at the same time.

2.2 Preferences

The representative household supplies inelastically one unit of labor per period, and her preferences are represented by the following utility function:

$$U_t = E_{\Omega(h,t)} \sum_{j=0}^{\infty} \beta^j \ln(c_{t+j})$$

where $\Omega(h,t)$ is her information set at date t, c_t her consumption at date t, and $\beta \in (0,1)$. We choose a logarithmic utility to simplify the algebra.

At each date, one representative entrepreneur is born, and lives for $N+1$ periods. She consumes only in her last period of life and has linear preferences. The utility of an entrepreneur born at date

t is given by:

$$V_t = \beta^N E_{\Omega(e,t)} c^e_{t+N}$$

where $\Omega(e,t)$ is her information set and c^e_t her consumption at date t. We assume that each generation contains a large number of entrepreneurs, so that the representative entrepreneur is price-taker.

2.3 Market organization

We present here the real economy under Laissez-Faire. We will later introduce public intervention, which will receive an interpretation as monetary policy. All markets (good, labor, bonds) are competitive. The final good is the numéraire. A newborn entrepreneur may want to borrow κ to undertake a production project. The return from this investment will be the profit she will obtain from production N periods onwards. We assume that the only financial market that is opened is a market for N-period bonds. Both the household and the entrepreneurs have access to this market, and there is secondary market for those bonds. We denote B_{t+N} the number of bonds that pay in period $t+N$, and that has been subscribed by the household in period t. Each of this bond will pay one unit of good in period $t+N$, and its price is denoted q_t. B^e_t is the number of bonds emitted by the entrepreneurs.

2.4 Resource constraints

The resource constraint on the good market states that the total number of goods consumed and invested cannot be larger than the total amount of goods available in a given period. Let $z_t \in \mathcal{F}_t$ denote the technology chosen by the entrepreneur at date t. We have:

$$c_t + c^e_t + \kappa(z_t) \leq Y_t$$

Labor services cannot exceed the total amount of labor that is supplied, so that

$$L_t \leq 1$$

2.5 Agents programs

The household enters period t with a portfolio $S_{t-1} = (B_t, ..., B_{t+n}, ..., B_{t+N-1})$ of bonds, that pay interest if at maturity. The household then decides how much to consume and how much to save,

supplying inelastically one unit of labor. The household program can be recursively written in the following way:

$$W(S_{t-1}) = \max_{c_t, B_{t+N}} \left\{ \ln(c_t) + \beta E_{\Omega(h,t)} W(S_t) \right\}$$
$$\text{subject to} \left\{ \begin{array}{l} c_t + q_t B_{t+N} \leq B_t + w_t L_t^s \\ L_t^s = 1 \end{array} \right. ,$$

where w_t is the wage rate at date t. The houshold's optimality conditions can be reduced to

$$\begin{aligned} q_t &= \beta^N E_{\Omega(h,t)} \left[\frac{c_t}{c_{t+N}} \right], \\ L_t^s &= 1, \end{aligned}$$

and a transversality condition.

Let us now consider the optimal behavior of the representative newborn entrepreneur of period t. She borrows $\kappa(z_t)$ in period t, hires L_{t+N} and produces Y_{t+N} in period $t+N$. Note that $z_t = 0$ is a possible option, which corresponds to no investment in t and no production in $t+N$. Production proceeds are used to pay wages, reimburse the debt and to consume. Budget constraints of an active entrepreneur born in period t are therefore

$$\begin{array}{lcl} \kappa(z_t) & \leq & q_t B_{t+N}^e, \\ c_{t+N} + B_{t+N}^e & \leq & \Pi_{t+N} = A(z_t) L_{t+N}^{\alpha} - w_{t+N} L_{t+N}. \end{array}$$

Labor demand L will be set such that marginal productivity of labor equalizes the real wage w:

$$w_{t+N} = \alpha A(z_t) L_{t+N}^{\alpha-1}.$$

Therefore, the intertemporal utility of a new-born entrepreneur is

$$V = max_{z_t \in \mathcal{F}_t} V^e(z_t) = max_{z_t \in \mathcal{F}_t} \beta^N E_{\Omega(e,t)} \left(\Pi_{t+N} - \frac{1}{q_t} \kappa(z_t) \right).$$

2.6 Competitive equilibrium

In this economy, a competitive equilibrium is a sequence of prices $\{q_t, w_t\}$ and quantities $\{B_t, B_t^e, c_t, c_t^e, L_t\}$ $\forall\ t \geq 1$ such that, given initial conditions and for an exogenous sequence of technological possibilities $\mathcal{F}_t$, *(i)* household's consumption and bonds holding solve her maximization problem given prices,

(ii) investment decision z_t maximizes expected intertemporal utility of newborn entrepreneurs given prices, *(iii)* labor demand L_t maximizes aged $N+1$ entrepreneurs profits given prices and *(iv)* labor, bonds and good markets clear.

2.7 Discussion

In the set of assumptions that we have made, some are needed for the analytical tractability of the model, while some are less restrictive than what they look. First, we consider risk-neutral entrepreneurs that consume only in the last period of their life. This assumption is crucial in order to solve the model when we introduce private information and potential informational cascades. Second, we have introduced bonds of maturity N only. This is not a restriction since other maturity bonds would not be traded. Third, we assume that only non-contingent debt contracts are possible. This assumption is *a priori* restrictive because, when we introduce entrepreneurs' private signals, households might prefer to propose contingent debt contracts to entrepreneurs in order to gain some information about their private signal. That would complicate the analysis since the desirability of a monetary policy intervention would then depend on the social cost of these contingent debt contracts. Fourth, we consider investment projects that pay only N periods ahead. This assumption will be crucial when we introduce private signals and the possibility of informational cascades.

3 Competitive equilibrium with public information

In this section, we consider economies with public information only. In tranquil times, there is only one technology available, and we study the existence, uniqueness and stability of the steady state. We then introduce technological change: a new technology is available today, that will happen to be more productive or not N periods onwards. We study the properties of the equilibrium path in such a case. The results that we obtain will be useful for the analysis of the private-information case.

3.1 Tranquil times

We assume that, in tranquil times, only one technology is available ($\forall t \in \mathbb{Z}$, $\mathcal{F}_t = \{0, \overline{z}\}$). We also assume that there exists $\overline{c} > 0$ such that there exists an equilibrium at which $\forall t \in \mathbb{Z}$, $z_t = \overline{z}$ and $c_t = \overline{c}$. A necessary and sufficient condition for that is

$$\beta^N (1-\alpha) A(\overline{z}) - \kappa(\overline{z}) > 0 \tag{1}$$

$$\text{and } \alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N}\kappa(\overline{z}) > 0. \tag{2}$$

Indeed, if $\forall t \in \mathbb{Z}$, $c_t = \overline{c}$, then $\forall t \in \mathbb{Z}$, $q_t = \beta^N \equiv \overline{q}$, *i.e.* the N-period interest factor is $R_t = \overline{q}^{-1} = \beta^{-N} \equiv \overline{R}$. If, moreover, $\forall t \in \mathbb{Z}$, $z_t = \overline{z}$, then the labor market equilibrium condition implies $\forall t \in \mathbb{Z}$, $w_t L_t = \alpha A(\overline{z})$ and $\Pi_t = (1-\alpha) A(\overline{z})$, from which we deduce $\overline{c} = \alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N}\kappa(\overline{z})$, which is strictly positive if and only if (2) holds. And entrepreneurs are active if and only if $V(\overline{z}) > V(0)$, that is to say if and only if (1) holds.

Note that, under conditions (1) and (2), this steady-state equilibrium is stable in the sense that if $\forall t \in \mathbb{Z}^-$, $z_t = \overline{z}$, $c_t = \overline{c}$ and $q_t = \overline{q}$, then there exists a unique equilibrium at which $\forall t \in \mathbb{N}^*$, $z_t = \overline{z}$, $c_t = \overline{c}$ and $q_t = \overline{q}$. Indeed, suppose that $\forall t \in \mathbb{Z}^-$, $z_t = \overline{z}$, $c_t = \overline{c}$ and $q_t = \overline{q}$. If there existed $t \in \{1, ..., N\}$ such that $z_t = 0$, then we would get $c_{t+N} = 0$. But since $c_t > 0$ due to (2), that would imply $q_t \longrightarrow +\infty$ and hence $z_t = \overline{z}$, which would be contradictory. Therefore, $\forall t \in \{1, ..., N\}$, $z_t = \overline{z}$, $c_t = \overline{c}$ and $q_t = \overline{q}$, and by recurrence we easily obtain that $\forall t \in \mathbb{N}^*$, $z_t = \overline{z}$, $c_t = \overline{c}$ and $q_t = \overline{q}$.

We also assume that this steady-state equilibrium is locally dynamically stable in the sense that there exist some neighborhoods $\mathcal{N}_{\overline{c}}$ of $\overline{c}$ and $\mathcal{N}_{\overline{q}}$ of $\overline{q}$ such that if $\forall t \in \mathbb{Z}^-$, $z_t = \overline{z}$, $c_t \in \mathcal{N}_{\overline{c}}$ and $q_t \in \mathcal{N}_{\overline{q}}$, then there exists an equilibrium at which $\forall t \in \mathbb{N}^*$, $z_t = \overline{z}$, $c_t \in \mathcal{N}_{\overline{c}}$, $q_t \in \mathcal{N}_{\overline{q}}$ and $(c_t, q_t) \longrightarrow (\overline{c}, \overline{q})$ as $t \longrightarrow +\infty$. A necessary and sufficient condition for that is

$$\beta^N > \frac{\kappa(\overline{z})}{|\alpha A(\overline{z}) - \kappa(\overline{z})|}. \tag{3}$$

Indeed, if $\forall t \in \mathbb{Z}$, $z_t = \overline{z}$, then $\forall t \in \mathbb{Z}$,

$$q_t = \beta^N \frac{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_{t-N}}}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_t}}$$

and hence

$$q_t - \beta^N = \frac{-\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \frac{q_{t-N} - \beta^N}{q_{t-N}},$$

so that any sequence $(q_t)_{t\in\mathbb{Z}}$ originating in the neighborhood of $\overline{q}$ will converge towards $\overline{q}$ if and only if (3) holds.

3.2 Technological change

We now consider the response of the economy to the unexpected availability of a new technology. We assume that the economy is at its steady state until date 0 included ($\forall t \in \mathbb{Z}^-$ and $\forall k \in \mathbb{N}$, $\mathcal{F}_t = \{0, \overline{z}\}$, $E_t(\mathcal{F}_{t+k}) = \{0, \overline{z}\}$, $z_t = \overline{z}$, $c_t = \overline{c}$ and $q_t = \overline{q}$) and that a new technology $z > \overline{z}$ becomes available from date 1 onwards ($\forall t \in \mathbb{N}^*$ and $\forall k \in \mathbb{N}$, $\mathcal{F}_t = \{0, \overline{z}, z\}$ and $E_t(\mathcal{F}_{t+k}) = \{0, \overline{z}, z\}$). This new technology requires more investment than the old one: $\kappa(z) > \kappa(\overline{z})$. It may be "good" and lead N periods later to a productivity parameter $A(z) > A(\overline{z})$, or it may be "bad" and lead N periods later to the same productivity parameter $A(\overline{z})$ as the old technology $\overline{z}$: in words, entrepreneurs invest more to raise labour productivity, but labour productivity remains unchanged. We note p the probability that the new technology is good and assume that this probability is common knowledge at dates 1 to N. We also assume that whether the new technology is good or bad becomes common knowledge at date $N+1$ even if there has been no investment in the new technology at date 1.

At each date $t \in \mathbb{N}^*$, the representative newborn entrepreneur chooses $z_t \in \{0, \overline{z}, z\}$ and borrows $q_t^{-1}\kappa(z_t)$. The representative entrepreneurs born at dates $-(N-1)$ to 0 pay back their debts at dates 1 to N at the interest factor $\overline{R}$. Therefore, $\forall t \in \{1, ..., N\}$, the sequence of the representative household's consumption is $c_t = \alpha A(\overline{z}) - \kappa(z_t) + \beta^{-N}\kappa(\overline{z})$. We restrict our analysis to equilibria at which $\forall t > N$, $z_t = z$ if the new technology turns out to be good and $z_t = \overline{z}$ otherwise. As a consequence, we have: $\forall t > N$, $c_t = \alpha A(z) - \kappa(z) + q_{t-N}^{-1}\kappa(z)$ if $z_{t-N} = z$ and the new technology is good, $c_t = \alpha A(\overline{z}) - \kappa(z) + q_{t-N}^{-1}\kappa(z)$ if $z_{t-N} = z$ and the new technology is bad, $c_t = \alpha A(\overline{z}) - \kappa(z) + q_{t-N}^{-1}\kappa(\overline{z})$ if $z_{t-N} = \overline{z}$ and the new technology is good, and $c_t = \alpha A(\overline{z}) - \kappa(\overline{z}) + q_{t-N}^{-1}\kappa(\overline{z})$ if $z_{t-N} = \overline{z}$ and the new technology is bad.

We focus on the real-interest-rate transmission channel of monetary policy, *i.e.* monetary policy has an effect on the economy only through its effect on the real interest rate. Technically speaking, this can be done by modeling monetary policy as a tax (or subsidy) on lending together with a positive (or negative) lump-sum transfer to the representative household. More precisely, at date t the representative household lends $q_t B_{t+N}$ to the entrepreneur and gives $(\tau_t - 1) q_t B_{t+N}$ to the central bank (when $\tau_t > 1$) or receives $-(\tau_t - 1) q_t B_{t+N}$ from the central bank (when $0 < \tau_t < 1$), while the central bank gives a lump-sum transfer $T_t \equiv (\tau_t - 1) q_t B_{t+N}$ to the representative household (when $\tau_t > 1$) or receives a lump-sum transfer $T_t \equiv -(\tau_t - 1) q_t B_{t+N}$ from the representative household (when $0 < \tau_t < 1$). The intertemporal budget constraint of the representative household thus becomes at date t

$$c_t + \tau_t q_t B_{t+N} \leq B_t + w_t L_t + T_t.$$

This implies

$$\tau_t q_t = \beta^N E_{\Omega(h,t)} \left[\frac{c_t}{c_{t+N}} \right].$$

In particular, for $t \in \{1, ..., N\}$, when $z_t = \overline{z}$, we get

$$\tau_t q_t = \beta^N \left[\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N} \right] \left[\frac{p}{\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{q_t}} + \frac{1-p}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_t}} \right]. \tag{4}$$

Noting

$$\forall x \geq \overline{z},\ \tau^u(x) \equiv \frac{\beta^N [\alpha A(\overline{z}) - \kappa(x)] + \kappa(\overline{z})}{\kappa(x)},$$

we obtain the following proposition concerning q_t for $t \in \{1, ..., N\}$ when $z_t = \overline{z}$:

Proposition 1: i) there exists a strictly positive real number q_t solution of (4) for all $p \in [0; 1]$ if and only if $\alpha A(\overline{z}) - \kappa(z) > 0$ and $\tau_t < \tau^u(\overline{z})$; ii) then q_t, which we note $q(z, \tau_t, p, 0)$, is unique, and $\frac{\partial q(z,\tau_t,p,0)}{\partial \tau_t} < 0$ and $\frac{\partial q(z,\tau_t,p,0)}{\partial p} > 0$.

Proof: *cf* appendix A. ■

We therefore make the following assumptions:

$$\alpha A(\overline{z}) - \kappa(z) > 0, \tag{5}$$

$$\forall t \in \{1, ..., N\}, \ \tau_t < \tau^u(\overline{z}). \tag{6}$$

Note that $\tau^u(\overline{z}) > 1$, so that (6) is satisfied in the absence of monetary policy intervention. The result $\frac{\partial q(z,\tau,p,0)}{\partial \tau} < 0$ is simply due to the fact that a positive tax on lending (*i.e.* a monetary policy tightening) raises the interest rate and therefore lowers q_t. The result $\frac{\partial q(z,\tau,p,0)}{\partial p} > 0$ is due to the fact that if entrepreneurs invest in the old technology at date t, then, as p increases, c_t remains unchanged but $E_t\{\frac{1}{c_{t+N}}\}$ increases (because the representative household is expected to lend more, and hence to consume less, at date $t+N$), so that q_t increases.

Alternatively, for $t \in \{1, ..., N\}$, when $z_t = z$, we get

$$\tau_t q_t = \beta^N \left[\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{\beta^N}\right] \left[\frac{p}{\alpha A(z) - \kappa(z) + \frac{\kappa(z)}{q_t}} + \frac{1-p}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(z)}{q_t}}\right] \tag{7}$$

and we obtain the following proposition concerning q_t for $t \in \{1, ..., N\}$ when $z_t = z$:

Proposition 2: i) there exists a strictly positive real number q_t solution of (7) for all $p \in [0; 1]$ if and only if $\tau_t < \tau^u(z)$; ii) then q_t, which we note $q(z, \tau_t, p, 1)$, is unique, and $\frac{\partial q(z,\tau_t,p,1)}{\partial \tau_t} < 0$, $\frac{\partial q(z,\tau_t,p,1)}{\partial p} > 0$ if $\alpha A(\overline{z}) - \kappa(\overline{z}) > \alpha A(z) - \kappa(z)$, $\frac{\partial q(z,\tau_t,p,1)}{\partial p} = 0$ if $\alpha A(\overline{z}) - \kappa(\overline{z}) = \alpha A(z) - \kappa(z)$ and $\frac{\partial q(z,\tau_t,p,1)}{\partial p} < 0$ if $\alpha A(\overline{z}) - \kappa(\overline{z}) < \alpha A(z) - \kappa(z)$.

Proof: *cf* appendix B. ■

We therefore make the following assumption:

$$\forall t \in \{1, ..., N\}, \ \tau_t < \tau^u(z). \tag{8}$$

Note that $\forall z > \overline{z}$, $\tau^u(\overline{z}) > \tau^u(z)$, so that (8) implies (6). As previously, the result $\frac{\partial q(z,\tau_t,p,1)}{\partial \tau_t} < 0$ is simply due to the fact that a positive tax on lending (*i.e.* a monetary policy tightening) raises the interest rate and therefore lowers q_t. The result $\frac{\partial q(z,\tau_t,p,1)}{\partial p} \lesseqgtr 0$ is due to the fact that if entrepreneurs

invest in the new technology at date t, then, as p increases, c_t remains unchanged but $E_t\{\frac{1}{c_{t+N}}\}$ either increases or decreases depending on the sign of $[\alpha A(z) - \alpha A(\overline{z})] - [\kappa(z) - \kappa(\overline{z})]$ (because the representative household is expected both to lend more, as $\kappa(z) > \kappa(\overline{z})$, and to receive a higher wage, as $\alpha A(z) > \alpha A(\overline{z})$, at date $t+N$), so that q_t either increases or decreases depending on the sign of $[\alpha A(z) - \alpha A(\overline{z})] - [\kappa(z) - \kappa(\overline{z})]$.

Assuming that $\forall t > N$, $\tau_t = 1$ (this assumption will be justified in the next section), we also obtain the following proposition concerning q_t for $t > N$:

Proposition 3: q_t is unique and strictly positive for all $t > N$ and all $p \in [0;1]$, and $\lim_{t \longrightarrow +\infty} q_t = \beta^N$ for all $p \in [0;1]$, if and only if conditions (17), (18), (19) and (20) are satisfied.

Proof: *cf* appendix C. ■

We therefore assume that parameters are such that (17), (18), (19) and (20) hold. Note that, given (5), this implies that c_t is unique and strictly positive for all $t > N$ and all $p \in [0;1]$.

Finally, noting $\tau^l(z) \equiv 0$ if $A(z)\kappa(\overline{z}) \geq A(\overline{z})\kappa(z)$ and

$$\tau^l(z) \equiv \frac{\kappa(\overline{z})[\kappa(z) - \kappa(\overline{z})] - [\alpha A(z) - \alpha A(\overline{z})]\left[\kappa(\overline{z})\left(1 - \beta^N\right) + \alpha A(\overline{z})\beta^N\right]}{\alpha A(\overline{z})\kappa(z) - \alpha A(z)\kappa(\overline{z})}$$

if, alternatively, $A(z)\kappa(\overline{z}) < A(\overline{z})\kappa(z)$, we obtain the following proposition:

Proposition 4: $q(z,\tau,p,0) > q(z,\tau,p',1)$ for all $(p,p') \in [0;1]^2$ if and only if $\tau > \tau^l(z)$.

Proof: *cf* appendix D. ■

We make the following assumption:

$$\forall t \in \{1,...,N\}, \tau_t > \tau^l(z), \tag{9}$$

so that we get $\forall (p,p') \in [0;1]^2$, $q(z,\tau,p,0) > q(z,\tau,p',1)$, which will simplify the analysis in the following section. In words, that means that the interest rate maximized over $p \in [0;1]$ that prevails

when the entrepreneurs borrow little (as they invest in the old technology) is strictly lower than the interest rate minimized over $p \in [0;1]$ that prevails when the entrepreneurs borrow much (as they invest in the new technology). Note that $\forall z > \overline{z}$, $\tau^l(z) < 1$, so that (9) is satisfied in the absence of monetary policy intervention.

4 Competitive equilibrium with private information

We assume that the economy is initially at the steady state with a technology $\overline{z}$. Therefore, the investment cost and the interest rate are respectively: $\kappa_{-i} = \kappa(\overline{z})$ and $q_{-i} = \beta^N$ for $i \geq 0$. Also recall that for simplicity we limit our analysis to outcomes symmetric across entrepreneurs and across households *i.e.* there is one representative household and, in each generation, one representative entrepreneur.

4.1 General case

Information accumulation is modelled as follows:

- The representative entrepreneur starts period t with the public information available at time $t-1$. Public information is derived from the observation of the investment behavior of the previous representative entrepreneurs. We note μ_{t-1} the probability that the new technology is good based on public information available at date $t-1$. μ_{t-1} is therefore the prior at time t.

- At time t, the representative entrepreneur receives a private signal S_t. This signal can be either good, then $S_t = 1$, or bad, then $S_t = 0$. This signal is then incorporated into the private information available for the representative entrepreneur at time t. We note $\widetilde{\mu}_t$ the probability that the new technology is good based on private information available at date t, which includes S_t if $t > 0$. $\widetilde{\mu}_t$ is therefore the posterior at time t. Let λ denote the probability that a signal, whether good or bad, is right. According to Bayes' theorem:

$$\widetilde{\mu}_t = S_t \frac{\mu_{t-1}\lambda}{\mu_{t-1}\lambda + \left(1-\mu_{t-1}\right)(1-\lambda)} + (1-S_t)\frac{\mu_{t-1}(1-\lambda)}{\mu_{t-1}(1-\lambda) + \left(1-\mu_{t-1}\right)\lambda}$$

Moreover, let us note $\widetilde{\mu}_t^0$ the value taken by $\widetilde{\mu}_t$ when $S_t = 0$ and $\widetilde{\mu}_t^1$ the value taken by $\widetilde{\mu}_t$ when $S_t = 1$ for $t \in \{1, ..., N\}$.

- Still at time t, investment decision I_t is taken. If the representative investor invests in the new technology then we note $I_t = 1$; if she invests the old technology, $I_t = 0$. This decision is public information that is available at time t. Therefore, the probability that the new technology is good based on public information available at date t, which we note μ_t includes I_t if $t > 0$. According to this information structure, the interest rate becomes: $q_t = q(z, \tau_t, \mu_t, I_t)$.

Noting $p_0 \equiv \mu_0$,

$$p_i \equiv \frac{p_{i-1}\lambda}{p_{i-1}\lambda + (1 - p_{i-1})(1 - \lambda)} \text{ and } p_{-i} \equiv \frac{p_{-i+1}(1 - \lambda)}{p_{-i+1}(1 - \lambda) + (1 - p_{-i+1})\lambda} \text{ for } i \in \mathbb{N}^*,$$
$$B(z) \equiv \frac{\kappa(z) - \kappa(\overline{z})}{(1 - \alpha)[A(z) - A(\overline{z})]} \text{ for } z > \overline{z} \text{ and } B(\overline{z}) \equiv \frac{\frac{d\kappa}{dz}\big|_{z=\overline{z}}}{(1 - \alpha)\frac{dA}{dz}\big|_{z=\overline{z}}},$$

we obtain the following proposition:

Proposition 5: in equilibrium, $\forall t \in \{1, ..., N\}$, there are only three possibilities, and these possibilities are mutually exclusive: i) either $\widetilde{\mu}_t^1 q\left(z, \tau_t, \mu_{t-1}, 0\right) < B(z)$, then $I_t = 0$ whatever $S_t \in \{0, 1\}$ and $\mu_t = \mu_{t-1}$; ii) or $\widetilde{\mu}_t^0 q\left(z, \tau_t, \mu_{t-1}, 1\right) > B(z)$, then $I_t = 1$ whatever $S_t \in \{0, 1\}$ and $\mu_t = \mu_{t-1}$; iii) or $\widetilde{\mu}_t^0 q\left(z, \tau_t, \widetilde{\mu}_t^0, 0\right) < B(z)$ and $\widetilde{\mu}_t^1 q\left(z, \tau_t, \widetilde{\mu}_t^1, 1\right) > B(z)$, then $I_t = S_t$ whatever $S_t \in \{0, 1\}$ and $\mu_t = \widetilde{\mu}_t$.

Proof: *cf* appendix E. ■

This implies in particular that $\forall t \in \{1, ..., N\}$, $\exists i \in \mathbb{Z}$, $\widetilde{\mu}_t = p_i$ and $\mu_t \in \{p_{i-1}, p_i, p_{i+1}\}$.

Note that we can rewrite the necessary and sufficient condition for the monetary policy intervention to ensure the absence of cascade at date t in an alternative way. Indeed, from (4) and (7), it is easy to see that, whatever $z \geq \overline{z}$, $p \in [0; 1]$ and $Q > 0$, there exists a unique $\tau > 0$ such that $q(z, \tau, p, 0) = Q$ and there exists a unique $\tau > 0$ such that $q(z, \tau, p, 1) = Q$. Let us note

$$\tau^{l*}(z, p) \equiv \beta^N \left[\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N}\right] \left[\frac{p}{[\alpha A(\overline{z}) - \kappa(z)]\frac{B(z)}{p} + \kappa(\overline{z})} + \frac{1 - p}{[\alpha A(\overline{z}) - \kappa(\overline{z})]\frac{B(z)}{p} + \kappa(\overline{z})}\right]$$

the unique value of τ such that $q(z,\tau,p,0)=\frac{B(z)}{p}$ and

$$\tau^{u*}(z,p)\equiv\beta^{N}\left[\alpha A(\overline{z})-\kappa(z)+\frac{\kappa(\overline{z})}{\beta^{N}}\right]\left[\frac{p}{[\alpha A(z)-\kappa(z)]\frac{B(z)}{p}+\kappa(z)}+\frac{1-p}{[\alpha A(\overline{z})-\kappa(\overline{z})]\frac{B(z)}{p}+\kappa(z)}\right]$$

the unique value of τ such that $q(z,\tau,p,1)=\frac{B(z)}{p}$. Since $\frac{\partial q(z,\tau,p,0)}{\partial\tau}<0$ and $\frac{\partial q(z,\tau,p,1)}{\partial\tau}<0$ (as stated in propositions 1 and 2), the necessary and sufficient condition for the monetary policy intervention to ensure the absence of cascade at date t can therefore be rewritten $\tau^{l*}\left(z,\widetilde{\mu}_t^0\right)<\tau_t<\tau^{u*}\left(z,\widetilde{\mu}_t^1\right)$.

In order to illustrate the mechanism of monetary policy intervention, suppose for a moment that there exists $t\in\{1,...,N\}$ at which there is an upward cascade under laisser-faire, *i.e.* that there exists $t\in\{1,...,N\}$ such that $\widetilde{\mu}_t^0 q\left(z,1,\mu_{t-1},1\right)>B(z)$. Then, as a direct application of proposition 5, a necessary condition for the monetary policy intervention to interrupt the cascade at date t is $\widetilde{\mu}_t^0 q\left(z,\tau_t,\widetilde{\mu}_t^0,0\right)<B(z)$. Given proposition 4, $\widetilde{\mu}_t^0 q\left(z,1,\mu_{t-1},1\right)>B(z)$ implies $\widetilde{\mu}_t^0 q\left(z,1,\widetilde{\mu}_t^0,0\right)>B(z)$. Since $\frac{\partial q(z,\tau,p,0)}{\partial\tau}<0$ (as stated in proposition 1), $\tau_t>1$ is therefore a necessary condition for the monetary policy intervention to interrupt the cascade at date t. In other words, monetary policy must be tightened to interrupt an upward cascade.

Lastly, two points are worth noting about monetary policy. First, our assumption that $\tau_t=1$ for all $t>N$ is not restrictive because, whatever happened until date N, there is no rationale for intervening after date N, as there is then no financial market imperfection anymore. Second, there is no problem of time-inconsistency for monetary policy since $\forall t\in\{1,...,N\}$, the private agents' decisions at date t depend on τ_t but not on $E_t\{\tau_{t+k}\}$ for $k>0$.

4.2 A simple case

In order to focus on analytically tractable informational cascades, we assume here that $N=3$ and z is arbitrarily close to $\overline{z}$. This implies that $\forall p\in[0;1]$, $\forall I\in\{0,1\}$, $q(z,1,p,I)$ is arbitrarily close to $q(\overline{z},1,p,I)=\beta^3$. We also impose the following condition on the parameters which, given proposition 5, is necessary for an arbitrarily small intervention to be enough to avoid the cascade with a prior p_1:

$$\left.\frac{dA}{dz}\right|_{z=\overline{z}}=\frac{1}{(1-\alpha)\,p_0\beta^3}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}. \tag{10}$$

We also assume that, under laisser-faire ($\tau_1 = \tau_2 = \tau_3 = 1$), there is no cascade at $t = 1$ and a cascade at $t = 2$ when $S_1 = 1$. The necessary and sufficient condition for that is

$$\frac{\beta^3 \left[(1-\alpha)\, \beta^3 p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2 \kappa}{dz^2}\right|_{z=\overline{z}} \right]}{2 \left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \right)^2} > \frac{1 + \beta^N (1 - p_1) + \frac{\alpha}{1-\alpha} \frac{p_1}{p_0}}{\alpha A(\overline{z}) - \kappa(\overline{z})}. \tag{11}$$

Indeed, given proposition 5, there is no cascade at $t = 1$ and a cascade at $t = 2$ when $S_1 = 1$ under laisser-faire if and only if $p_{-1} q(z, 1, p_{-1}, 0) < B(z)$, $p_1 q(z, 1, p_1, 1) > B(z)$ and $p_0 q(z, 1, p_1, 1) > B(z)$, that is to say, given proposition 4, if and only if $p_{-1} q(z, 1, p_{-1}, 0) < B(z)$ and $p_0 q(z, 1, p_1, 1) > B(z)$. The first condition is satisfied since $B(\overline{z}) = p_0 \beta^3$, due to (10), and since $q(z, 1, p_{-1}, 0)$ is arbitrarily close to β^3. The second condition is satisfied if and only if

$$p_0 \left.\frac{\partial q(z, 1, p_1, 1)}{\partial z}\right|_{z=\overline{z}} > \left.\frac{dB}{dz}\right|_{z=\overline{z}}. \tag{12}$$

At date $t = 2$, under laisser-faire ($\tau_2 = 1$), the derivation of (7) with respect to z, taken at point $z = \overline{z}$, and the use of (10) lead to

$$\left.\frac{\partial q(z, 1, p_1, 1)}{\partial z}\right|_{z=\overline{z}} = -\frac{\left[1 + \beta^N (1 - p_1)\right] + \frac{\alpha p_1}{(1-\alpha) p_0}}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}.$$

Besides, using (10), we also get

$$\left.\frac{dB}{dz}\right|_{z=\overline{z}} = \frac{\beta^N p_0 \left[\left.\frac{d^2 \kappa}{dz^2}\right|_{z=\overline{z}} - (1-\alpha)\, \beta^N p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} \right]}{2 \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}. \tag{13}$$

Using the last two results to rewrite (12) leads to condition (11).

We focus on the case where $S_1 = 1$, which implies $\mu_1 = \widetilde{\mu}_1 = p_1$ (and, if $\tau_2 = 1$, $\mu_2 = p_1$). For simplicity, we consider the possibility of intervention only at time $t = 2$ ($\tau_1 = \tau_3 = 1$)[5], and we focus on the "minimal" intervention to avoid a cascade at date $t = 2$, that is to say, of all the values of τ_2 such that there is no cascade at time $t = 2$, the one closest to 1 or, equivalently here, of all the values of $\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}}$ such that there is no cascade at time $t = 2$, the one closest to 0. Our objective is

[5] There is no rationale to intervene at date $t = 1$, since there is no cascade at that date, but there could be a rationale to intervene at date $t = 3$, even though making S_3 public would not benefit future entrepreneurs (since the true productivity of the new technology is known at date $t = 4$ anyway): indeed, making S_3 public could benefit the household.

to examine whether this particular intervention is welfare-improving, not to determine the optimal monetary policy[6]. Since $\tau^{l*}(\overline{z}, p_0) = 1 < \tau^{u*}(\overline{z}, p_2)$, this "minimal" intervention is

$$\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = \left.\frac{\partial \tau^{l*}(z, p_0)}{\partial z}\right|_{z=\overline{z}}$$

which, using (13), leads to

$$\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = \frac{\beta^N \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{\beta^N \left[\alpha A(\overline{z}) - \kappa(\overline{z})\right] + \kappa(\overline{z})} \left[p_0 - \frac{\left[\alpha A(\overline{z}) - \kappa(\overline{z})\right] \left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}} - (1-\alpha)\beta^N p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} \right]}{2\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2} \right]. \tag{14}$$

From proposition 5 we also get

$$\left.\frac{dq(z, \tau_2(z), p_0, 0)}{dz}\right|_{z=\overline{z}} = \frac{1}{p_0} \left.\frac{dB}{dz}\right|_{z=\overline{z}} = \frac{\beta^N \left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}} - (1-\alpha)\beta^N p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} \right]}{2 \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}. \tag{15}$$

Finally, the derivation of (7) at date $t = 2$ with respect to z, taken at point $z = \overline{z}$, and the use of (14) lead to

$$\left.\frac{dq(z, \tau_2(z), p_2, 1)}{dz}\right|_{z=\overline{z}} =$$
$$-\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \left\{ \frac{1 + \beta^N (1 + p_0 - p_2) + \frac{\alpha}{1-\alpha}\frac{p_2}{p_0}}{\alpha A(\overline{z}) - \kappa(\overline{z})} + \frac{\beta^N \left[(1-\alpha)\beta^N p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}} \right]}{2\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2} \right\}. \tag{16}$$

In order to determine whether such an intervention increases welfare, we compare the welfare of both investors and households under this intervention (superscript I) and under laisser-faire (superscript LF). To consider social welfare, we also introduce weights applied to households and entrepreneurs such that aggregate utility corresponds to GDP in this linearized case. Let p_A be the probability of receiving a signal $S_2 = 1$, *i.e.* $p_A = p_1\lambda + (1 - p_1)(1 - \lambda)$, and p_B be the probability of receiving a signal $S_3 = 1$ once having received $S_2 = 0$, ie. $p_B = p_0\lambda + (1 - p_0)(1 - \lambda)$.

In the case of laissez-faire (*cf* appendix F), we obtain:

$$\frac{\kappa(\overline{z}) + \beta^3 \left[\alpha A(\overline{z}) - \kappa(\overline{z})\right]}{\beta^3} \left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} + \left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta} \left[\frac{p_1}{(1-\alpha)p_0} - 1 + \beta^3 (1 - p_1) \right] > 0,$$

[6] Note that this "minimal" intervention may not be the optimal intervention even among the set of interventions at date $t = 2$. Indeed, we know that social welfare is maximal under laisser-faire in the fictitious situation of public (rather than private) signals; any arbitrarily small intervention reduces social welfare in this fictitious situation, *i.e.* there is a neighbourhood of laisser-faire within which increasing intervention decreases social welfare in this situation; but as z becomes arbitrarily close to $\overline{z}$ the size of this neighbourhood may tend towards zero; hence our "minimal" intervention (not to mention interventions of greater size), which reveals the private signal, may fall outside this neighbourhood.

$$\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} > 0,$$

and we find that $\left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}}$ is positive for $\frac{p_1}{p_0}$ sufficiently large (as can be easily checked, using (1)) and negative for p_0 sufficiently close to 1. How can we get $\left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}} < 0$ though each entrepreneur individually gains from investing in the new technology? There are three possible reasons: i) entrepreneurs investing at dates 1 to N do not internalize the possible costs (in terms of interest-rate fluctuations) that their actions will impose on the following entrepreneurs; ii) entrepreneurs investing at dates $t > N$ do not internalize the possible costs (again, via the interest rate) that their actions impose on the entrepreneurs investing at dates 1 to N; iii) if there were only one entrepreneur per generation, then she would choose between borrowing little at a low rate or borrowing much at a high rate, and might prefer to borrow little at a low rate; but there are many of them, so that each of them, taking the interest rate as given, has either to choose between borrowing little or much at a low rate, or to choose between borrowing little or much at a high rate; if in both cases she prefers to borrow much, then the only stable symmetric equilibrium is that all entrepreneurs borrow much at a high rate.

In the case of intervention (*cf* appendix G), we obtain:

$$\frac{\kappa(\overline{z}) + \beta^3 [\alpha A(\overline{z}) - \kappa(\overline{z})]}{\beta^3} \left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}} + \left.\frac{dV_1^I}{dz}\right|_{z=\overline{z}} = \left\{ \left[\frac{p_1}{(1-\alpha)p_0} - 1\right] + \frac{\beta^3}{1-\beta} p_1 \left[\frac{1}{(1-\alpha)p_0} - 1\right] \right.$$
$$\left. + \beta(1+\beta) p_A \left[\frac{p_2}{(1-\alpha)p_0} - 1\right] + \beta^2 (1-p_A) p_B \left[\frac{p_1}{(1-\alpha)p_0} - 1\right] \right\} \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} > 0,$$

$$\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}} > 0,$$

and we find again that $\left.\frac{dV_1^I}{dz}\right|_{z=\overline{z}}$ is positive if the ratio between the discounted sum of future probabilities for the technology to be good $(p_1, p_A p_2, (1-p_A) p_B p_1)$ and p_0 is sufficiently large, and negative for p_0 sufficiently close to 1. Reasons for having $\left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}} < 0$ are pretty much the same as under laissez-faire.

Lastly, we compare laissez-faire and intervention by computing:

$$\frac{\kappa(\overline{z})+\beta^3\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}{\beta^3}\left(\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}}-\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}}\right)+\left(\left.\frac{dV_1^I}{dz}\right|_{z=\overline{z}}-\left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}}\right)=$$
$$\frac{\beta(1-p_A)}{1-\alpha}\left[\beta\left(\frac{p_1}{p_0}-1\right)p_B-\alpha\left[1+\beta(1-p_B)\right]\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}},$$

$$\begin{aligned}\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}}-\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} &= \frac{\beta^3\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{\kappa(\overline{z})+\beta^3\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}\left\{\frac{1-p_A}{(1-\alpha)\beta}\left[\frac{p_B(p_1-p_0)}{p_0}\left[\alpha\beta^3+\frac{\left(1-\beta^3\right)\kappa(\overline{z})}{A(\overline{z})}\right]\right.\right.\\ &\quad\left.-\frac{1+\beta(1-p_B)}{\beta}\left[\alpha\beta^3+\frac{\kappa(\overline{z})}{\alpha A(\overline{z})}\left[(1-\alpha)+\alpha\left(1-\beta^3\right)\right]\right]\right]+\frac{\beta p_0\kappa(\overline{z})}{\alpha A(\overline{z})}\\ &\quad\left.+\frac{\beta\kappa(\overline{z})\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}{2\alpha A(\overline{z})}\left[\frac{(1-\alpha)\beta^3p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}-\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right]\right\},\end{aligned}$$

$$\begin{aligned}\left.\frac{dV_1^I}{dz}\right|_{z=\overline{z}}-\left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}} &= \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\left\{\frac{1-p_A}{(1-\alpha)\beta}\left[\frac{p_B(p_1-p_0)}{p_0}\left[(1-\alpha)\beta^3-\frac{\left(1-\beta^3\right)\kappa(\overline{z})}{A(\overline{z})}\right]\right.\right.\\ &\quad\left.+\frac{1+\beta(1-p_B)}{\beta}\frac{\kappa(\overline{z})}{\alpha A(\overline{z})}\left[(1-\alpha)+\alpha\left(1-\beta^3\right)\right]\right]-\frac{\beta p_0\kappa(\overline{z})}{\alpha A(\overline{z})}\\ &\quad\left.-\frac{\beta\kappa(\overline{z})\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}{2\alpha A(\overline{z})}\left[\frac{(1-\alpha)\beta^3p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}-\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right]\right\}.\end{aligned}$$

The conditions imposed on the parameters are listed in appendix H. It is clear that there exist some calibrations of the parameters satisfying all these conditions and such that p_0 (the initial prior on the success of the new technology) is arbitrarily close to zero, λ (the precision of private signals) is arbitrarily close to one, and $\alpha < \frac{\beta}{1+\beta}$. The results above imply that, for these calibrations, the monetary policy intervention considered increases social welfare. Of course, when p_0 is low, so is the probability that $S_1 = S_2 = 1$ and therefore so is the probability to intervene.

4.3 Numerical simulations

In this section we plan to run numerical simulations in order to rank different monetary policy interventions according to welfare criteria for various calibrations satisfying the conditions listed in appendix H (in the general case). We aim at numerically generalizing the analytical results obtained in the previous subsection by relaxing the assumptions that $N = 3$, that z is arbitrarily close to $\overline{z}$, that (10) holds and that monetary policy is limited to the "minimal" intervention to avoid a cascade

at date $t = 2$. On the one hand, we expect the relaxation of the assumption $N = 3$ to spread the gains of the monetary policy intervention over more periods and therefore to increase the desirability of this intervention, *i.e.* to make the conditions for its desirability less stringent. Moreover, the relaxation of the assumption that z is arbitrarily close to $\overline{z}$ will make households' risk-aversion matter in welfare computations and may therefore also be expected to increase the desirability of the monetary policy intervention. On the other hand, the relaxation of the assumption that parameters are such that the monetary policy intervention can be arbitrarily small will increase the distortion caused by this intervention and will therefore tend to decrease its desirability. Sensitivity analyses will be carried out for each parameter. Lastly, we will consider the case where the central bank does not observe μ_0 and therefore can neither identify a cascade with certainty nor know how much intervention exactly is needed to prevent a cascade. This will amount to extend our work in a similar way as Bernanke and Gertler (2001) extend on Bernanke and Gertler (1999), *i.e.* to unconditionally assess the desirability of a given monetary policy intervention.

5 Conclusion

This paper studies the interaction between monetary policy and asset prices using a simple general equilibrium model in which asset-price bubbles may form due to herd behavior in investment in a new technology whose productivity is uncertain. Monetary policy tightening, by making borrowing dearer for the entrepreneurs, can make them invest in the new technology if and only if they receive an encouraging private signal about its productivity. Monetary policy intervention can therefore prevent herd behavior and the asset-price bubble. We identify conditions under which such a policy intervention is socially desirable. We obtain in particular that if booms in new-tech equity prices are best modeled as the result of herd behavior, then the conditions most commonly stressed by central bankers for the desirability of a monetary policy reaction to these booms over and above their short-term effects on the business cycle may well prove less demanding than they seem at first sight.

6 References

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7 Appendix

A Proof of proposition 1

Let us note, for all $z > \overline{z}$ and $\tau_t > 0$,

$$D_0(\tau_t) \equiv \frac{\beta^N}{\tau_t}\left[\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N}\right], \; F_0(z) \equiv \alpha A(\overline{z}) - \kappa(z), \; G_0 \equiv \alpha A(\overline{z}) - \kappa(\overline{z}) \text{ and } H_0 \equiv \kappa(\overline{z}),$$

so that (4) corresponds to

$$q_t = D_0(\tau_t)\left[\frac{p}{F_0(z) + \frac{H_0}{q_t}} + \frac{1-p}{G_0 + \frac{H_0}{q_t}}\right].$$

Note that conditions (2) and (3) together imply $G_0 > 0$.

Suppose first that (4) admits a strictly positive solution q_t for all $p \in [0;1]$. Then, (4) admits a strictly positive solution q_t in particular for $p = 0$, which implies that $D_0(\tau_t) > H_0$, and for $p = 1$, which implies that $F_0(z) > 0$ (given that $D_0(\tau_t) > H_0$). Now suppose conversely that $F_0(z) > 0$ and $D_0(\tau_t) > H_0$. Then, when $p \in \{0;1\}$, (4) admits a unique solution q_t and this solution is strictly

positive. When $p \notin \{0;1\}$, (4) is equivalent to

$$\Phi_0(z)\, q_t^2 + \Psi_0(z,\tau_t,p)\, q_t + \Omega_0(\tau_t) = 0$$

where, for all $z > \overline{z}$, $\tau_t > 0$ and $p \in]0;1[$, $\Phi_0(z) \equiv F_0(z)\, G_0$, $\Psi_0(z,\tau_t,p) \equiv [F_0(z) + G_0]\, H_0 - D_0(\tau_t)\,[G_0 p + F_0(z)(1-p)]$ and $\Omega_0(\tau_t) \equiv H_0\,[H_0 - D_0(\tau_t)]$. We have: $\forall p \in]0;1[$, $[\Psi_0(z,\tau_t,p)]^2 - 4\Phi_0(z)\,\Omega_0(\tau_t) \geq -4\Phi_0(z)\,\Omega_0(\tau_t) > 0$, so that (4) admits two distinct real-number solutions and, since $\frac{\Omega_0(\tau_t)}{\Phi_0(z)} < 0$, one solution is strictly negative and the other strictly positive. Point (i) of proposition 1 follows.

From the previous paragraph, we also get that if $F_0(z) > 0$ and $D_0(\tau_t) > H_0$, then (4) admits a unique strictly positive solution q_t for all $p \in [0;1]$, which we note $q(z,\tau_t,p,0)$. When $p \in]0;1[$, the derivation of $\Phi_0(z)\, q(z,\tau_t,p,0)^2 + \Psi_0(z,\tau_t,p)\, q(z,\tau_t,p,0) + \Omega_0(\tau_t) = 0$ with respect to $x \in \{\tau_t, p\}$ leads to

$$[2\Phi_0(z)\, q(z,\tau_t,p,0) + \Psi_0(z,\tau_t,p)]\, \frac{\partial q(z,\tau_t,p,0)}{\partial x} + q(z,\tau_t,p,0)\, \frac{\partial \Psi_0(z,\tau_t,p)}{\partial x} = 0,$$

where $2\Phi_0(z)\, q(z,\tau_t,p,0) + \Psi_0(z,\tau_t,p) > 0$ by definition of $q(z,\tau_t,p,0)$. Given that $\frac{\partial \Psi_0(z,\tau_t,p)}{\partial \tau_t} = \frac{D_0(\tau_t)}{\tau_t}\,[G_0 p + F_0(z)(1-p)] > 0$ and $\frac{\partial \Psi_0(z,\tau_t,p)}{\partial p} = D_0(\tau_t)\,[F_0(z) - G_0] < 0$, we therefore obtain that $\frac{\partial q(z,\tau_t,p,0)}{\partial \tau_t} < 0$ and $\frac{\partial q(z,\tau_t,p,0)}{\partial p} > 0$ for $p \in]0;1[$ and by continuity for $p \in \{0;1\}$ as well. Point (ii) of proposition 1 follows.

B Proof of proposition 2

Let us note, for all $z > \overline{z}$ and $\tau_t > 0$,

$$D_1(z,\tau_t) \equiv \frac{\beta^N}{\tau_t}\left[\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{\beta^N}\right],\ F_1(z) \equiv \alpha A(z) - \kappa(z),\ G_1 \equiv \alpha A(\overline{z}) - \kappa(\overline{z}) \text{ and } H_1(z) \equiv \kappa(z),$$

so that (7) can be rewritten as

$$q_t = D_1(z,\tau_t)\left[\frac{p}{F_1(z) + \frac{H_1(z)}{q_t}} + \frac{1-p}{G_1 + \frac{H_1(z)}{q_t}}\right].$$

Recall that $G_1 > 0$ and note moreover that $F_1(z) > 0$, given condition (5).

Suppose first that (7) admits a strictly positive solution q_t for all $p \in [0;1]$. Then, (7) admits a strictly positive solution q_t in particular for $p = 0$, which implies that $D_1(z, \tau_t) > H_1(z)$. Now suppose conversely that $D_1(z, \tau_t) > H_1(z)$. Then, when $p \in \{0;1\}$ or $F_1(z) = G_1$, (7) admits a unique solution q_t and this solution is strictly positive. When $p \notin \{0;1\}$ and $F_1(z) \neq G_1$, (7) is equivalent to

$$\Phi_1(z) q_t^2 + \Psi_1(z, \tau_t, p) q_t + \Omega_1(z, \tau_t) = 0$$

where, for all $z > \overline{z}$, $\tau_t > 0$ and $p \in]0;1[$, $\Phi_1(z) \equiv F_1(z) G_1$, $\Psi_1(z, \tau_t, p) \equiv [F_1(z) + G_1] H_1(z) - D_1(z, \tau_t)[G_1 p + F_1(z)(1-p)]$ and $\Omega_1(z, \tau_t) \equiv H_1(z)[H_1(z) - D_1(z, \tau_t)]$. We have: $\forall p \in]0;1[$, $[\Psi_1(z, \tau_t, p)]^2 - 4\Phi_1(z)\Omega_1(z, \tau_t) \geq -4\Phi_1(z)\Omega_1(z, \tau_t) > 0$, so that (7) admits two distinct real-number solutions and, since $\frac{\Omega_1(z,\tau_t)}{\Phi_1(z)} < 0$, one solution is strictly negative and the other strictly positive. Point (i) of proposition 2 follows.

From the previous paragraph, we also get that if $D_1(z, \tau_t) > H_1(z)$, then (7) admits a unique strictly positive solution q_t for all $p \in [0;1]$, which we note $q(z, \tau_t, p, 1)$. When $p \in]0;1[$, the derivation of $\Phi_1(z) q(z, \tau_t, p, 1)^2 + \Psi_1(z, \tau_t, p) q(z, \tau_t, p, 1) + \Omega_1(z, \tau_t) = 0$ with respect to $x \in \{\tau_t, p\}$ leads to

$$[2\Phi_1(z) q(z, \tau_t, p, 1) + \Psi_1(z, \tau_t, p)] \frac{\partial q(z, \tau_t, p, 1)}{\partial x} + q(z, \tau_t, p, 1) \frac{\partial \Psi_1(z, \tau_t, p)}{\partial x} = 0,$$

where $2\Phi_1(z) q(z, \tau_t, p, 1) + \Psi_1(z, \tau_t, p) > 0$ by definition of $q(z, \tau_t, p, 1)$. Given that $\frac{\partial \Psi_1(z,\tau_t,p)}{\partial \tau_t} = \frac{D_1(z,\tau_t)}{\tau_t}[G_1 p + F_1(z)(1-p)] > 0$ and $\frac{\partial \Psi_1(z,\tau_t,p)}{\partial p} = D_1(z, \tau_t)[F_1(z) - G_1] < 0$, we therefore obtain that, for $p \in]0;1[$ and by continuity for $p \in \{0;1\}$ as well, $\frac{\partial q(z,\tau_t,p,1)}{\partial \tau_t} < 0$ and $\frac{\partial q(z,\tau_t,p,1)}{\partial p} < 0$ if $F_1(z) > G_1$, $\frac{\partial q(z,\tau_t,p,1)}{\partial p} = 0$ if $F_1(z) = G_1$ and $\frac{\partial q(z,\tau_t,p,1)}{\partial p} > 0$ if $F_1(z) < G_1$. Point (ii) of proposition 2 follows.

C Proof of proposition 3

We have $\forall t \in \{N+1, ...2N\}$,

$$q_t = \beta^N \frac{\alpha A(\overline{z}) - \kappa(z)}{\alpha A(z) - \kappa(z)} + \frac{1}{\alpha A(z) - \kappa(z)} \left[\frac{\beta^N}{q(z, \tau_{t-N}, p, 0)} \kappa(\overline{z}) - \kappa(z) \right]$$

if $z_{t-N} = \overline{z}$ and the new technology is good,

$$q_t = \beta^N + \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \left[\frac{\beta^N}{q(z, \tau_{t-N}, p, 1)} - 1 \right]$$

if $z_{t-N} = z$ and the new technology is good,

$$q_t = \beta^N + \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[\frac{\beta^N}{q(z, \tau_{t-N}, p, 0)} - 1 \right]$$

if $z_{t-N} = \overline{z}$ and the new technology is bad, and

$$q_t = \beta^N + \frac{1}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[\frac{\beta^N}{q(z, \tau_{t-N}, p, 1)} \kappa(z) - \kappa(\overline{z}) \right]$$

if $z_{t-N} = z$ and the new technology is bad. Note that, given condition (5), if $q_t > 0$ in the first case ($z_{t-N} = \overline{z}$ and the new technology is good), then $q_t > 0$ in the third case ($z_{t-N} = \overline{z}$ and the new technology is bad). Besides, from propositions 1 and 2 we get that $\forall p \in [0; 1]$,

$$\begin{aligned} q(z, \tau_{t-N}, p, 0) &\leq \frac{\beta^N [\alpha A(\overline{z}) - \kappa(\overline{z})] + \kappa(\overline{z})(1 - \tau_{t-N})}{\tau_{t-N} [\alpha A(\overline{z}) - \kappa(z)]}, \\ q(z, \tau_{t-N}, p, 1) &\leq \frac{\beta^N [\alpha A(\overline{z}) - \kappa(z)] + \kappa(\overline{z}) - \tau_{t-N}\kappa(z)}{\tau_{t-N} \min[\alpha A(z) - \kappa(z), \alpha A(\overline{z}) - \kappa(\overline{z})]}, \end{aligned}$$

each of these two inequalities being an equality either for $p = 0$ or for $p = 1$. Therefore, $q_t > 0$ for all $t \in \{N+1, ...2N\}$ and all $p \in [0; 1]$ if and only if

$$\beta^N [\alpha A(\overline{z}) - \kappa(z)] + \frac{\beta^N \kappa(\overline{z}) [\alpha A(\overline{z}) - \kappa(z)] \tau_{t-N}}{\beta^N [\alpha A(\overline{z}) - \kappa(\overline{z})] + \kappa(\overline{z})(1 - \tau_{t-N})} > \kappa(z), \quad (17)$$

$$\beta^N [\alpha A(z) - \kappa(z)] + \frac{\beta^N \kappa(z) \min[\alpha A(z) - \kappa(z), \alpha A(\overline{z}) - \kappa(\overline{z})] \tau_{t-N}}{\beta^N [\alpha A(\overline{z}) - \kappa(z)] + \kappa(\overline{z}) - \tau_{t-N}\kappa(z)} > \kappa(z), \quad (18)$$

$$\text{and } \beta^N [\alpha A(\overline{z}) - \kappa(\overline{z})] + \frac{\beta^N \kappa(z) \min[\alpha A(z) - \kappa(z), \alpha A(\overline{z}) - \kappa(\overline{z})] \tau_{t-N}}{\beta^N [\alpha A(\overline{z}) - \kappa(z)] + \kappa(\overline{z}) - \tau_{t-N}\kappa(z)} > \kappa(\overline{z}). \quad (19)$$

Now suppose that (17), (18), (19) and

$$\beta^N > \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \quad (20)$$

hold. Then, from the previous paragraph we get that $q_t > 0$ for all $t \in \{N+1, ...2N\}$ and all $p \in [0; 1]$. Consider first the case where the new technology is good. Then, we have $\forall t > 2N$,

$$q_t - \beta^N = \frac{\beta^N \kappa(z)}{\alpha A(z) - \kappa(z)} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N} \right). \quad (21)$$

By recurrence, we therefore get $\forall t > 2N$,

$$q_t > \beta^N + \frac{\beta^N \kappa(z)}{\alpha A(z) - \kappa(z)} \inf_{x \in \mathbb{R}^{+*}} \left(\frac{1}{x} - \frac{1}{\beta^N} \right) = \beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} > 0$$

due to (20). Moreover, we have $\forall t > 3N$,

$$q_t - \beta^N = \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2 q_{t-N} q_{t-2N}} \left(q_{t-2N} - \beta^N \right), \tag{22}$$

Using $[\forall t > N,\, q_t > 0]$ and (21), we get for all $t > 3N$,

$$\begin{aligned}
&\left| \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2 q_{t-N} q_{t-2N}} \right| < 1 \Longleftrightarrow q_{t-N} q_{t-2N} > \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2} \\
&\Longleftrightarrow \left[\beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \right] q_{t-2N} + \frac{\beta^N \kappa(z)}{\alpha A(z) - \kappa(z)} > \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2} \\
&\Longleftrightarrow \left[\beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \right] \left[q_{t-2N} + \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \right] > 0 \\
&\Longleftrightarrow \beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} > 0,
\end{aligned}$$

so that (20) implies $\lim_{t \longrightarrow +\infty} q_t = \beta^N$. Consider then the case where the new technology is bad. Using

$$\begin{aligned}
\forall t &> 2N,\, q_t - \beta^N = \frac{\beta^N \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N} \right), \\
\forall t &> 3N,\, q_t - \beta^N = \frac{[\kappa(\overline{z})]^2}{[\alpha A(\overline{z}) - \kappa(\overline{z})]^2 q_{t-N} q_{t-2N}} \left(q_{t-2N} - \beta^N \right)
\end{aligned} \tag{23}$$

and (3) instead of (21), (22) and (20) respectively, we similarly obtain that $\forall t > 2N$, $q_t > 0$ and $\lim_{t \longrightarrow +\infty} q_t = \beta^N$.

Conversely, suppose that $q_t > 0$ for all $t > N$ and all $p \in [0;1]$, and $\lim_{t \longrightarrow +\infty} q_t = \beta^N$ for all $p \in [0;1]$. Then, from the first paragraph we get that (17), (18) and (19) hold. Moreover, using $\lim_{t \longrightarrow +\infty} q_t = \beta^N$ and the first-order Taylor developments of (21) for q_t and q_{t-N} close to β^N, we get that (20) holds.

D Proof of proposition 4

From (4) and (7) we easily get, using the notations of appendices A and B:

$$q(z, \tau, 0, 0) = \frac{D_0(\tau) - H_0}{G_0},\; q(z, \tau, 0, 1) = \frac{D_1(z, \tau) - H_1(z)}{G_1} \text{ and } q(z, \tau, 1, 1) = \frac{D_1(z, \tau) - H_1(z)}{F_1(z)}.$$

Since $D_0(\tau) > D_1(z,\tau)$, $H_0 < H_1(z)$ and $G_0 = G_1 > 0$, we conclude that

$$q(z,\tau,0,0) > q(z,\tau,0,1). \tag{24}$$

Moreover, simple computations lead to

$$q(z,\tau,0,0) - q(z,\tau,1,1) = \frac{f(z,\tau)}{\tau[\alpha A(\overline{z}) - \kappa(\overline{z})][\alpha A(z) - \kappa(z)]}$$

where $f(z,\tau) \equiv [\tau\alpha A(\overline{z}) - \kappa(\overline{z})][\kappa(z) - \kappa(\overline{z})] + \left[\kappa(\overline{z})\left(1 - \beta^N - \tau\right) + \alpha A(\overline{z})\beta^N\right][\alpha A(z) - \alpha A(\overline{z})]$,

so that $q(z,\tau,0,0) > q(z,\tau,1,1)$ if and only if $f(z,\tau) > 0$. We have in particular:

$$\begin{aligned} f(z,1) &= [\alpha A(\overline{z}) - \kappa(\overline{z})]\left[[\kappa(z) - \kappa(\overline{z})] + \beta^N[\alpha A(z) - \alpha A(\overline{z})]\right] \\ &> 0 \end{aligned}$$

due to condition (5), from which we conclude that $\forall z > \overline{z}$ such that $A(z)\kappa(\overline{z}) < A(\overline{z})\kappa(z)$, $\tau^l(z) < 1$. We also have in particular, using $\tau^u(\overline{z}) > \tau^u(z) > 1$:

$$\begin{aligned} f(z,\tau^u(z)) &= [\tau^u(z)\alpha A(\overline{z}) - \kappa(\overline{z})][\kappa(z) - \kappa(\overline{z})] + \left[\kappa(\overline{z})\left(1 - \beta^N - \tau^u(z)\right) + \alpha A(\overline{z})\beta^N\right][\alpha A(z) - \alpha A(\overline{z})] \\ &> [\tau^u(z)\alpha A(\overline{z}) - \kappa(\overline{z})][\kappa(z) - \kappa(\overline{z})] + \left[\kappa(\overline{z})\left(1 - \beta^N - \tau^u(\overline{z})\right) + \alpha A(\overline{z})\beta^N\right][\alpha A(z) - \alpha A(\overline{z})] \\ &= [\tau^u(z)\alpha A(\overline{z}) - \kappa(\overline{z})][\kappa(z) - \kappa(\overline{z})] \\ &> 0 \end{aligned}$$

due to condition (5), from which we conclude, since $\frac{\partial f(z,\tau)}{\partial \tau}$ does not depend on τ and since (8) holds, that $\forall z > \overline{z}$ such that $\frac{\partial f(z,\tau)}{\partial \tau} \leq 0$ or $\left[\frac{\partial f(z,\tau)}{\partial \tau} > 0 \text{ and } f(z,0) \geq 0\right]$, $f(z,\tau) > 0$, and therefore

$$\forall z > \overline{z} \text{ such that } \frac{\partial f(z,\tau)}{\partial \tau} \leq 0 \text{ or } \left[\frac{\partial f(z,\tau)}{\partial \tau} > 0 \text{ and } f(z,0) \geq 0\right], q(z,\tau,0,0) > q(z,\tau,1,1). \tag{25}$$

Results (24) and (25) together with propositions 1 and 2 then imply that $\forall z > \overline{z}$ such that $\frac{\partial f(z,\tau)}{\partial \tau} \leq 0$ or $\left[\frac{\partial f(z,\tau)}{\partial \tau} > 0 \text{ and } f(z,0) \geq 0\right]$, $\forall (p,p') \in [0;1]^2$,

$$q(z,\tau,p,0) \geq q(z,\tau,0,0) > \max[q(z,\tau,0,1), q(z,\tau,1,1)] \geq q\left(z,\tau,p',1\right).$$

We similarly obtain that, alternatively, $\forall z > \overline{z}$ such that $\frac{\partial f(z,\tau)}{\partial \tau} > 0$ and $f(z,0) < 0$, $f(z,\tau) > 0 \Longleftrightarrow \tau > \tau^l(z)$, and therefore

$$\forall z > \overline{z} \text{ such that } \frac{\partial f(z,\tau)}{\partial \tau} > 0 \text{ and } f(z,0) < 0,\ q(z,\tau,0,0) > q(z,\tau,1,1) \Longleftrightarrow \tau > \tau^l(z). \tag{26}$$

Results (24) and (26) together with propositions 1 and 2 then imply that $\forall z > \overline{z}$ such that $\frac{\partial f(z,\tau)}{\partial \tau} > 0$ and $f(z,0) < 0$,

$$\left[\forall (p,p') \in [0;1]^2,\ q(z,\tau,p,0) > q(z,\tau,p',1)\right] \Longleftrightarrow q(z,\tau,0,0) > \max\left[q(z,\tau,0,1), q(z,\tau,1,1)\right] \Longleftrightarrow \tau > \tau^l(z).$$

E Proof of proposition 5

Let us note μ_t^0 the value taken by μ_t when $I_t = 0$ and μ_t^1 the value taken by μ_t when $I_t = 1$ for $t \in \{1, ..., N\}$. Since entrepreneurs take the interest rate as given when deciding in which technology to invest, $I_t = 0$ is supported by an equilibrium only if

$$(1-\alpha)A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z,\tau_t,\mu_t^0,0)} > \widetilde{\mu}_t\left[(1-\alpha)A(z) - \frac{\kappa(z)}{q(z,\tau_t,\mu_t^0,0)}\right] + (1-\widetilde{\mu}_t)\left[(1-\alpha)A(\overline{z}) - \frac{\kappa(z)}{q(z,\tau_t,\mu_t^0,0)}\right],$$

that is to say only if

$$\widetilde{\mu}_t q(z,\tau_t,\mu_t^0,0) < B(z), \tag{27}$$

while $I_t = 1$ is supported by an equilibrium only if

$$(1-\alpha)A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z,\tau_t,\mu_t^1,1)} < \widetilde{\mu}_t\left[(1-\alpha)A(z) - \frac{\kappa(z)}{q(z,\tau_t,\mu_t^1,1)}\right] + (1-\widetilde{\mu}_t)\left[(1-\alpha)A(\overline{z}) - \frac{\kappa(z)}{q(z,\tau_t,\mu_t^1,1)}\right],$$

that is to say only if

$$\widetilde{\mu}_t q(z,\tau_t,\mu_t^1,1) > B(z). \tag{28}$$

Proposition 4 implies that $\forall (\mu_t^0, \mu_t^1) \in [0;1]^2$, conditions (27) and (28) cannot hold for the same values of the parameters. This implies that at most one of the following four cases can occur in equilibrium at a given date $t \in \{1, ..., N\}$: $S_t = 0 \Longrightarrow I_t = 0$ and $S_t = 1 \Longrightarrow I_t = 0$ (case a), $S_t = 0 \Longrightarrow I_t = 1$ and $S_t = 1 \Longrightarrow I_t = 1$ (case b), $S_t = 0 \Longrightarrow I_t = 0$ and $S_t = 1 \Longrightarrow I_t = 1$ (case c), $S_t = 0 \Longrightarrow I_t = 1$ and $S_t = 1 \Longrightarrow I_t = 0$ (case d). Note that case d is actually impossible, as it

would require $\widetilde{\mu}_t^0 q\left(z,\tau_t,\mu_t^1,1\right) > B\left(z\right)$ and $\widetilde{\mu}_t^1 q\left(z,\tau_t,\mu_t^0,0\right) < B\left(z\right)$ where $\widetilde{\mu}_t^0 \leq \widetilde{\mu}_t^1$, which contradicts proposition 4. Note also that cases a and b both lead to $\mu_t = \mu_{t-1}$, while case c leads to $\mu_t = \widetilde{\mu}_t$. As a consequence, taking the households' rational expectations into account, case a is supported by an equilibrium if and only if $\widetilde{\mu}_t^0 q\left(z,\tau_t,\mu_{t-1},0\right) < B\left(z\right)$ and $\widetilde{\mu}_t^1 q\left(z,\tau_t,\mu_{t-1},0\right) < B\left(z\right)$, that is to say if and only if

$$\widetilde{\mu}_t^1 q\left(z,\tau_t,\mu_{t-1},0\right) < B\left(z\right), \tag{29}$$

case b is supported by an equilibrium if and only if $\widetilde{\mu}_t^0 q\left(z,\tau_t,\mu_{t-1},1\right) > B\left(z\right)$ and $\widetilde{\mu}_t^1 q\left(z,\tau_t,\mu_{t-1},1\right) > B\left(z\right)$, that is to say if and only if

$$\widetilde{\mu}_t^0 q\left(z,\tau_t,\mu_{t-1},1\right) > B\left(z\right), \tag{30}$$

and case c is supported by an equilibrium if and only if

$$\widetilde{\mu}_t^0 q\left(z,\tau_t,\widetilde{\mu}_t^0,0\right) < B\left(z\right) \text{ and } \widetilde{\mu}_t^1 q\left(z,\tau_t,\widetilde{\mu}_t^1,1\right) > B\left(z\right). \tag{31}$$

Given that $\widetilde{\mu}_t^0 \leq \widetilde{\mu}_t^1$, proposition 4 implies that at most one of the three conditions (29), (30) and (31) holds for some given values of the parameters. We therefore conclude that, in equilibrium, $\forall t \in \{1,...,N\}$, there exist only three possibilities and these possibilities are mutually exclusive: either condition (29) holds, then $I_t = 0$ whatever $S_t \in \{0,1\}$ and $\mu_t = \mu_{t-1}$; or condition (30) holds, then $I_t = 1$ whatever $S_t \in \{0,1\}$ and $\mu_t = \mu_{t-1}$; or condition (31) holds, then $I_t = S_t$ whatever $S_t \in \{0,1\}$ and $\mu_t = \widetilde{\mu}_t$.

F Welfare analysis under laissez-faire in the simple case

We focus on the case in which the first investor receives a positive signal ($S_1 = 1$). The value taken by the representative household's utility function at date 1 is then:

$$\begin{aligned} U_1^{LF}(z) &= (1+\beta+\beta^2)\ln\left[\alpha A\left(\overline{z}\right)+\frac{\kappa\left(\overline{z}\right)}{\beta^3}-\kappa\left(z\right)\right]+p_1\beta^3\textstyle\sum_{i=0}^{+\infty}\beta^i\ln\left[\alpha A\left(z\right)+\frac{\kappa\left(z\right)}{q_{i+1}^{(1)}\left(z\right)}-\kappa\left(z\right)\right] \\ &\quad +(1-p_1)\beta^3\textstyle\sum_{i=0}^{2}\beta^i\ln\left[\alpha A\left(\overline{z}\right)+\frac{\kappa\left(z\right)}{q_{i+1}^{(2)}\left(z\right)}-\kappa\left(\overline{z}\right)\right]+(1-p_1)\beta^3\textstyle\sum_{i=3}^{+\infty}\beta^i\ln\left[\alpha A\left(\overline{z}\right)+\frac{\kappa\left(\overline{z}\right)}{q_{i+1}^{(2)}\left(z\right)}-\kappa\left(\overline{z}\right.\right. \end{aligned}$$

where superscripts (1) (resp. (2)) indicates that technology turns out to be good (resp. bad). Computations lead to

$$\begin{aligned}
q_i^{(1)}(z) &= q_i^{(2)}(z) = q(z,1,p_1,1) \text{ for } i \in \{1,2,3\}, \\
q_i^{(1)}(z) &= \beta^3 + \frac{\beta^3 \kappa(z)}{\alpha A(z) - \kappa(z)} \left[\frac{1}{q_{i-3}^{(1)}(z)} - \frac{1}{\beta^3} \right] \text{ for } i \geq 4, \\
q_i^{(2)}(z) &= \beta^3 + \frac{\beta^3 \kappa(z)}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[\frac{1}{q_{i-3}^{(2)}(z)} - \frac{1}{\beta^3} \right] + \frac{\kappa(z) - \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \text{ for } i \in \{4,5,6\}, \\
q_i^{(2)}(z) &= \beta^3 + \frac{\beta^3 \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[\frac{1}{q_{i-3}^{(2)}(z)} - \frac{1}{\beta^3} \right] \text{ for } i \geq 7.
\end{aligned}$$

Using in particular $q_i^{(1)}(\overline{z}) = q_i^{(2)}(\overline{z}) = \beta^3 \ \forall i \geq 0$ and (10):

$$\begin{aligned}
\left.\frac{dq_j^{(k)}}{dz}\right|_{z=\overline{z}} &= \frac{-1}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[1 + \beta^3(1-p_1) + \frac{\alpha p_1}{(1-\alpha)p_0} \right] \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \text{ for } j \in \{1,2,3\} \text{ and } k \in \{1,2\}, \\
\left.\frac{dq_{3i+j}^{(1)}}{dz}\right|_{z=\overline{z}} &= \left[\frac{-\kappa(\overline{z})}{\beta^3 [\alpha A(\overline{z}) - \kappa(\overline{z})]} \right]^i \left.\frac{dq_j^{(1)}}{dz}\right|_{z=\overline{z}} \text{ for } i \geq 1 \text{ and } j \in \{1,2,3\}, \\
\left.\frac{dq_{3i+j}^{(2)}}{dz}\right|_{z=\overline{z}} &= \left[\frac{-\kappa(\overline{z})}{\beta^3 [\alpha A(\overline{z}) - \kappa(\overline{z})]} \right]^{i-1} \left\{ \frac{1}{[\alpha A(\overline{z}) - \kappa(\overline{z})]} \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} - \frac{\kappa(\overline{z})}{\beta^3 [\alpha A(\overline{z}) - \kappa(\overline{z})]} \left.\frac{dq_j^{(1)}}{dz}\right|_{z=\overline{z}} \right\} \\
&\quad \text{for } i \geq 1 \text{ and } j \in \{1,2,3\}.
\end{aligned}$$

We end up with:

$$\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{(1-\beta)\left[\kappa(\overline{z}) + \beta^3 [\alpha A(\overline{z}) - \kappa(\overline{z})]\right]} \left\{ \frac{\alpha\beta^3 p_1}{(1-\alpha)p_0} + \frac{\kappa(\overline{z})\left(1-\beta^3\right)}{\alpha A(\overline{z})} \left[1 + \frac{\alpha p_1}{(1-\alpha)p_0} \right] \right\}.$$

For the entrepreneurs:

$$\begin{aligned}
V_1^{LF}(z) &= p_1 \textstyle\sum_{i=0}^{+\infty} \beta^{3+i} \left[(1-\alpha) A(z) - \frac{\kappa(z)}{q_{i+1}^{(1)}(z)} \right] \\
&\quad + (1-p_1) \textstyle\sum_{i=0}^{2} \beta^{3+i} \left[(1-\alpha) A(\overline{z}) - \frac{\kappa(z)}{q_{i+1}^{(2)}(z)} \right] + (1-p_1) \textstyle\sum_{i=3}^{+\infty} \beta^{3+i} \left[(1-\alpha) A(\overline{z}) - \frac{\kappa(\overline{z})}{q_{i+1}^{(2)}(z)} \right].
\end{aligned}$$

Using (10), we end up with

$$\left.\frac{dV_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{-\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta} \left\{ 1 - \beta^3 + \beta^3 p_1 - \frac{p_1}{p_0} + \frac{\kappa(\overline{z})\left(1-\beta^3\right)}{\alpha A(\overline{z})\beta^3} \left[1 + \frac{\alpha p_1}{(1-\alpha)p_0} \right] \right\}.$$

G Welfare analysis under intervention in the simple case

We focus on the case in which the first investor receives a positive signal ($S_1 = 1$) and there is only a "minimal" intervention at date 2. The value taken by the representative household's utility function at date 1 is then:

$$
\begin{aligned}
U_1^I(z) \;=\; & \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z)\right] \\
& + p_A \left\{ \sum_{i=1}^{2} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z)\right] + p_2 \sum_{i=3}^{+\infty} \beta^i \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_{i-2}^{(1)}(z)} - \kappa(z)\right] \right. \\
& \left. + (1-p_2)\left[\sum_{i=3}^{5} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_{i-2}^{(2)}(z)} - \kappa(\overline{z})\right] + \sum_{i=6}^{+\infty} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(2)}(z)} - \kappa(\overline{z})\right]\right]\right\} \\
& + (1-p_A)\left\{ \beta \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(\overline{z})\right] + p_B \left[\beta^2 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z)\right]\right.\right. \\
& + p_1 \left[\sum_{i\in\mathbb{N}\setminus\{0,1,2,4\}} \beta^i \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_{i-2}^{(3)}(z)} - \kappa(z)\right] + \beta^4 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_2^{(3)}(z)} - \kappa(z)\right]\right] \\
& + (1-p_1)\left[\sum_{i\in\{3,5\}} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_{i-2}^{(4)}(z)} - \kappa(\overline{z})\right] + \sum_{i\in\mathbb{N}\setminus\{0,1,2,3,5\}} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(4)}(z)} - \kappa(\overline{z})\right.\right. \\
& + (1-p_B)\left[\beta^2 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(\overline{z})\right] + p_{-1}\left[\beta^3 \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_1^{(5)}(z)} - \kappa(z)\right]\right.\right. \\
& \sum_{i=4}^{5} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(5)}(z)} - \kappa(z)\right] + \sum_{i=6}^{+\infty} \beta^i \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_{i-2}^{(5)}(z)} - \kappa(z)\right]\Bigg] \\
& \left.\left.\left. + (1-p_{-1})\left[\beta^3 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_1^{(6)}(z)} - \kappa(\overline{z})\right] + \sum_{i=4}^{+\infty} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(6)}(z)} - \kappa(\overline{z})\right]\right]\right]\right\},
\end{aligned}
$$

where superscripts (1) to (6) correspond to the following cases:

Superscript	S_2	S_3	Technology
(1)	1	0 or 1	good
(2)	1	0 or 1	bad
(3)	0	1	good
(4)	0	1	bad
(5)	0	0	good
(6)	0	0	bad

Computations lead to

$$q_1^{(1)}(z) = q(z,1,p_1,1),\ q_2^{(1)}(z) = q(z,\tau_2(z),p_2,1),\ q_3^{(1)}(z) = q(z,1,p_2,1),$$

$$q_i^{(1)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z) - \kappa(z)}\left[\frac{1}{q_{i-3}^{(1)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i \geq 4,$$

$$q_1^{(2)}(z) = q(z,1,p_1,1),\ q_2^{(2)}(z) = q(z,\tau_2(z),p_2,1),\ q_3^{(2)}(z) = q(z,1,p_2,1),$$

$$q_i^{(2)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z}) - \kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(2)}(z)} - \frac{1}{\beta^3}\right] + \frac{\kappa(z) - \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \text{ for } 4 \leq i \leq 6,$$

$$q_i^{(2)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(2)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i \geq 7,$$

$$q_1^{(3)}(z) = q(z,1,p_1,1),\ q_2^{(3)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(3)}(z) = q(z,1,p_1,1),$$

$$q_i^{(3)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z) - \kappa(z)}\left[\frac{1}{q_{i-3}^{(3)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i = 4 \text{ and } i \geq 6,$$

$$q_5^{(3)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(z) - \kappa(z)}\left[\frac{1}{q_2^{(3)}(z)} - \frac{1}{\beta^3}\right] - \frac{\kappa(z) - \kappa(\overline{z}) + \beta^3\left[\alpha A(z) - \alpha A(\overline{z})\right]}{\alpha A(z) - \kappa(z)},$$

$$q_1^{(4)}(z) = q(z,1,p_1,1),\ q_2^{(4)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(4)}(z) = q(z,1,p_1,1),$$

$$q_i^{(4)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z}) - \kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(4)}(z)} - \frac{1}{\beta^3}\right] + \frac{\kappa(z) - \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \text{ for } i \in \{4,6\},$$

$$q_i^{(4)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(4)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i = 5 \text{ and } i \geq 7,$$

$$q_1^{(5)}(z) = q(z,1,p_1,1),\ q_2^{(5)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(5)}(z) = q(z,1,p_{-1},0),$$

$$q_i^{(5)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z) - \kappa(z)}\left[\frac{1}{q_{i-3}^{(5)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i = 4 \text{ and } i \geq 7,$$

$$q_i^{(5)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(z) - \kappa(z)}\left[\frac{1}{q_{i-3}^{(5)}(z)} - \frac{1}{\beta^3}\right] - \frac{\kappa(z) - \kappa(\overline{z}) + \beta^3\left[\alpha A(z) - \alpha A(\overline{z})\right]}{\alpha A(z) - \kappa(z)} \text{ for } i \in \{5,6\},$$

$$q_1^{(6)}(z) = q(z,1,p_1,1),\ q_2^{(6)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(6)}(z) = q(z,1,p_{-1},0),$$

$$q_4^{(6)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z}) - \kappa(\overline{z})}\left[\frac{1}{q_1^{(6)}(z)} - \frac{1}{\beta^3}\right] + \frac{\kappa(z) - \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})},$$

$$q_i^{(6)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(6)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i \geq 5.$$

Using (10), (15) and (16), we get:

$$\left.\frac{dq_1^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left[1+\beta^3(1-p_1)+\frac{\alpha p_1}{\left(1-\alpha\right)p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{1,...,6\},$$

$$\left.\frac{dq_2^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A(\overline{z})-\kappa\left(\overline{z}\right)}\left[1+\beta^3\left(1+p_0-p_2\right)+\frac{\alpha}{1-\alpha}\frac{p_2}{p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}},$$

$$+\frac{\beta^3}{2\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}\left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}-\left(1-\alpha\right)\beta^3 p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}\right] \quad \text{for } k\in\{1,2\},$$

$$\left.\frac{dq_2^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{\beta^3}{2\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}\left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}-\left(1-\alpha\right)\beta^3 p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}\right] \quad \text{for } k\in\{3,...,6\},$$

$$\left.\frac{dq_3^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left[1+\beta^3(1-p_2)+\frac{\alpha p_2}{\left(1-\alpha\right)p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{1,2\},$$

$$\left.\frac{dq_3^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left[1+\beta^3(1-p_1)+\frac{\alpha p_1}{\left(1-\alpha\right)p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{3,4\},$$

$$\left.\frac{dq_3^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{\beta^3 p_{-1}}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{5,6\},$$

$$\left.\frac{dq_{3i+j}^{(1)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_j^{(1)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1 \text{ and } j\in\{1,2,3\},$$

$$\left.\frac{dq_{3i+j}^{(2)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left[\left.\frac{dq_j^{(2)}}{dz}\right|_{z=\overline{z}}-\frac{\beta^3}{\kappa(\overline{z})}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right] \quad \text{for } i\geq 1 \text{ and } j\in\{1,2,3\},$$

$$\left.\frac{dq_{3i+j}^{(3)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_j^{(3)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1 \text{ and } j\in\{1,3\},$$

$$\left.\frac{dq_{3i+2}^{(3)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left\{\left.\frac{dq_2^{(3)}}{dz}\right|_{z=\overline{z}}+\frac{\beta^3}{\kappa\left(\overline{z}\right)}\left[1+\frac{\alpha}{\left(1-\alpha\right)p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right\} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+j}^{(4)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left[\left.\frac{dq_j^{(4)}}{dz}\right|_{z=\overline{z}}-\frac{\beta^3}{\kappa(\overline{z})}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right] \quad \text{for } i\geq 1 \text{ and } j\in\{1,3\},$$

$$\left.\frac{dq_{3i+2}^{(4)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_2^{(4)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+1}^{(5)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_1^{(5)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+j}^{(5)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left\{\left.\frac{dq_j^{(5)}}{dz}\right|_{z=\overline{z}}+\frac{\beta^3}{\kappa(\overline{z})}\left[1+\frac{\alpha}{\left(1-\alpha\right)p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right\} \quad \text{for } i\geq 1 \text{ and } j\in\{2,3\},$$

$$\left.\frac{dq_{3i+1}^{(6)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left\{\left.\frac{dq_1^{(6)}}{dz}\right|_{z=\overline{z}}-\frac{\beta^3}{\kappa(\overline{z})}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right\} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+j}^{(6)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_j^{(6)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1 \text{ and } j\in\{2,3\}.$$

Using $p_1 = p_A p_2 + (1-p_A)\, p_0$, we end up with:

$$\begin{aligned}
\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}} &= \frac{\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{\kappa(\overline{z}) + \beta^3 [\alpha A(\overline{z}) - \kappa(\overline{z})]} \left\{ \frac{\kappa(\overline{z})}{\alpha A(\overline{z})} \left[1 + p_0\beta^4 + p_A\beta(1+\beta) + (1-p_A)\, p_B\beta^2\right] \right. \\
&+ \frac{\kappa(\overline{z})}{(1-\alpha)\, p_0 A(\overline{z})} \left[\left(1+\beta^4+\beta^5\right) p_1 + p_A p_2 \beta(1+\beta)\left(1-\beta^3\right) + (1-p_A)\, p_B p_1 \beta^2 \left(1-\beta^3\right)\right] \\
&+ \frac{\beta^3\alpha}{(1-\beta)(1-\alpha)\, p_0} \left[p_1\left(1-\beta+\beta^3\right) + p_A p_2 \beta\left(1-\beta^2\right) + (1-p_A)\, p_B p_1 \beta^2 (1-\beta)\right] \\
&\left. + \frac{\beta^4 \kappa(\overline{z}) [\alpha A(\overline{z}) - \kappa(\overline{z})]}{2\alpha A(\overline{z})} \left[\frac{(1-\alpha)\beta^3 p_0 \left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2} \right] \right\}.
\end{aligned}$$

For the entrepreneurs:

$$\begin{aligned}
V_1^I(z) &= p_A\beta^3 \left\{ p_2 \textstyle\sum_{i=0}^{+\infty} \beta^i \left[(1-\alpha) A(z) - \dfrac{\kappa(z)}{q_{i+1}^{(1)}(z)}\right] \right. \\
&+ (1-p_2) \left[\textstyle\sum_{i=0}^{2} \beta^i \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(z)}{q_{i+1}^{(2)}(z)}\right] + \sum_{i=3}^{+\infty} \beta^i \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(\overline{z})}{q_{i+1}^{(2)}(z)}\right] \right] \Bigg\} \\
&+ (1-p_A)\beta^3 \left\{ p_B \left[p_1 \left[\textstyle\sum_{i\in\mathbb{N}\setminus\{1\}} \beta^i \left[(1-\alpha) A(z) - \dfrac{\kappa(z)}{q_{i+1}^{(3)}(z)}\right] + \beta \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(\overline{z})}{q_2^{(3)}(z)}\right] \right] \right. \right. \\
&+ (1-p_1) \left[\textstyle\sum_{i\in\{0,2\}} \beta^i \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(z)}{q_{i+1}^{(4)}(z)}\right] + \sum_{i\in\mathbb{N}\setminus\{0,2\}} \beta^i \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(\overline{z})}{q_{i+1}^{(4)}(z)}\right] \right] \Bigg] \\
&+ (1-p_B) \left[p_{-1} \left[\textstyle\sum_{i\in\mathbb{N}\setminus\{1,2\}} \beta^i \left[(1-\alpha) A(z) - \dfrac{\kappa(z)}{q_{i+1}^{(5)}(z)}\right] + \sum_{i=1}^{2} \beta^i \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(\overline{z})}{q_{i+1}^{(5)}(z)}\right] \right] \right. \\
&\left.\left. + (1-p_{-1}) \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(z)}{q_1^{(6)}(z)} + \sum_{i=1}^{+\infty} \beta^i \left[(1-\alpha) A(\overline{z}) - \dfrac{\kappa(\overline{z})}{q_{i+1}^{(6)}(z)}\right] \right] \right] \right\}.
\end{aligned}$$

Using (10) and $p_1 = p_A p_2 + (1-p_A)\, p_0$, we end up with:

$$\begin{aligned}
\left.\frac{dV_1^I}{dz}\right|_{z=\overline{z}} &= \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \left\{ \frac{-1}{1-\beta} \left[(1-\beta) + p_1\beta^3 + (1-p_A)\, p_B\beta^2(1-\beta) + p_A\beta\left(1-\beta^2\right)\right] \right. \\
&+ \frac{1}{p_0(1-\beta)} \left[p_1\left(1-\beta+\beta^3\right) + p_A p_2 \beta\left(1-\beta^2\right) + (1-p_A)\, p_B p_1 \beta^2 (1-\beta)\right] \\
&- \frac{\kappa(\overline{z})}{A(\overline{z})\,\alpha\beta^3} \left[1 + p_0\beta^4 + p_A\beta(1+\beta) + (1-p_A)\, p_B\beta^2\right] \\
&- \frac{\kappa(\overline{z})}{A(\overline{z})(1-\alpha)\, p_0\beta^3} \left[\left(1+\beta^4+\beta^5\right) p_1 + p_A p_2 \beta(1+\beta)\left(1-\beta^3\right) + (1-p_A)\, p_B p_1 \beta^2 \left(1-\beta^3\right)\right] \\
&\left. - \frac{\beta\kappa(\overline{z}) [\alpha A(\overline{z}) - \kappa(\overline{z})]}{2\alpha A(\overline{z})} \left[\frac{(1-\alpha)\beta^3 p_0 \left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2} \right] \right\}.
\end{aligned}$$

H Conditions imposed on the parameters

In the general case (subsections 4.1 and 4.3), the relevant parameters are α, β, $\kappa(\overline{z})$, $\kappa(z)$, $A(\overline{z})$, $A(z)$, p_0, λ, N and τ_t for $t \in \{1, ..., N\}$. The conditions imposed on them are $0 < \alpha < 1$, $0 < \beta < 1$, $\kappa(z) > \kappa(\overline{z}) > 0$, $A(z) > A(\overline{z}) > 0$, $0 < p_0 < 1$, $0 < \lambda < 1$, $N \in \mathbb{N}^*$, $\tau_t > 0$ for $t \in \{1, ..., N\}$, (1), (2), (3), (5), (6), (8), (9), (17), (18), (19), (20), as well as the necessary and sufficient condition to get: $\forall t > N$, $z_t = z$ if the new technology turns out to be good, and $z_t = \overline{z}$ otherwise, whatever $(z_t, c_t, q_t) \in \mathcal{F}_t \times \mathbb{R}^+ \times \mathbb{R}^{+*}$ for $t \in \{1, ..., N\}$.

In the simple case (subsection 4.2), the relevant parameters are α, β, $\kappa(\overline{z})$, $\frac{d\kappa}{dz}\big|_{z=\overline{z}}$, $A(\overline{z})$, $\frac{dA}{dz}\big|_{z=\overline{z}}$, p_0, λ, N and $\frac{d\tau_t}{dz}\big|_{z=\overline{z}}$ for $t \in \{1, ..., N\}$. The conditions imposed on them are those corresponding to the conditions imposed in the general case (listed above), to which should be added the following conditions: $N = 3$, $\left.\frac{d\tau_1}{dz}\right|_{z=\overline{z}} = \left.\frac{d\tau_3}{dz}\right|_{z=\overline{z}} = 0$, (10), (11) and (14). Together, these conditions are equivalent to the following ones[7]: $0 < \alpha < 1$, $0 < \beta < 1$, $\kappa(\overline{z}) > 0$, $\frac{d\kappa}{dz}\big|_{z=\overline{z}} > 0$, $A(\overline{z}) > 0$, $\frac{dA}{dz}\big|_{z=\overline{z}} > 0$, $0 < p_0 < 1$, $0 < \lambda < 1$, $N = 3$, $\left.\frac{d\tau_1}{dz}\right|_{z=\overline{z}} = \left.\frac{d\tau_3}{dz}\right|_{z=\overline{z}} = 0$, (1), (2), (3), (10), (11) and (14). It is easy to see that the set of parameters satisfying all these conditions is not empty. As a consequence, neither is the set of parameters satisfying all the conditions imposed in the general case.

[7] In particular, the conditions $p_0 < 1$ and (10) imply that the last condition in the general case is satisfied in the simple case. Indeed, on the one hand, $\forall (z_t, c_t, q_t) \in \mathcal{F}_t \times \mathbb{R}^+ \times \mathbb{R}^{+*}$ for $t \in \{1, ..., N\}$, we have: $\forall t > N$, q_t is close to β^3. On the other hand, given (10) and $p_0 < 1$, we have $\beta^3 > B(\overline{z}) > 0$. These two results together imply that $\forall t > N$, $q_t > B(\overline{z}) > 0$ and therefore (following a reasoning similar to that of the beginning of appendix F) $\forall t > N$, $I_t = 1$ if the new technology is good and $I_t = 0$ otherwise.